CHAPTER 10

Introduction to Power Electronics

Until the last few decades of the twentieth century, ac machines tended to be employed primarily as single-speed devices. Typically they were operated from fixed-frequency sources (in most cases this was the 50- or 60-Hz power grid). In the case of motors, control of motor speed requires a variable-frequency source, and such sources were not readily available. Thus, applications requiring variable speed were serviced by dc machines, which can provide highly flexible speed control, although at some cost since they are more complex, more expensive, and require more maintenance than their ac counterparts.

The availability of solid-state power switches changed this picture immensely. It is now possible to build power electronics capable of supplying the variable-voltage/current, variable-frequency drive required to achieve variable-speed performance from ac machines. Ac machines have now replaced dc machines in many traditional applications, and a wide range of new applications have been developed.

As is the case with electromechanics and electric machinery, power electronics is a discipline which can be mastered only through significant study. Many books have been written on this subject, a few of which are listed in the bibliography at the end of this chapter. It is clear that a single chapter in a book on electric machinery cannot begin to do justice to this topic. Thus our objectives here are limited. Our goal is to provide an overview of power electronics and to show how the basic building blocks can be assembled into drive systems for ac and dc machines. We will not focus much attention on the detailed characteristics of particular devices or on the many details required to design practical drive systems. In Chapter 11, we will build on the discussion of this chapter to examine the characteristics of some common drive systems.
10.1 POWER SWITCHES

Common to all power-electronic systems are switching devices. Ideally, these devices control current much like valves control the flow of fluids: turn them “ON,” and they present no resistances to the flow of current; turn them “OFF,” and no current flow is possible. Of course, practical switches are not ideal, and their specific characteristics significantly affect their applicability in any given situation. Fortunately, the essential performance of most power-electronic circuits can be understood assuming the switches to be ideal. This is the approach which we will adopt in this book. In this section we will briefly discuss some of the common switching devices and present simplified, idealized models for them.

10.1.1 Diodes

Diodes constitute the simplest of power switches. The general form of the $v$-$i$ characteristics of a diode is shown in Fig. 10.1.

The essential features of a diode are captured in the idealized $v$-$i$ characteristic of Fig. 10.2a. The symbol used to represent a diode is shown in Fig. 10.2b along with the reference directions for the current $i$ and voltage $v$. Based upon terminology developed when rectifier diodes were electron tubes, diode current flows into the anode and flows out of the cathode.

Figure 10.1 $v$-$i$ characteristic of a diode.

Figure 10.2 (a) $v$-$i$ characteristic of an ideal diode. (b) Diode symbol.
We can see that the ideal diode blocks current flow when the voltage is negative \((i = 0\) for \(v < 0\)) and passes positive current without voltage drop \((v = 0\) for \(i \geq 0\)). We will refer to the negative-voltage region as the diode’s OFF state and the positive-current region as the diode’s ON state. Comparison with the \(v-i\) characteristic shows that a practical diode varies from an ideal diode in that:

- There is a finite forward voltage drop, labeled \(V_F\) in Fig. 10.1, for positive current flow. For low-power devices, this voltage range is typically on the order of 0.6–0.7 V while for high-power devices it can exceed 3 V.
- Corresponding to this voltage drop is a power dissipation. Practical diodes have a maximum power dissipation (and a corresponding maximum current) which must not be exceeded.
- A practical diode is limited in the negative voltage it can withstand. Known as the reverse-breakdown voltage and labeled \(V_{RB}\) in Fig. 10.1, this is the maximum reverse voltage that can be applied to the diode before it starts to conduct reverse current.

The diode is the simplest power switch in that it cannot be controlled; it simply turns ON when positive current begins to flow and turns OFF when the current attempts to reverse. In spite of this simple behavior, it is used in a wide variety of applications, the most common of which is as a rectifier to convert ac to dc.

The basic performance of a diode can be illustrated by the simple example shown in Example 10.1.

**EXAMPLE 10.1**

Consider the half-wave rectifier circuit of Fig. 10.3a in which a resistor \(R\) is supplied by a voltage source \(v_s(t) = V_0 \sin \omega t\) through a diode. Assume the diode to be ideal. (a) Find the resistor voltage \(v_R(t)\) and current \(i_R(t)\). (b) Find the dc average resistor voltage \(V_{dc}\) and current \(I_{dc}\).

![Figure 10.3](image-url)
Solution

a. This is a nonlinear problem in that it is not possible to write an analytic expression for the $v$-$i$ characteristic of the ideal diode. However, it can readily be solved using the *method-of-assumed-states* in which, for any given value of the source voltage, the diode is alternately assumed to be ON (a short-circuit) or OFF (an open-circuit) and the current is found. One of the two solutions will violate the $v$-$i$ characteristic of the diode (i.e., there will be negative current flow through the short-circuit or positive voltage across the open-circuit) and must be discarded; the remaining solution will be the correct one.

Following the above procedure, we find that the solution is given by

$$v_R(t) = \begin{cases} v_s(t) = V_0 \sin \omega t & v_s(t) \geq 0 \\ 0 & v_s(t) < 0 \end{cases}$$

This voltage is plotted in Fig. 10.3b. The current is identical in form and is found simply as $i_R(t) = v_R(t)/R$. The terminology *half-wave rectification* is applied to this system because voltage is applied to the resistor during only the half cycle for which the supply voltage waveform is positive.

b. The dc or average value of the voltage waveform is equal to

$$V_{dc} = \frac{\omega}{\pi} \int_0^{\pi} V_0 \sin(\omega t) \, dt = \frac{V_0}{\pi}$$

and hence the dc current through the resistor is equal to

$$I_{dc} = \frac{V_0}{\pi R}$$

Practice Problem 10.1

Calculate the average voltage across the resistor of Fig. 10.3 if the sinusoidal voltage source of Example 10.1 is replaced by a source of the same frequency but which produces a square wave of zero average value and peak-peak amplitude $2V_0$.

Solution

$$V_{dc} = \frac{V_0}{2}$$

10.1.2 Silicon Controlled Rectifiers and TRIACs

The characteristics of a *silicon controlled rectifier*, or SCR, also referred to as a *thyristor*, are similar to those of a diode. However, in addition to an anode and a cathode, the SCR has a third terminal known as the *gate*. Figure 10.4 shows the form of the $v$-$i$ characteristics of a typical SCR.

As is the case with a diode, the SCR will turn ON only if the anode is positive with respect to the cathode. Unlike a diode, the SCR also requires a pulse of current $i_G$ into the gate to turn ON. Note however that once the SCR turns ON, the gate signal
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ON state
OFF state

$V_{RB}$

$V_F$

$V_{FB}$

Figure 10.4 $v$-$i$ characteristic of an SCR.

(a) (b)

Figure 10.5 (a) Idealized SCR $v$-$i$ characteristic.
(b) SCR symbol.

As can be seen from Fig. 10.4, the ON-state characteristic of an SCR is similar to that of a diode, with a forward voltage drop $V_F$ and a reverse-breakdown voltage $V_{RB}$. When the SCR is OFF, it does not conduct current over its normal operating range of positive voltage. However, it will conduct if this voltage exceeds a characteristic voltage, labeled $V_{FB}$ in the figure and known as the forward-breakdown voltage. As is the case for a diode, a practical SCR is limited in its current-carrying capability.

For our purposes, we will simplify these characteristics and assume the SCR to have the idealized characteristics of Fig. 10.5a. Our idealized SCR appears as an open-circuit when it is OFF and a short-circuit when it is ON. It also has a holding current of zero; i.e., it will remain ON until the current drops to zero and attempts to go negative. The symbol used to represent an SCR is shown in Fig. 10.5b.

Care must be taken in the design of gate-drive circuitry to insure that an SCR turns on properly; e.g., the gate pulse must inject enough charge to fully turn on the SCR, and so forth. Similarly, an additional circuit, typically referred to as a snubber circuit, may be required to protect an SCR from being turned on inadvertently, such as might occur if the rate of rise of the anode-to-cathode voltage is excessive. Although
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these details must be properly accounted for to achieve successful SCR performance in practical circuits, they are not essential for the present discussion.

The basic performance of an SCR can be understood from the following example.

**EXAMPLE 10.2**

Consider the half-wave rectifier circuit of Fig. 10.6 in which a resistor $R$ is supplied by a voltage source $v_s(t) = V_0 \sin \omega t$ through an SCR. Note that this is identical to the circuit of Example 10.1, with the exception that the diode has been replaced by an SCR.

Assume that a pulse of gate current is applied to the SCR at time $t_0$ ($0 \leq t_0 < \pi / \omega$) following each zero-crossing of the source voltage, as shown in Fig. 10.7a. It is common to describe this firing-delay time in terms of a firing-delay angle, $\alpha_0 \equiv \omega t_0$. Find the resistor voltage $v_R(t)$ as a function of $\alpha_0$. Assume the SCR to be ideal and that the gate pulses supply sufficient charge to properly turn ON the SCR.

**Solution**

The solution follows that of Example 10.1 with the exception that, independent of the polarity of the voltage across it, once the SCR turns OFF, it will remain OFF until both the SCR voltage becomes positive and a pulse of gate current is applied. Once a gate pulse has been applied, the method-of-assumed-states can be used to solve for the state of the SCR.

Following the above procedure, we find the solution is given by

$$v_R(t) = \begin{cases} 
0 & v_s(t) \geq 0 \text{ (prior to the gate pulse)} \\
v_s(t) = V_0 \sin \omega t & v_s(t) \geq 0 \text{ (following the gate pulse)} \\
0 & v_s(t) < 0 
\end{cases}$$

This voltage is plotted in Fig. 10.7b. Note that this system produces a half-wave rectified voltage similar to that of the diode system of Example 10.1. However, in this case, the dc value of the rectified voltage can be controlled by controlling the timing of the gate pulse. Specifically, it is given by

$$V_{dc} = \frac{V_0}{2\pi} (1 + \cos \alpha_0)$$

Note that when there is no delay in firing the SCR ($\alpha_0 = 0$), this system produces a dc voltage of $V_0/\pi$, equal to that of the diode rectifier system of Example 10.1. However, as the gate pulse of the SCR is delayed (i.e., by increasing $\alpha_0$), the dc voltage can be reduced. In fact, by delaying the gate pulse a full half cycle ($\alpha_0 = \pi$) the dc voltage can be reduced to

![Figure 10.6](image)
zero. This system is known as a phase-controlled rectifier because the dc output voltage can be varied by controlling the phase angle of the gate pulse relative to the zero crossing of the source voltage.

**Practice Problem 10.2**

Calculate the resistor average voltage as a function of the delay angle $\alpha_0$ if the sinusoidal source of Example 10.2 is replaced by a source of the same frequency, but which produces a square wave of zero average value and peak-peak amplitude $2V_0$.

**Solution**

$$V_{dc} = \frac{V_0}{2} \left( 1 - \frac{\alpha_0}{\pi} \right)$$

Example 10.2 shows that the SCR provides a significant advantage over the diode in systems where voltage control is desired. However, this advantage comes at the additional expense of the SCR as well as the circuitry required to produce the gate pulses used to fire the SCR.
Another phase-controlled device is the TRIAC, which behaves much like two back-to-back SCRs sharing a common gate. The idealized $v$-$i$ characteristic of a TRIAC is shown in Fig. 10.8a and its symbol in Fig. 10.8b. As with an SCR, TRIACs can be turned ON by the application of a pulse of current at their gate. Unlike an SCR, provided the current pulses inject sufficient charge, both positive and negative gate current pulses can be used to turn ON a TRIAC.

The use of a TRIAC is illustrated in the following example.

**EXAMPLE 10.3**

Consider the circuit of Fig. 10.9 in which the SCR of Example 10.2 has been replaced by a TRIAC.

Assume again that a short gate pulse is applied to the SCR at a delay angle $\alpha_0 (0 \leq \alpha_0 < \pi)$ following each zero-crossing of the source voltage, as shown in Fig. 10.10a. Find the resistor voltage $v_R(t)$ and its rms value $V_{R,\text{rms}}$ as a function of $\alpha_0$. Assume the TRIAC to be ideal and that the gate pulses inject sufficient charge to properly turn it ON.

**Solution**

The solution to this example is similar to that of Example 10.2 with the exception that the TRIAC, which will permit current to flow in both directions, turns on each half cycle of the source-voltage waveform.

$$v_R(t) = \begin{cases} 0 & \text{(prior to the gate pulse)} \\ V_s(t) = V_0 \sin \omega t & \text{(following the gate pulse)} \end{cases}$$

Unlike the rectification of Example 10.2, in this case the resistor voltage, shown in Fig. 10.10b, has no dc component. However, its rms value varies with $\alpha_0$:

$$V_{R,\text{rms}} = V_0 \sqrt{\frac{\omega}{\pi} \int_{0}^{\pi/2} \sin^3 (\omega t) \, dt}$$

$$= V_0 \sqrt{\frac{1}{2} - \frac{\alpha_0}{2\pi} + \frac{1}{4\pi} \sin (2\alpha_0)}$$
Figure 10.9 Circuit for Example 10.3.

Figure 10.10 (a) Gate pulses for Example 10.3. (b) Resistor voltage.

Notice that when \( \alpha_0 = 0 \), the TRIAC is ON all the time and it appears that the resistor is connected directly to the voltage source. In this case, \( V_{R \text{rms}} = V_0 / \sqrt{2} \) as expected. As \( \alpha_0 \) is increased to \( \pi \), the rms voltage decreases towards zero.

This simple type of controller can be applied to an electric light bulb (in which case it serves as *light dimmer*) as well as to a resistive heater. It is also used to vary the speed of a
universal motor and finds widespread application as a speed-controller in small ac hand tools, such as hand drills, as well as in small appliances, such as electric mixers, where continuous speed variation is desired.

### Practice Problem 10.3
Find the rms resistor voltage for the system of Example 10.3 if the sinusoidal source has been replaced by a source of the same frequency but which produces a square wave of zero average value and peak-peak amplitude $2V_0$.

**Solution**

$$V_{R\text{,rms}} = V_0 \sqrt{1 - \frac{\alpha_0}{\pi}}$$

### 10.1.3 Transistors

For power-electronic circuits where control of voltages and currents is required, power transistors have become a common choice for the controllable switch. Although a number of types are available, we will consider only two: the **metal-oxide-semiconductor field effect transistor (MOSFET)** and the **insulated-gate bipolar transistor (IGBT)**.

MOSFETs and IGBTs are both three-terminal devices. Figure 10.11a shows the symbols for n- and p-channel MOSFETs, while Fig. 10.11b shows the symbol for n- and p-channel IGBTs. In the case of the MOSFET, the three terminals are referred to as the **source**, **drain**, and **gate**, while in the case of the IGBT the corresponding

![Figure 10.11](a) Symbols for n- and p-channel MOSFETs. (b) Symbols for n- and p-channel IGBTs.)
terminals are the *emitter*, *collector*, and *gate*. For the MOSFET, the control signal is the gate-source voltage, $v_{GS}$. For the IGBT, it is the gate-emitter voltage, $v_{GE}$. In both the MOSFET and the IGBT, the gate electrode is capacitively coupled to the remainder of the device and appears as an open circuit at dc, drawing no current, and drawing only a small capacitive current under ac operation.

Figure 10.12a shows the $v$-$i$ characteristic of a typical n-channel MOSFET. The characteristic of the corresponding p-channel device looks the same, with the exception that the signs of the voltages and the currents are reversed. Thus, in an n-channel device, current flows from the drain to the source when the drain-source and

![Diagram](image-url)

**Figure 10.12** (a) Typical $v$-$i$ characteristic for an n-channel MOSFET. (b) Typical $v$-$i$ characteristic for an n-channel IGBT.
gate-source voltages are positive, while in a p-channel device current flows from the source to the drain when the drain-source and gate-source voltages are negative.

Note the following features of the MOSFET and IGBT characteristics:

■ In the case of the MOSFET, for positive drain-source voltage $v_{DS}$, no drain current will flow for values of $v_{GS}$ less than a threshold voltage which we will refer to by the symbol $V_T$. Once $v_{GS}$ exceeds $V_T$, the drain current $i_D$ increases as $v_{GS}$ is increased.

  In the case of the IGBT, for positive collector-emitter voltage $v_{CE}$, no collector current will flow for values of $v_{GE}$ less than a threshold voltage $V_T$. Once $v_{GE}$ exceeds $V_T$, the collector current $i_C$ increases as $v_{GE}$ is increased.

■ In the case of the MOSFET, no drain current flows for negative drain-source voltage.

■ In the case of the IGBT, no collector current flows for negative collector-emitter voltage.

■ Finally, the MOSFET will fail if the drain-source voltage exceeds its breakdown limits; in Fig. 10.12a, the forward breakdown voltage is indicated by the symbol $(V_{DS})_{FB}$ while the reverse breakdown voltage is indicated by the symbol $(V_{DS})_{RB}$.

  Similarly, the IGBT will fail if the collector-emitter voltage exceeds its breakdown values; in Fig. 10.12b, the forward breakdown voltage is indicated by the symbol $(V_{CE})_{FB}$ while the reverse breakdown voltage is indicated by the symbol $(V_{CE})_{RB}$.

■ Although not shown in the figure, a MOSFET will fail due to excessive gate-source voltage as well as excessive drain current which leads to excessive power dissipation in the device. Similarly an IGBT will fail due to excessive gate-emitter voltage and excessive collector current.

Note that for small values of $v_{CE}$, the IGBT voltage approaches a constant value, independent of the drain current. This saturation voltage, labeled $V_{CE,sat}$ in the figure, is on the order of a volt or less in small devices and a few volts in high-power devices. Correspondingly, in the MOSFET, for small values of $v_{DS}$, $v_{DS}$ is proportional to the drain current and the MOSFET behaves as a small resistance whose value decreases with increasing $v_{GS}$.

Fortunately, for our purposes, the details of these characteristics are not important. As we will see in the following example, with a sufficient large gate signal, the voltage drop across both the MOSFET and the IGBT can be made quite small. In this case, these devices can be modeled as a short circuit between the drain and the source in the case of the MOSFET and between the collector and the emitter in the case of the IGBT. Note, however, these “switches” when closed carry only unidirectional current, and hence we will model them as a switch in series with an ideal diode. This ideal-switch model is shown in Fig. 10.13a.

In many cases, these devices are commonly protected by reverse-biased protection diodes connected between the drain and the source (in the case of a MOSFET) or between the collector and emitter (in the case of an IGBT). These protection
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Figure 10.13  (a) Ideal-switch model for a MOSFET or an IGBT showing the series ideal diode which represents the unidirectional-current device characteristic. (b) Ideal-switch model for devices which include a reverse-biased protection diode. The symbols G, D, and S apply to the MOSFET while the symbols B, C, and E apply to the IGBT.

Devices are often included as integral components within the device package. If these protection diodes are included, there is actually no need to include the series diode, in which case the model can be reduced to that of Fig. 10.13b.

EXAMPLE 10.4

Consider the circuit of Fig. 10.14a. Here we see an IGBT which is to be used to control the current through the resistor $R$ as supplied from a dc source $V_0$. Assume that the IGBT characteristics are those of Fig. 10.12b and that $V_0$ is significantly greater than the saturation voltage. Show a graphical procedure that can be used to find $v_{CE}$ as a function of $v_{GE}$.

Figure 10.14  (a) Circuit for Example 10.4. (b) IGBT characteristic showing load line and operating point.
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Solution

Writing KVL for the circuit of Fig. 10.14a gives

\[ V_0 = i_C R + v_{CE} \]

Solving for \( i_C \) gives

\[ i_C = \frac{(V_0 - v_{CE})}{R} \]

Note that this linear relationship, referred to as the load line, represents a constraint imposed by the external circuit on the relationship between the IGBT terminal variables \( i_C \) and \( v_{CE} \). The corresponding constraint imposed by the IGBT itself is given by the \( v-i \) characteristic of Fig. 10.12b.

The operating point of the circuit is that point at which both these constraints are simultaneously satisfied. It can be found most easily by plotting the load line on the \( v-i \) relationship of the IGBT. This is done in Fig. 10.14b. The operating point is then found from the intersection of the load line with the \( v-i \) characteristic of the IGBT.

Consider the operating point labeled A in Fig. 10.14b. This is the operating point corresponding to values of \( v_{GE} \) less than or equal to the threshold voltage \( V_T \). Under these conditions, the IGBT is OFF; there is no collector current, and hence \( v_{CE} = V_0 \). As \( v_{GE} \) is increased past \( V_T \), collector current begins to flow, the operating point begins to climb up the load line, and \( v_{CE} \) decreases; the operating point labeled B is a typical example.

Note however that as \( v_{GE} \) is further increased, the operating point approaches that portion of the IGBT characteristic for which the curves crowd together (see the operating point labeled C in Fig. 10.14b). Once this point is reached, any further increase in \( v_{GE} \) will result in only a minimal decrease in \( v_{CE} \). Under this condition, the voltage across the IGBT is approximately equal to the saturation voltage \( (V_{CE})_{sat} \).

If the IGBT of this example were to be replaced by a MOSFET the result would be similar. As the gate-source voltage \( v_{GS} \) is increased, a point is reached where the voltage drop across approaches a small constant value. This can be seen by plotting the load line on the MOSFET characteristic of Fig. 10.12a.

The load line intersects the vertical axis at a collector current of \( i_C = V_0/R \). Note that the larger the resistance, the lower this intersection and hence the smaller the value of \( v_{GE} \) required to saturate the transistor. Thus, in systems where the transistor is to be used as a switch, it is necessary to insure that the device is capable of carrying the required current and that the gate-drive circuit is capable of supplying sufficient drive to the gate.

Example 10.4 shows that when a sufficiently large gate voltage is applied, the voltage drop across a power transistor can be reduced to a small value. Under these conditions, the IGBT will look like a constant voltage while the MOSFET will appear as a small resistance. In either case, the voltage drop will be small, and it is sufficient to approximate it as a closed switch (i.e., the transistor will be ON). When the gate drive is removed (i.e., reduced below \( V_T \)), the switch will open and the transistor will turn OFF.
10.2 RECTIFICATION: CONVERSION OF AC TO DC

The power input to many motor-drive systems comes from a constant-voltage, constant-frequency source (e.g., a 50- or 60-Hz power system), while the output must provide variable-voltage and/or variable-frequency power to the motor. Typically such systems convert power in two stages: the input ac is first rectified to dc, and the dc is then converted to the desired ac output waveform. We will thus begin with a discussion of rectifier circuits. We will then discuss inverters, which convert dc to ac, in Section 10.3.

10.2.1 Single-Phase, Full-Wave Diode Bridge

Example 10.1 illustrates a half-wave rectifier circuit. Such rectification is typically used only in small, low-cost, low-power applications. Full-wave rectifiers are much more common. Consider the full-wave rectifier circuit of Fig. 10.15a. Here the resistor $R$ is supplied from a voltage source $v_s(t) = V_0 \sin \omega t$ through four diodes connected in a full-wave bridge configuration.

If we assume the diodes to be ideal, we can use the method-of-assumed states to show that the allowable diode states are:

- Diodes D1 and D3 ON, diodes D2 and D4 OFF for $v_s(t) > 0$
- Diodes D2 and D4 ON, diodes D1 and D3 OFF for $v_s(t) < 0$

The resistor voltage, plotted in Fig. 10.15b, is then given by

$$v_R(t) = \begin{cases} v_s(t) = V_0 \sin \omega t & v_s(t) \geq 0 \\ -v_s(t) = -V_0 \sin \omega t & v_s(t) < 0 \end{cases}$$

(10.1)

Now notice that the resistor voltage is positive for both polarities of the source voltage, hence the terminology full-wave rectification. The dc or average value of this waveform can be seen to be twice that of the half-wave rectified waveform of Example 10.1.

$$V_{dc} = \left( \frac{2}{\pi} \right) V_0$$

(10.2)

Figure 10.15 (a) Full-wave bridge rectifier. (b) Resistor voltage.
The rectified waveforms of Figs. 10.3b and 10.15b are clearly not the sort of “dc” waveforms that are considered desirable for most applications. Rather, to be most useful, the rectified dc should be relatively constant and ripple free. Such a waveform can be achieved using a filter capacitor, as illustrated in Example 10.5.

**EXAMPLE 10.5**

As shown in Fig. 10.16, a filter capacitor has been added in parallel with the load resistor in the full-wave rectifier system of Fig. 10.15. For the purposes of this example, assume that \( v_s(t) = V_0 \sin \omega t \) with \( V_0 = \sqrt{2} (120) \text{ V} \), \( \omega = (2\pi)60 \approx 377 \text{ rad/sec} \) and that \( R = 10 \Omega \) and \( C = 10^4 \mu \text{F} \). Plot the resistor voltage, \( v_R(t) \), current, \( i_R(t) \), and the total bridge current, \( i_B(t) \).

![Figure 10.16](image)

**Figure 10.16** Full-wave bridge rectifier with capacitive filter for Example 10.5.

**Solution**

The addition of the filter capacitor will tend to maintain the resistor voltage \( v_R(t) \) as the source voltage drops. The diodes will remain ON as long as the bridge output current remains positive and will switch OFF when this current starts to reverse.

This example can be readily solved using MATLAB. Figure 10.17a shows the resistor voltage \( v_R(t) \) plotted along with the rectified source voltage. During the time that the bridge is ON, i.e., one pair of diodes is conducting, the resistor voltage is equal to the rectified source voltage. When the bridge is OFF, the resistor voltage decays exponentially.

Notice that because the capacitor is relatively large (the \( RC \) time constant is 100 msec as compared to the period of the rectified source voltage, which is slightly over 8.3 msec) the diodes conduct only for a short amount of time around the peak of the rectified-source-voltage waveform. This can be readily seen from the expanded plots of the resistor current and the bridge current in Fig. 10.17b. Although the resistor current remains continuous and relatively constant, varying between 15.8 and 17 A, the bridge output current consists essentially of a

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† MATLAB is a registered trademark of The MathWorks, Inc.
**10.2 Rectification: Conversion of AC to DC**

Figure 10.17 Example 10.5. (a) Resistor voltage and rectified source voltage (dashed). (b) Resistor current and total bridge current (dashed).
current pulse which flows for less than 0.9 msec near the peak of the rectified voltage waveform and has a peak value of 250 A. It should be pointed out that the peak current in a practical circuit will be smaller than 250 A, being limited by circuit impedances, diode drops, and so on.

Using MATLAB, it is possible to calculate the rms value of the resistor current to be 16.4 A while that of the bridge current is 51.8 A. We see therefore that the bridge diodes in such a system must be rated for rms currents significantly in excess of that of the load. The data sheets for power-supply diodes typically indicate their rms current ratings, specifically with these sorts of applications in mind. Such peaked supply currents are characteristic of rectifier circuits with capacitive loads and can significantly affect the voltage waveforms on ac power systems when they become a significant fraction of the overall system load.

The ripple voltage in the resistor voltage is defined as the difference between its maximum and minimum values. In this example, the maximum value is equal to the peak value of the source voltage, or 169.7 V. The minimum value can be found from the MATLAB solution to be 157.8 V. Thus the ripple voltage is 11.9 V. Clearly the ripple voltage can be decreased by increasing the value of the filter capacitor. Note however that this comes at the expense of increased cost as well as shorter current pulses and higher rms current through the rectifier diodes.

Here is the MATLAB script for Example 10.5.

```matlab
clear
%parameters
omega = 2*pi*60;
R = 10;
C = 0.01;
V0 = 120*sqrt(2);
tau = R*C;
Nmax = 800;
% diode = 1 when rectifier bridge is conducting
diode = 1;
%Here is the loop that does the work.
for n = 1:Nmax+1
    t(n) = (2.5*pi/omega)*(n-1)/Nmax; %time
    vs(n) = V0*cos(omega*t(n)); %source voltage
    vrect(n) = abs(vs(n)); %full-wave rectified source voltage
%Calculations if the rectifier bridge is ON
if diode == 1
    %If the bridge is ON, the resistor voltage is equal to the rectified
    %source voltage.
    vR(n) = vrect(n);
    %Check the total current out of rectifiers
    if (omega*t(n)) <= pi/2.
        iB(n) = vR(n)/R - V0*C*omega*sin(omega*t(n));
    elseif (omega*t(n)) <= 3.*pi/2.
        iB(n) = vR(n)/R - V0*C*omega*sin(omega*t(n));
    else
        iB(n) = vR(n)/R - V0*C*omega*sin(omega*t(n));
    end
end
```
\[ i_B(n) = \frac{v_R(n)}{R} + V_0C\omega \sin(\omega t(n)) \]  
\[ \text{else} \]  
\[ i_B(n) = \frac{v_R(n)}{R} - V_0C\omega \sin(\omega t(n)) \]  
\[ \text{end} \]  
% If the current tries to go negative, the diodes will switch OFF  
if \( i_B(n) < 0 \);  
\( \text{diode} = 0; \)  
\( \text{toff} = t(n); \)  
\( \text{Voff} = v_{\text{rect}}(n); \)  
\[ \text{end} \]  
\[ \text{else} \]  
% When the diodes are off, the resistor/capacitor voltage decays exponentially.  
\[ v_R(n) = V_{\text{off}}\exp(-t(n)-\text{toff})/\tau; \]  
\( i_B(n) = 0; \)  
if \( (v_{\text{rect}}(n) - v_R(n)) > 0 \);  
\( \text{diode} = 1; \)  
\[ \text{end} \]  
\[ \text{end} \]  
% Calculate the resistor current  
\( i_R = v_R/R; \)  
% Now plot \( v_R \) as a solid line and \( v_{\text{rect}} \) as a dashed line  
plot(1000\*t,v_R)  
xlabel('time [msec]')  
ylabel('Voltage [V]')  
axis ([0 22 0 180])  
hold  
plot(1000\*t,v_{\text{rect}},'--')  
hold  
fprintf('
Hit any key to continue\n')  
pause  
% Now plot \( i_R \) as a solid line and \( i_L \) as a dashed line  
plot(1000\*t,i_R)  
xlabel('time [msec]')  
ylabel('Source current [A]')  
axis ([0 22 -50 250])  
hold  
plot(1000\*t,i_B,'--')  
hold
CHAPTER 10  Introduction to Power Electronics

Practice Problem 10.4

Use MATLAB to calculate the ripple voltage and rms diode current for the system of Example 10.5 for (a) \( C = 5 \times 10^4 \, \mu F \) and (b) \( C = 5 \times 10^3 \, \mu F \). (Hint: Note that the rms current must be calculated over an integral number of cycles of the current waveform).

Solution

a. 2.64 V and 79.6 A rms  
b. 21.6 V and 42.8 A rms

In Example 10.5 we have seen that a capacitor can significantly decrease the ripple voltage across a resistive load. However, this comes at the cost of large bridge current pulses since the current must be delivered to the capacitor in the short time period during which the rectified source voltage is near its peak value.

Figure 10.18 shows the addition of an inductor \( L \) at the output of the bridge, in series with the filter capacitor and its load. If the impedance of the inductor is chosen to be large compared to that of the capacitor/load combination at the frequency of the rectified source voltage, very little of the ac component of the rectified source voltage will appear across the capacitor, and thus the resultant \( L\)-\( C \) filter will produce low voltage ripple while drawing a relatively constant current from the diode bridge.

We have seen how the addition of a filter capacitor across a dc load can significantly reduce the ripple voltage seen by the load. In fact, the addition of significant capacitance can “stiffen” the rectified voltage to the point that it appears as a constant-voltage dc source to a load. In an analogous fashion, an inductor in series with a load will reduce the current ripple out of a rectifier. Under these conditions, the rectified source will appear as a constant-current dc source to a load.

The combination of a rectifier and an inductor at the output to supply a constant dc current to a load is of significant importance in power-electronic applications. It can be used, for example, as the front end of a current-source inverter which can be used to supply ac current waveforms to a load. We will investigate the behavior of such rectifier systems in the next section.

Figure 10.18  Full-wave bridge rectifier with an \( L\)-\( C \) filter supplying a resistive load.
10.2.2 Single-Phase Rectifier with Inductive Load

In this section we will examine the performance of a single-phase rectifier driving an inductive load. This situation covers both the case where the inductor is included as part of the rectifier system as a filter to smooth out current pulses as well as the case where the load itself is primarily inductive.

Let us examine first the half-wave rectifier circuit of Fig. 10.19. Here, the load consists of an inductor $L$ in series with a resistor $R$. The source voltage is equal to $v_s(t) = V_0 \cos \omega t$.

Consider first the case where $L$ is small ($\omega L \ll R$). In this case, the load looks essentially resistive and the load current $i_L(t)$ will vary only slightly from the current for a purely resistive load as seen in Example 10.1. This current, obtained from a detailed analytical solution, is plotted in Fig. 10.20a, along with the current for a purely resistive load.

Note that the effect of the inductance is to decrease both the initial rate of rise of the current and the peak current. More significantly, the diode conduction angle increases; current flows for longer than the half-period that is the case for a purely resistive load. As can be seen in Fig. 10.20a, this effect increases as the inductance is increased; current flows for a greater fraction of the cycle, and the peak current as well as the current ripple is reduced.

Figure 10.20b, which shows the inductor voltage, illustrates an important point that applies to all situations in which an inductor is subjected to steady-state, periodic conditions: the time-averaged voltage across the inductor must equal zero. This can be readily seen from the basic $v$-$i$ relationship for an inductor

$$v = L \frac{di}{dt} \quad (10.3)$$

If we consider the operation of an inductor over a period of the excitation frequency and recognize that, under steady-state conditions, the change in the inductor current over that period must equal zero (i.e., it must have the same value at time $t$ at the beginning of the period as it does one period later at time $t + T$), then we can write

$$i(t + T) - i(T) = 0 = \frac{1}{L} \int_{t}^{t+T} v \, dt \quad (10.4)$$

![Figure 10.19 Half-wave rectifier with an inductive load.](image-url)
from which we can see that the net volt-seconds (and correspondingly the average voltage) across the inductor during a cycle must equal zero

\[ \int_{t}^{t+T} v \, dt = 0 \]  \hspace{1cm} (10.5)

For this half-wave rectifier, note that as the inductance increases both the ripple current and the dc current will decrease. In fact, for large inductance (\( \omega L \gg R \)) the dc load current will tend towards zero. This can be easily seen by the following
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argument:

As the inductance increases, the conduction angle of the diode will increase from 180° and approach 360° for large values of \( L \).

In the limit of a 360° conduction angle, the diode can be replaced by a continuous short circuit, in which case the circuit reduces to the ac voltage source connected directly across the series combination of the resistor and the inductor.

Under this situation, no dc current will flow since the source is purely ac. In addition, since the impedance \( Z = R + j\omega L \) becomes large with large \( L \), the ac (ripple) current will also tend to zero.

Figure 10.21a shows a simple modification which can be made to the half-wave rectifier circuit of Fig. 10.19. The free-wheeling diode D2 serves as an alternate path for inductor current.

To understand the behavior of this circuit, consider the condition when the source voltage is positive, and the rectifier diode D1 is ON. The equivalent circuit for this operating condition is shown in Fig. 10.21b. Note that under this condition, the voltage across diode D2 is equal to the negative of the source voltage and diode D2 is turned OFF.

This operating condition will remain in effect as long as the source voltage is positive. However, as soon as the source voltage begins to go negative, the voltage across diode D2 will begin to go positive and it will turn ON. Since the load is

Figure 10.21 (a) Half-wave rectifier with an inductive load and a free-wheeling diode. (b) Equivalent circuit when \( v_s(t) > 0 \) and diode D1 is conducting. (c) Equivalent circuit when \( v_s(t) < 0 \) and the free-wheeling diode D2 is conducting.
inductive, a positive load current will be flowing at this time, and that load current will immediately transfer to the short circuit corresponding to diode D2. At the same time, the current through diode D1 will immediately drop to zero, diode D1 will be reverse biased by the source voltage, and it will turn OFF. This operating condition is shown in Fig. 10.21c. Thus, the diodes in this circuit alternately switch ON and OFF each half cycle: D1 is ON when \( v_s(t) \) is positive, and D2 is ON when it is negative.

Based upon this discussion, we see that the voltage \( v_L(t) \) across the load (equal to the negative of the voltage across diode D2) is a half-wave rectified version of \( v_s(t) \) as seen in Fig. 10.22a. As shown in Example 10.1, the average of this voltage is \( V_{dc} = V_0/\pi \). Furthermore, the average of the steady-state voltage across the inductor must equal zero, and hence the average of the voltage \( v_L(t) \) will appear across the resistor. Thus the dc load current will equal \( V_0/(\pi R) \). This value is independent of the inductor value and hence does not approach zero as the inductance is increased.

Figure 10.22b shows the diode and load currents for a relatively small value of inductance \( \omega L < R \), and Fig. 10.22c shows these same currents for a large inductance \( \omega L \gg R \). In each case we see the load current, which must be continuous due to the presence of the inductor, instantaneously switching between the diodes depending on the polarity of the source voltage. We also see that during the time diode D1 is ON, the load current increases due to the application of the sinusoidal source voltage, while during the time diode D2 is ON, the load current simply decays with the \( L/R \) time constant of the load itself.

As expected, in each case the average current through the load is equal to \( V_0/(\pi R) \). In fact, the presence of a large inductor can be seen to reduce the ripple current to the point that the load current is essentially a dc current equal to this value.

Let us now consider the case where the half-wave bridge of Fig. 10.19 is replaced by a full-wave bridge as in Fig. 10.23a. In this circuit, the voltage applied to the load is the full-wave-rectified source voltage as shown in Fig. 10.15 and the average (dc) voltage applied to the load will equal \( 2V_0/\pi \). Here again, the presence of the inductor will tend to reduce the ac ripple. Figure 10.24, again obtained from a detailed analytical solution, shows the current through the resistor as the inductance is increased.

If we assume a large inductor \( \omega L \gg R \), the load current will be relatively ripple free and constant. It is therefore common practice to analyze the performance of this circuit by replacing the inductor by a dc current source \( I_{dc} \) as shown in Fig. 10.23b, where \( I_{dc} = 2V_0/(\pi R) \). This is a commonly used technique in the analysis of power-electronic circuits which greatly simplifies their analysis.

Under this assumption, we can easily show that the diode and source currents of this circuit are given by waveforms of Fig. 10.25. Figure 10.25a shows the current through one pair of diodes (e.g., diodes D1 and D3), and Fig. 10.25b shows the source current \( i_s(t) \). The essentially constant load current \( I_{dc} \) flows through each pair of diodes for one half cycle and appears as a square wave of amplitude \( I_{dc} \) at the source.

In a fashion similar to that which we saw in the half-wave rectifier circuit with the free-wheeling diode, here a pair of diodes (e.g., diodes D1 and D3) are carrying current when the source voltage reverses, turning ON the other pair of diodes and switching OFF the pair that were previously conducting. In this fashion, the load current remains continuous and simply switches between the diode pairs.
Figure 10.22 (a) Voltage applied to the load by the circuit of Fig. 10.21. (b) Load and diode currents for small $L$. (c) Load and diode currents for large $L$. 

\[ v_{L}(t) \]

\[ V_0 \]

\[ 0 \quad \pi \quad 2\pi \quad 3\pi \quad \omega t \]

\[ i_{D1} \]

\[ i_{D2} \]

\[ i_L(t) \]

\[ i_{D1} \]

\[ i_{D2} \]

\[ 0 \quad \pi \quad 2\pi \quad 3\pi \quad \omega t \]
10.2.3 Effects of Commutating Inductance

Our analysis and the current waveforms of Fig. 10.25 show that the current commutes instantaneously from one diode pair to the next. In practical circuits, due to the presence of source inductance, *commutation* of the current between the diode pairs does not occur instantaneously. The effect of source inductance, typically referred to as *commutating inductance*, will be examined by studying the circuit of Fig. 10.26 in which a source inductance $L_s$ has been added in series with the voltage source in the full-wave rectifier circuit of Fig. 10.23b. We have again assumed that the load time constant is large ($\omega L / R \gg 1$) and have replaced the inductor with a dc current source $I_{dc}$.

Figure 10.27a shows the situation which occurs when diodes D2 and D4 are ON and carrying current $I_{dc}$ and when $v_s < 0$. Commutation begins when $v_s$ reaches zero and begins to go positive, turning ON diodes D1 and D3. Note that because the current in the source inductance $L_s$ cannot change instantaneously, the circuit condition at
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This time is described by Fig. 10.27b: the current through $L_s$ is equal to $-I_{dc}$, the current through diodes D2 and D4 is equal to $I_{dc}$, while the current through diodes D1 and D3 is zero.

Under this condition with all four diodes ON, the source voltage $v_s(t)$ appears directly across the source inductance $L_s$. Noting that commutation starts at the time
when \( v_s(t) = 0 \), the current through \( L_s \) can be written as

\[
    i_s(t) = -I_{dc} + \frac{1}{L_s} \int_0^t v_s(t) \, dt
    = -I_{dc} + \left( \frac{V_0}{\omega L_s} \right) \left( 1 - \cos \omega t \right)
\]  

(10.6)

Noting that \( i_s = i_{D1} - i_{D4} \), that \( i_{D1} + i_{D2} = I_{dc} \) and, that by symmetry, \( i_{D4} = i_{D2} \), we can write that

\[
    i_{D2} = \frac{I_{dc} - i_s(t)}{2}
\]  

(10.7)

Diode D2 (and similarly diode D4) will turn OFF when \( i_{D2} \) reaches zero, which will occur when \( i_s(t) = I_{dc} \). In other words, commutation will be completed at time \( t_c \), when the current through \( L_s \) has completely reversed polarity and when all of the load current is flowing through diodes D1 and D3.
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Setting \( i_s(t_c) = I_{dc} \) and solving Eq. 10.6 gives an expression for the commutation interval \( t_c \) as a function of \( I_{dc} \)

\[
t_c = \frac{1}{\omega} \cos^{-1} \left[ 1 - \left( \frac{2I_{dc}L_s}{V_0} \right) \right]
\]  

(10.8)

Figure 10.28(a) shows the currents through diodes D1 and D2 as the current commutates between them. The finite commutation time \( t_c \) can clearly be seen. There is a second effect of commutation which can be clearly seen in Fig. 10.28b which shows the rectified load voltage \( v_L(t) \). Note that during the time of commutation, with all the diodes on, the rectified load voltage is zero. These intervals of zero voltage on the rectified voltage waveform are known as commutation notches.

Comparing the ideal full-wave rectified voltage of Fig. 10.15b to the waveform of Fig. 10.28b, we see that the effect of the commutation notches is to reduce the dc output of the rectifier. Specifically, the dc voltage in this case is given by

\[
V_{dc} = \left( \frac{\omega}{\pi} \right) \int_{t_c}^{\pi/2} V_0 \sin \omega t \, dt \\
= \frac{V_0}{\pi} (1 + \cos \omega t_c)
\]  

(10.9)

where \( t_c \) is the commutation interval as calculated by Eq. 10.8.
Finally, the dc load current can be calculated as function of $t_c$

$$I_{dc} = \frac{V_{dc}}{R} = \frac{V_0}{\pi R} (1 + \cos\omega t_c) \quad (10.10)$$

Substituting Eq. 10.8 into Eq. 10.10 gives a closed-form solution for $I_{dc}$

$$I_{dc} = \frac{2V_0}{\pi R + 2\omega L_s} \quad (10.11)$$

and hence

$$V_{dc} = I_{dc} R = \frac{2V_0}{\pi + \frac{2\omega L_s}{R}} \quad (10.12)$$

We have seen that commutating inductance (which is to a great extent unavoidable in practical circuits) gives rise to a finite commutation time and produces commutation notches in the rectified-voltage waveform which reduces the dc voltage applied to the load.

**EXAMPLE 10.6**

Consider a full-wave rectifier driving an inductive load as shown in Fig. 10.29. For a 60-Hz, 230-V rms source voltage, $R = 5.6 \, \Omega$ and large $L$ ($\omega L \gg R$), plot the dc current through the load $I_{dc}$ and the commutation time $t_c$ as the source inductance $L_s$ varies from 1 to 100 mH.

**Solution**

The solution can be obtained by substitution into Eqs. 10.8 and 10.11. This is easily done using MATLAB, and the plots of $I_{dc}$ and $t_c$ are shown in Figs. 10.30a and b respectively. Note that the maximum achievable dc current, corresponding to $L_s = 0$, is equal to $2V_0/(\pi R) = 37 \, A$. Thus, commutating inductances on the order of 1 mH can be seen to have little effect on the performance of the rectifier and can be ignored. On the other hand, a commutating inductance of 100 mH can be seen to reduce the dc current to approximately 7 A, significantly reducing the capability of the rectifier circuit.

![Figure 10.29](image-url) Full-wave bridge rectifier with source inductance for Example 10.6.
Figure 10.30 (a) Dc current $I_{dc}$ and (b) commutation time $t_c$ for Example 10.6.
Here is the MATLAB script for Example 10.6.

clc

clear

%parameters
omega = 2*pi*60;
R = 5.6;
V0 = 230*sqrt(2);

for n = 1:100
    Ls(n) = n*1e-3;
    Idc(n) = 2*V0/(pi*R + 2*omega*Ls(n));
    tc(n) = (1/omega)*acos(1-(2*Idc(n)*omega*Ls(n))/V0);
end

plot(Ls*1000,Idc)
xlabel('Commutating inductance Ls [mH]')
ylabel('Idc')

fprintf('
Hit any key to continue
')
pause

plot(Ls*1000,tc*1000)
xlabel('Commutating inductance Ls [mH]')
ylabel('tc [msec]')

Practice Problem 10.5

Calculate the commutating inductance and the corresponding commutation time for the circuit of Example 10.6 if the dc load current is observed to be 29.7 A.

Solution

\[ L_s = 5.7 \text{ mH} \text{ and } t_c = 2.4 \text{ msec} \]

10.2.4 Single-Phase, Full-Wave, Phase-Controlled Bridge

Figure 10.31 shows a full-wave bridge in which the diodes of Fig. 10.15 have been replaced by SCRs. We will assume that the load inductance \( L \) is sufficiently large that the load current is essentially constant at a dc value \( I_{dc} \). We will also ignore any effects of commutating inductance, although they clearly would play the same role in a phase-controlled rectifier system as they do in a diode-rectifier system.

Figure 10.32 shows the source voltage and the timing of the SCR gate pulses under a typical operating condition for this circuit. Here we see that the firing pulses are delayed by an angle \( \alpha_D \) after the zero-crossing of the source-voltage waveform, with the firing pulses for SCRs T1 and T3 occurring after the positive-going transition of \( v_S(t) \) and those for SCRs T2 and T4 occurring after the negative-going transition.
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Figure 10.31 Full-wave, phase-controlled SCR bridge.

Figure 10.32 Source voltage and firing pulses for the phase-controlled SCR bridge of Fig. 10.31.

Figure 10.33a shows the current through SCRs T1 and T3. Note that these SCRs do not turn ON until they receive firing pulses at angle $\alpha_d$ after they are forward biased following positive-going zero crossing of the source voltage. Furthermore, note that SCRs T2 and T4 do not turn ON following the next zero crossing of the source voltage. Hence, SCRs T1 and T3 remain ON, carrying current until SCRs T2 and T4 are turned ON by gate pulses. Rather, T2 and T4 turn ON only after they receive their respective gate pulses (for example, at angle $\pi + \alpha_d$ in Fig. 10.33). This is an example of forced commutation, in that one pair of SCRs does not naturally commutate OFF but rather must be forcibly commutated when the other pair is turned ON.

Figure 10.33b shows the resultant load voltage $v_L(t)$. We see that the load voltage now has a negative component, which will increase as the firing-delay angle $\alpha_d$ is increased. The dc value of this waveform is equal to

$$V_{dc} = \left( \frac{2V_0}{\pi} \right) \cos \alpha_d \quad (0 \leq \alpha_d \leq \pi) \quad (10.13)$$
Figure 10.33 Waveforms for the phase-controlled SCR bridge of Fig. 10.31. (a) Current through SCRs T1 and T3. (b) Load voltage. (c) Source voltage and current.
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from which the firing-delay angle corresponding to a given value of dc voltage can be seen to be

$$\alpha_d = \cos^{-1} \left( \frac{\pi V_{dc}}{2V_0} \right)$$ (10.14)

From Eq. 10.13 we see that the dc voltage applied to the load can vary from $2V_0/\pi$ to $-2V_0/\pi$. This is a rather surprising result in that it is hard to understand how a rectifier bridge can supply negative voltage. However, in this case, it is necessary to recognize that this result applies to an inductive load which maintains positive current flow through the SCRs in spite of the reversal of polarity of the source voltage. If the load were purely resistive, the current through the conducting SCRs would go to zero as the source-voltage reversed polarity, and they would simply turn OFF; no load current would flow until the next pair of SCRs is turned ON.

Figure 10.33c shows the source voltage and current waveforms for the phase-controlled SCR-bridge. We see that the square-wave source current is out of phase with the source voltage. Its fundamental-harmonic is given by

$$i_{s,1}(t) = \left( \frac{4}{\pi} \right) I_{dc} \sin(\omega t - \alpha_d)$$ (10.15)

and thus the real power supplied to the load is given by

$$P = V_{dc}I_{dc} = \frac{2}{\pi} V_0I_{dc} \cos \alpha_d$$ (10.16)

and the reactive power supplied is

$$Q = -\frac{2}{\pi} V_0I_{dc} \sin \alpha_d$$ (10.17)

Under steady-state operation at a load current $I_{dc}$, $V_{dc} = I_{dc}R$ and the steady-state firing-delay angle can be found from Eq. 10.14 to be $\alpha_{ss} = \cos^{-1} \left( \frac{\pi I_{dc}R}{2V_0} \right)$. Under this condition, the real power simply supplies the losses in the resistor and hence $P = I_{dc}^2 R$. It may seem strange to be supplying reactive power to a “dc” load. However, careful analysis will show that this reactive power supplies the energy associated with the small but finite ripple current through the inductor.

If the delay angle is suddenly reduced ($\alpha_d < \alpha_{ss}$), the dc voltage applied to the load will increase (see Eq. 10.13) as will the power supplied to the load (see Eq. 10.16). As a result, $I_{dc}$ will begin to increase and the increased power will increase the energy storage in the inductor. Similarly, if the delay time is suddenly increased ($t_d > t_{d0}$), $V_{dc}$ will decrease (it may even go negative) and the power into the load will decrease, corresponding to a decrease in $I_{dc}$ and a decrease in the energy storage in the inductor.

Note that if $\alpha_d > \pi/2$, $V_{dc}$ will be negative, a condition which will continue until $I_{dc}$ reaches zero at which time the SCR bridge will turn OFF. Under this condition, the real power $P$ will also be negative. Under this condition, power is being supplied from the load to the source and the system is said to be regenerating.
A small superconducting magnet has an inductance $L = 1.2$ H. Although the resistance of the magnet itself is essentially zero, the resistance of the external leads is 12.5 mΩ. Current is supplied to the magnet from a 60-Hz, 15-V peak, single-phase source through a phase-controlled SCR bridge as in Fig. 10.31.

a. The magnet is initially operating in the steady state at a dc current of 35 A. Calculate the dc voltage applied to the magnet, the power supplied to the magnet, and the delay angle $\alpha_d$ in msec. Plot the magnet voltage $v_L(t)$.

b. In order to quickly discharge the magnet, the delay angle is suddenly increased to $\alpha_d = 0.9\pi = 162^\circ$. Plot the corresponding magnet voltage. Calculate the time required to discharge the magnet and the maximum power regenerated to the source.

**Solution**

The example is most easily solved using MATLAB, which can easily produce the required plots.

a. Under this steady-state condition, $V_{dc} = I_{dc}R = 35 \times 0.0125 = 0.438$ V. The power supplied to the magnet is equal to $P = V_{dc}I_{dc} = 0.438 \times 35 = 15.3$ W, all of which is going into supplying losses in the lead resistance. The delay angle can be found from Eq. 10.14.

$$\alpha_d = \cos^{-1}\left(\frac{\pi RI_{dc}}{2V_0}\right) = \cos^{-1}\left(\frac{\pi \times 0.0125 \times 35}{2 \times 15}\right)$$

$$= 1.52 \text{ rad} = 87.4^\circ$$

A plot of $v_L(t)$ for this condition is given in Fig. 10.34a.

b. For a delay angle of $0.9\pi$, the dc load voltage will be

$$V_{dc} = \left(\frac{2V_0}{\pi}\right) \cos \alpha_d = \left(\frac{2 \times 15}{\pi}\right) \cos (0.9\pi) = -9.1 \text{ V}$$

A plot of $v_L(t)$ for this condition is given in Fig. 10.34b.

The magnet current $i_m$ can be calculated from the differential equation

$$V_{dc} = i_mR + L \frac{di_m}{dt}$$

subject to the initial condition that $i_m(0) = 35$ A. Thus

$$i_m(t) = \frac{V_{dc}}{R} + \left(\frac{i_m(0) - V_{dc}}{R}\right) e^{-\left(\frac{t}{\tau}\right)}$$

From this equation we find that the magnet current will reach zero at time $t = 4.5$ seconds, at which time the bridge will shut off. The power regenerated to the source will be given by $-V_{dc}i_m(t)$. It has a maximum value of $9.1 \times 35 = 318$ W at time $t = 0$. 
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Figure 10.34 Waveforms for Example 10.7. (a) Magnet-voltage for \( \alpha_d = 87.4^\circ \), \( V_{dc} = 0.438 \) V. (b) Magnet-voltage for \( \alpha_d = 162^\circ \), \( V_{dc} = -9.1 \) V.
Here is the MATLAB script for Example 10.7.

```matlab
clc
clear

% system parameters
R = 12.5e-3;
L = 1.2;
V0 = 15;
omega = 120*pi;

% part (a)
% dc current
Idc = 35;
% dc voltage
Vdc_a = R*Idc;
% Power
P = Vdc_a*Idc;

% Calculate the delay angle
alpha_da = acos(pi*R*Idc/(2*V0));

% Now calculate the load voltage
for n = 1:1300
    theta(n) = 2*pi*(n-1)/1000;
    t(n) = theta(n)/omega;
    vs(n) = V0*sin(theta(n));
    if theta(n) < alpha_da
        vL(n) = -vs(n);
    elseif (theta(n) < pi + alpha_da)
        vL(n) = vs(n);
    elseif theta(n) < 2*pi + alpha_da
        vL(n) = -vs(n);
    elseif theta(n) < 3*pi + alpha_da
        vL(n) = vs(n);
    elseif theta(n) < 4*pi + alpha_da
        vL(n) = -vs(n);
    else
        vL(n) = vs(n);
    end
end
plot(1000*t,vL)
xlabel('time [msec]')
ylabel('Load voltage [V]')
pause

% part (b)
```
% delay angle
alpha_db = 0.9*pi;
% Find the new dc voltage
Vdc_b = (2*V0/pi)*cos(alpha_db);
% Time constants
tau = L/R;
% Initial current
im0 = Idc;
% Calculate the time at which the current reaches zero
tzero = -tau*log((-Vdc_b/R)/(im0-Vdc_b/R));
% Now plot the load voltage
for n = 1:1300
    theta(n) = 2*pi*(n-1)/1000;
    t(n) = theta(n)/omega;
    vs(n) = V0*sin(theta(n));
    if theta(n) < alpha_db
        vL(n) = -vs(n);
    elseif (theta(n) < pi + alpha_db)
        vL(n) = vs(n);
    elseif theta(n) < 2*pi + alpha_db
        vL(n) = -vs(n);
    elseif theta(n) < 3*pi + alpha_db
        vL(n) = vs(n);
    elseif theta(n) < 4*pi + alpha_db
        vL(n) = -vs(n);
    else
        vL(n) = vs(n);
    end
end
plot(1000*t,vL)
xlabel('time [msec]')
ylabel('Load voltage [V]')
fprintf('part (a):

Vdc_a = %g [mV]',1000*Vdc_a)
fprintf('Power = %g [W]',P)
fprintf('alpha_a=%g [rad]=%g [degrees]',alpha_a,180*alpha_a/pi)
fprintf('part (b):

Vdc_b=%g [V]',Vdc_b)
fprintf('Current will reach zero at %g [sec]',tzero)
fprintf('n')
Practice Problem 10.6

The field winding of a small synchronous generator has a resistance of 0.3 Ω and an inductance of 250 mH. It is fed from a 24-V peak, single-phase 60-Hz source through a full-wave phase-controlled SCR bridge. (a) Calculate the dc voltage required to achieve a dc current of 18 A through the field winding and the corresponding firing-delay angle. (b) Calculate the field current corresponding to a delay angle of 45°.

Solution

a. 5.4 V, 69°
b. 36.0 A

10.2.5 Inductive Load with a Series DC Source

As we have seen in Chapter 9, dc motors can be modeled as dc voltage sources in series with an inductor and a resistor. Thus, it would be useful to briefly investigate the case of a dc voltage source in series with an inductive load.

Let us examine the full-wave, phase-controlled SCR rectifier system of Fig. 10.35. Here we have added a dc source $E_{dc}$ in series with the load. Again assuming that $\omega L \gg R$ so that the load current is essentially dc, we see that the load voltage $v_L(t)$ depends solely on the timing of the SCR gate pulses and hence is unchanged by the presence of the dc voltage source $E_{dc}$. Thus the dc value of $v_L(t)$ is given by Eq. 10.13 as before.

In the steady-state, the dc current through the load can be found from the net dc voltage across the resistor as

$$I_{dc} = \frac{V_{dc} - E_{dc}}{R} \quad (V_{dc} \geq E_{dc}) \quad (10.18)$$

where $V_{dc}$ is found from Eq. 10.13. Under transient conditions, it is the difference voltage, $V_{dc} - E_{dc}$, that drives a change in the dc current through the series $R$-$L$ combination, in a fashion similar to that illustrated in Example 10.7.

![Figure 10.35](image)

Figure 10.35 Full-wave, phase-controlled SCR bridge with an inductive load including a dc voltage source.
A small permanent-magnet dc motor is to be operated from a phase-controlled bridge. The 60-Hz ac waveform has an rms voltage of 35 volts. The dc motor has an armature resistance of 3.5 Ω and an armature inductance of 17.5 mH. It achieves a no-load speed of 8000 r/min at an armature voltage of 50 V.

Calculate the no-load speed in r/min of the motor as a function of the firing delay angle \( \alpha_d \).

**Solution**

In Section 7.7 we see that the equivalent circuit for a permanent-magnet dc motor consists of a dc source (proportional to motor speed) in series with an inductance and a resistance. Thus, the equivalent circuit of Fig. 10.35 applies directly to the situation of this problem.

From Eq. 7.26, the generated voltage from the dc motor (\( E_{dc} \) in Fig. 10.35) is proportional to the speed of the dc motor. Thus,

\[
E_{dc} = \frac{8000}{50} E_{dc} \text{ r/min}
\]

Under steady state operation, the dc voltage drop across the armature inductance will be zero. In addition, at no load, the armature current will be sufficiently small that the voltage drop across the armature resistance can be neglected. Thus, setting \( E_{dc} = V_{dc} \) and substituting the expression for \( V_{dc} \) from Eq. 10.13 give,

\[
E_{dc} = V_{dc} = \left( \frac{2V_0}{\pi} \right) \cos \alpha_d
\]

\[
= \left( \frac{2 \sqrt{2} \times 35}{\pi} \right) \cos \alpha_d = 31.5 \cos \alpha_d
\]

Note that because the bridge can only supply positive current to the dc motor (and hence, in the steady-state, the dc voltage must be positive), this expression is valid only for \( 0 \leq \alpha_d \leq \pi/2 \).

Finally, substituting the expression for the speed \( n \) in terms of \( E_{dc} \) gives

\[
n = 160 \times (31.5 \cos \alpha_d) = 5040 \cos \alpha_d \text{ r/min} \quad (0 \leq \alpha_d \leq \pi/2)
\]

**Practice Problem 10.7**

The dc-motor of Example 10.8 is observed to be operating at a speed of 3530 r/min and drawing a dc current of 1.75 ampere. Calculate the corresponding firing delay angle \( \alpha_d \).

**Solution**

\[
\alpha_d = 0.15\pi \text{ rad} = 27^\circ
\]
full-wave bridges apply directly to situations with three-phase bridges. As a result, we will discuss three-phase bridges only briefly.

Figure 10.36a shows a system in which a resistor $R$ is supplied from a three-phase source through a three-phase, six-pulse diode bridge. Figure 10.36b shows the three-phase line-to-line voltages (peak value $\sqrt{2} V_{l-l,\text{rms}}$ where $V_{l-l,\text{rms}}$ is the rms value of the line-to-line voltage) and the resistor voltage $v_R(t)$, found using the method-of-assumed-states and assuming that the diodes are ideal.

Note that $v_R$ has six pulses per cycle. Unlike the single-phase, full-wave bridge of Fig. 10.15a, the resistor voltage does not go to zero. Rather, the three-phase diode bridge produces an output voltage equal to the instantaneous maximum of the absolute value of the three line-to-line voltages. The dc average of this voltage (which can...
be obtained by integrating over $1/6$ of a cycle) is given by

$$V_{dc} = \frac{3\omega}{\pi} \int_{0}^{\frac{\pi}{3}} -v_{bc}(t) \, dt$$

$$= -\frac{3\omega}{\pi} \int_{0}^{\frac{\pi}{3}} \sqrt{2}V_{l-1,\text{rms}} \sin \left( \omega t - \frac{2\pi}{3} \right) \, dt$$

$$= \left( \frac{3\sqrt{2}}{\pi} \right) V_{l-1,\text{rms}}$$

(10.19)

where $V_{l-1,\text{rms}}$ is the rms value of the line-to-line voltage.

Table 10.1 shows the diode-switching sequence for the three-phase bridge of Fig. 10.36a corresponding to a single period of the three-phase voltage of waveforms of Fig. 10.36b. Note that only two diodes are on at any given time and that each diode is on for $1/3$ of a cycle ($120^\circ$).

Analogous to the single-phase, full-wave, phase-controlled SCR bridge of Figs. 10.31 and 10.35, Fig. 10.37 shows a three-phase, phase-controlled SCR bridge. Assuming continuous load current, corresponding for example to the condition $\omega L \gg R$, in which case the load current will be essentially a constant dc current $I_{dc}$, this bridge is capable of applying a negative voltage to the load and of regenerating power in a fashion directly analogous to the single-phase, full-wave, phase-controlled SCR bridge which we discussed in Section 10.2.4.

<table>
<thead>
<tr>
<th>$\alpha_d$</th>
<th>$0-\pi/3$</th>
<th>$\pi/3-2\pi/3$</th>
<th>$2\pi/3-\pi$</th>
<th>$\pi-4\pi/3$</th>
<th>$4\pi/3-5\pi/3$</th>
<th>$5\pi/3-2\pi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>D1</td>
<td>OFF</td>
<td>ON</td>
<td>ON</td>
<td>OFF</td>
<td>OFF</td>
<td>OFF</td>
</tr>
<tr>
<td>D2</td>
<td>OFF</td>
<td>OFF</td>
<td>OFF</td>
<td>ON</td>
<td>ON</td>
<td>OFF</td>
</tr>
<tr>
<td>D3</td>
<td>ON</td>
<td>OFF</td>
<td>OFF</td>
<td>OFF</td>
<td>OFF</td>
<td>ON</td>
</tr>
<tr>
<td>D4</td>
<td>OFF</td>
<td>OFF</td>
<td>OFF</td>
<td>OFF</td>
<td>ON</td>
<td>ON</td>
</tr>
<tr>
<td>D5</td>
<td>ON</td>
<td>ON</td>
<td>OFF</td>
<td>OFF</td>
<td>OFF</td>
<td>OFF</td>
</tr>
<tr>
<td>D6</td>
<td>OFF</td>
<td>OFF</td>
<td>ON</td>
<td>ON</td>
<td>OFF</td>
<td>OFF</td>
</tr>
</tbody>
</table>

**Figure 10.37** Three-phase, phase-controlled SCR bridge circuit with an inductive load.
It is a relatively straightforward matter to show that maximum output voltage of this bridge configuration will occur when the SCRs are turned ON at the times when the diodes in a diode bridge would naturally turn ON. These times can be found from Table 10.1. For example, we see that SCR T5 must be turned ON at angle $\alpha_d = 0$ (i.e., at the positive zero crossing of $v_{ab}(t)$). Similarly, SCR T1 must be turned ON at time $\alpha_d = \pi/3$, and so on.

Thus, one possible scheme for generating the SCR gate pulses is to use the positive-going zero crossing of $v_{ab}(t)$ as a reference from which to synchronize a pulse train running at six times the fundamental frequency (i.e., there will be six uniformly-spaced pulses in each cycle of the applied voltage). SCR T5 would be fired first, followed by SCRs T1, T6, T2, T4, and T3 in that order, each separated by $60^\circ$ in phase delay.

If the firing pulses are timed to begin immediately following the zero crossing of $v_{ab}(t)$ the load voltage waveform $v_{L}(t)$ will be that of Fig. 10.36b. If the firing pulses are delayed by an angle $\alpha_d$, then the load-voltage waveforms will appear as in Fig. 10.38a (for $\alpha_d = 0.1\pi$) and Fig. 10.38b (for $\alpha_d = 0.9\pi$).

![Figure 10.38](image)

**Figure 10.38** Typical load voltages for delayed firing of the SCRs in the three-phase, phase-controlled rectifier of Fig. 10.37; (a) $\alpha_d = 0.1\pi$, (b) $\alpha_d = 0.9\pi$. 
The dc average of the output voltage of the phase-controlled bridge can be found as

\[ V_{dc} = \frac{3\omega}{\pi} \int_{\alpha_d}^{\alpha_d + \pi/3} -v_{bc}(t) \, dt \]

\[ = -\frac{3\omega}{\pi} \int_{\alpha_d}^{\alpha_d + \pi/3} \sqrt{2} V_{l-1,\text{rms}} \sin \left(\omega t - \frac{2\pi}{3}\right) \, dt \]

\[ = \left(\frac{3\sqrt{2}}{\pi}\right) V_{l-1,\text{rms}} \cos \alpha_d \quad (0 \leq \alpha_d \leq \pi) \quad (10.20) \]

where \( V_{l-1,\text{rms}} \) is the rms value of the line-to-line voltage.

**EXAMPLE 10.9**

A large magnet with an inductance of 14.7 H and resistance of 68 \( \Omega \) is to be supplied from a 60-Hz, 460-V, three-phase source through a phase-controlled SCR bridge as in Fig. 10.37. Calculate (a) the maximum dc voltage \( V_{dc,\text{max}} \) and current \( I_{dc,\text{max}} \) which can be supplied from this source and (b) the delay angle \( \alpha_d \) required to achieve a magnet current of 2.5 A.

**Solution**

a. From Eq. 10.20, the maximum voltage (corresponding to \( \alpha_d = 0 \)) is equal to

\[ V_{dc,\text{max}} = \left(\frac{3\sqrt{2}}{\pi}\right) V_{l-1,\text{rms}} = \left(\frac{3\sqrt{2}}{\pi}\right) 460 = 621 \text{ V} \]

and \( I_{dc,\text{max}} = V_{dc,\text{max}} / R = 9.1 \text{ A} \)

b. The delay angle for a current of 2.5 A, corresponding to \( V_{dc} = I_d R = 170 \text{ V} \), can be found from inverting Eq. 10.20 as

\[ \alpha_d = \cos^{-1} \left[ \left(\frac{\pi}{3\sqrt{3}}\right) \left(\frac{V_{dc}}{V_{l-1,\text{rms}}}\right) \right] = 1.29 \text{ rad} = 74.1^\circ \]

**Practice Problem 10.8**

Repeat Example 10.9 for the case in which the 60-Hz source is replaced by a 50-Hz, 400-V, three-phase source.

**Solution**

a. \( V_{dc,\text{max}} = 540 \text{ V}, I_{dc,\text{max}} = 7.94 \text{ A} \)

b. \( \alpha_d = 1.25 \text{ rad} = 71.6^\circ \)

The derivations for three-phase bridges presented here have ignored issues such as the effect of commutating inductance, which we considered during our examination of single-phase rectifiers. Although the limited scope of our presentation does not
permit us to specifically discuss them here, the effects in three-phase rectifiers are similar to those for single-phase systems and must be considered in the design and analysis of practical three-phase rectifier systems.

10.3 INVERSION: CONVERSION OF DC TO AC

In Section 10.2 we discussed various rectifier configurations that can be used to convert ac to dc. In this section, we will discuss some circuit configurations, referred to as inverters, which can be used to convert dc to the variable-frequency, variable-voltage power required for many motor-drive applications. Many such configurations and techniques are available, and we will not attempt to discuss them all. Rather, consistent with the aims of this chapter, we will review some of the common inverter configurations and highlight their basic features and characteristics.

For the purposes of this discussion, we will assume the inverter is preceded by a “stiff” dc source. For example, in Section 10.2, we saw how an LC filter can be used to produce a relatively constant dc output voltage from a rectifier. Thus, as shown in Fig. 10.39a, for our study of inverters we will represent such rectifier systems by a constant dc voltage source $V_0$, known as the dc bus voltage at the inverter input. We will refer to such a system, with a constant-dc input voltage, as a voltage-source inverter.

Similarly, we saw that a “large” inductor in series with the rectifier output produces a relatively constant dc current, known as the dc link current. We will therefore

![Figure 10.39](image)

**Figure 10.39** Inverter-input representations. (a) Voltage source. (b) Current source.
represent such a rectifier system by a current source $I_0$ at the inverter input. We will refer to this type of inverter as a current-source inverter.

Note that, as we have seen in Section 10.2, the values of these dc sources can be varied by appropriate controls applied to the rectifier stage, such as the timing of gate pulses to SCRs in the rectifier bridge. Control of the magnitude of these sources in conjunction with controls applied to the inverter stage provides the flexibility required to produce a wide variety of output waveforms for various motor-drive applications.

## 10.3.1 Single-Phase, H-bridge Step-Waveform Inverters

Figure 10.40a shows a single-phase inverter configuration in which a load (consisting here of a series $RL$ combination) is fed from a dc voltage source $V_0$ through a set of four IGBTs in what is referred to as an H-bridge configuration. MOSFETs or other switching devices are equally applicable to this configuration. As we discussed in Section 10.1.3, the IGBTs in this system are used simply as switches. Because the IGBTs in this H-bridge include protection diodes, we can analyze the performance of this circuit by replacing the IGBTs by the ideal-switch model of Fig. 10.13b as shown in Fig. 10.40b.

For our analysis of this inverter, we will assume that the switching times of this inverter (i.e., the length of time the switches remain in a constant state) are much longer than the load time constant $L/R$. Hence, on the time scale of interest, the load current will simply be equal to $V_L/R$, with $V_L$ being determined by the state of the switches.

Let us begin our investigation of this inverter configuration assuming that switches S1 and S3 are ON and that $i_L$ is positive, as shown in Fig. 10.41a. Under this condition the load voltage is equal to $V_0$ and the load current is thus equal to $V_0/R$.

Let us next assume that switch S1 is turned OFF, while S3 remains ON. This will cause the load current, which cannot change instantaneously due to the presence of the inductor, to commutate from switch S1 to diode D2, as shown in Fig. 10.41b. Note that under this condition, the load voltage is zero and hence there will be zero load current. Note also that this same condition could have been reached by turning switch S3 OFF with S1 remaining ON.

![Figure 10.40](image-url)  
*Figure 10.40* Single-phase H-bridge inverter configuration. (a) Typical configuration using IGBTs. (b) Generic configuration using ideal switches.
At this point, it is possible to reverse the load voltage and current by turning ON switches S2 and S4, in which case $V_L = -V_0$ and $i_L = -V_0/R$. Finally, the current can be again brought to zero by turning OFF either switch S2 or switch S4. At this point, one cycle of an applied load-voltage waveform of the form of Fig. 10.42 has been completed.

A typical waveform produced by the switching sequence described above is shown in Fig. 10.42, with an ON time of $\Delta_1 T$ and OFF time of $\Delta_2 T$ ($\Delta_2 = 0.5 - \Delta_1$) for both the positive and negative portions of the cycle. Such a waveform consists of a fundamental ac component of frequency $f_0 = 1/T$, where $T$ is the period of the switching sequence, and components at odd-harmonics frequencies of that fundamental.

The waveform of Fig. 10.42 can be considered a simple one-step approximation to a sinusoidal waveform. Fourier analysis can be used to show that it has a fundamental component of peak amplitude

$$V_{L,1} = \left( \frac{4}{\pi} \right) V_0 \sin (\Delta_1 \pi) \quad (10.21)$$
and \( n \)'th-harmonic components \( (n = 3, 5, 7, \ldots) \) of peak amplitude

\[
V_{L,n} = \left( \frac{4}{n\pi} \right) V_0 \sin (n \Delta_1 \pi) \quad (10.22)
\]

Although this stepped waveform appears to be a rather crude approximation to a sinusoid, it clearly contains a significant fundamental component. In many applications it is perfectly adequate as the output voltage of a motor-drive. For example, three-such waveforms, separated by 120° in time phase, could be used to drive a three-phase motor. The fundamental components would combine to produce a rotating flux wave as discussed in Chapter 4. In some motor-drive systems, LC filters, consisting of shunt capacitors operating in conjunction with the motor phase inductances, are used to reduce the harmonic voltages applied to the motor phase windings.

In general, the higher-order harmonics, whose amplitudes vary inversely with their harmonic number, as seen from Eq. 10.22, will produce additional core loss in the stator as well as dissipation in the rotor. Provided that these additional losses are acceptable both from the point of view of motor heating as well as motor efficiency, a drive based upon this switching scheme will be quite adequate for many applications.

### EXAMPLE 10.10

A three-phase, H-bridge, voltage-source, step-waveform inverter will be built from three H-bridge inverter stages of the type shown in Fig. 10.40b. Each phase will be identical, with the exception that the switching pattern of each phase will be displaced by 1/3 of a period in time phase. This system will be used to drive a three-phase, four-pole motor with \( N_{ph} = 34 \) turns per phase and winding factor \( k_w = 0.94 \). The motor is Y-connected, and the inverters are each connected phase-to-neutral.

For a dc supply voltage of 125 V, a switching period \( T \) of 20 msec and with \( \Delta_1 = 0.44 \), calculate (a) the frequency and synchronous speed in rpm of the resultant air-gap flux wave and (b) the rms amplitude of the line-to-neutral voltage applied to the motor.

**Solution**

a. The frequency \( f_e \) of the fundamental component of the drive voltage will equal

\[
f_e = 1/T = 50 \text{ Hz}
\]

From Eq. 4.41 this will produce an air-gap flux wave which rotates at

\[
n_s = \left( \frac{120}{\text{poles}} \right) f_e = \left( \frac{120}{4} \right) 50 = 1500 \text{ r/min}
\]

b. The peak of the fundamental component of the applied line-to-neutral voltage can be found from Eq. 10.21.

\[
V_{a,\text{peak}} = \left( \frac{4}{\pi} \right) V_0 \sin (\Delta_1 \pi) = \left( \frac{4}{\pi} \right) 125 \sin (0.44 \pi) = 156 \text{ V}
\]

The resultant rms, line-to-line voltage is thus given by

\[
V_{l-l,\text{rms}} = \sqrt{\frac{3}{2}} V_{a,\text{peak}} = 191 \text{ V}
\]
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Figure 10.43 (a) H-bridge inverter configuration fed by a current source. (b) Typical stepped load-current waveform.

Practice Problem 10.9

For the three-phase inverter system of Example 10.10, (a) find the ON-time fraction $\Delta_1$ for which the 5'th-harmonic component of the applied voltage will be zero. (b) Calculate the corresponding peak amplitude of the fundamental component of the line-to-neutral voltage.

Solution

a. 0.2
b. 93 V

Figure 10.43a shows an H-bridge current-source inverter. This inverter configuration is analogous to the voltage-source configuration of Fig. 10.40. In fact, the discussion of the voltage-source inverter applies directly to the current-source configuration with the exception that the switches control the load current instead of the load voltage. Thus, again assuming that the load time constant ($L/R$) is much shorter than the switching time, a typical load current waveform would be similar to that shown in Fig. 10.43b.

Example 10.11

Determine a switching sequence for the inverter of Fig. 10.43a that will produce the stepped waveform of Fig. 10.43b.

Solution

Table 10.2 shows one such switching sequence, starting at time $t = 0$ at which point the load current $i_L(t) = -I_0$. Note that zero load current is achieved by turning on two switches so as to bypass the load and to directly short the current source. When this is done, the load current will quickly decay to zero, flowing through one of the switches and one of the reverse-polarity diodes. In general, one would not apply such a direct short across the voltage source in a voltage-source inverter.
Table 10.2 Switching sequence used to produce the load current waveform of Fig. 10.43b.

<table>
<thead>
<tr>
<th>$i_L(t)$</th>
<th>S1</th>
<th>S2</th>
<th>S3</th>
<th>S4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-I_0$</td>
<td>OFF</td>
<td>ON</td>
<td>OFF</td>
<td>ON</td>
</tr>
<tr>
<td>0</td>
<td>ON</td>
<td>ON</td>
<td>OFF</td>
<td>OFF</td>
</tr>
<tr>
<td>$I_0$</td>
<td>ON</td>
<td>OFF</td>
<td>ON</td>
<td>OFF</td>
</tr>
<tr>
<td>0</td>
<td>OFF</td>
<td>OFF</td>
<td>ON</td>
<td>ON</td>
</tr>
<tr>
<td>$-I_0$</td>
<td>OFF</td>
<td>ON</td>
<td>OFF</td>
<td>ON</td>
</tr>
</tbody>
</table>

because the resultant current would most likely greatly exceed the ratings of the switches. However, in the case of a current-source inverter, the switch current cannot exceed that of the current source, and hence the direct short can be (and, in fact, must be) maintained for as long as it is desired to maintain zero load current.

EXAMPLE 10.12

Consider the current-source inverter of Fig. 10.44a. Here the load consists of a sinusoidal voltage source $V_a \cos \omega t$. Assume the inverter switches are controlled such that the load current is a square-wave, also at frequency $f = \omega / (2\pi)$, as shown in Fig. 10.44b. Calculate the time-average power delivered to the load as a function of the delay angle $\alpha_d$ as defined in Fig. 10.44b.

**Solution**

Because the load voltage is sinusoidal, time-average power will only be produced by the fundamental component of the load current. By analogy to Eq. 10.21, with $I_0$ replacing $V_0$ and with $\Delta_1 = 0.5$, the amplitude of the fundamental-component of the load current is

$$I_{L,1} = \left(\frac{4}{\pi}\right) I_0$$

![Figure 10.44](image-url)  
(a) Current-source inverter for Example 10.12. (b) Load-current waveform.
and therefore the fundamental component of the load current is equal to
\[ i_{L,1}(t) = I_{L,1} \cos(\omega t - \alpha_d) = \left( \frac{4}{\pi} \right) I_0 \cos(\omega t - \alpha_d) \]

The complex amplitude of the load voltage is thus given by \( \hat{V}_L = V_a \) and that of the load current is \( \hat{I}_L = I_{L,1} e^{-j\alpha_d} \). Thus the time average power is equal to
\[ P = \frac{1}{2} \text{Re}[\hat{I}_L \hat{V}_L^*] = \left( \frac{2}{\pi} \right) V_a I_0 \cos \alpha_d \]

By varying the firing-delay angle \( \alpha_d \), the power transferred from the source to the load can be varied. In fact, as \( \alpha_d \) is varied over the range \( 0 \leq \alpha_d \leq \pi \), the power will vary over the range
\[ \left( \frac{2}{\pi} \right) V_a I_0 \geq P \geq -\left( \frac{2}{\pi} \right) V_a I_0 \]

Note that this inverter can regenerate; i.e., for \( \pi/2 < \alpha_d \leq \pi \), \( P < 0 \) and hence power will flow from the load back into the inverter.

**Practice Problem 10.10**

The inverter of Example 10.12 is operated with a fixed delay angle \( \alpha_d = 0 \) but with a variable ON-time fraction \( \Delta_1 \). Find an expression for the time-average power delivered to the load as a function of \( \Delta_1 \).

**Solution**

\[ P = \left( \frac{2}{\pi} \right) V_a I_0 \sin(\Delta_1\pi) \]

### 10.3.2 Pulse-Width-Modulated Voltage-Source Inverters

Let us again consider the H-bridge configuration of Fig. 10.40b, repeated again in Fig. 10.45. Again, an RL load is fed from a voltage source through the H-bridge. However, in this case, let us assume that the switching time of the inverter is much shorter than the load time constant \( L/R \).

Consider a typical operating condition as shown in Fig. 10.46. Under this condition, the switches are operated with a period \( T \) and a duty cycle \( D \) (0 \( \leq D \leq 1 \)). As can be seen from Fig. 10.46a, for a time \( DT \) switches S1 and S3 are ON, and the load voltage is \( V_0 \). This is followed by a time \( (1 - D)T \) during which switches S1 and S3 are OFF, and the current is transferred to diodes D2 and D4, setting the load voltage equal to \( -V_0 \). The duty cycle \( D \) is thus a fraction of the total period, in this case the fraction of the period during which the load voltage is \( V_0 \).
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Figure 10.45 Single-phase H-bridge inverter configuration.

Note that although switches S2 and S4 would normally be turned ON after switches S1 and S3 are turned OFF (but not before they are turned OFF, to avoid a direct short across the voltage source) because they are actually semiconductor devices, they will not carry any current unless the load current goes negative. Rather, the current will flow through the protection diodes D1 and D3. Alternatively, if the load current is negative, then the current will be controlled by the operation of switches S2 and S4 in conjunction with diodes D1 and D3. Under this condition, switches S1 and S3 will not carry current.

This type of control is referred to as pulse-width modulation, or PWM, because it is implemented by varying the width of the voltage pulses applied to the load. As can be seen from Fig. 10.46a, the average voltage applied to the load is equal to

\[(v_L)_{\text{avg}} = (2D - 1)V_0 \quad (10.23)\]

As we will now show, varying the duty cycle under PWM control can produce a continuously varying load current.

A typical load-current waveform is shown in Fig. 10.46b. In the steady state, the average current through the inductor will be constant and hence voltage across the inductor must equal zero. Thus, the average load current will equal the average
voltage divided by the resistance or

\[ (i_L)_{\text{avg}} = \frac{(v_L)_{\text{avg}}}{R} = \frac{[2D - 1]V_0}{R} \]  
(10.24)

Thus we see that by varying the duty cycle \( D \) over the range of 0 to 1, we can vary the load current over the range \(-V_0/R \leq (i_L)_{\text{avg}} \leq V_0/R\).

Because the current waveform is periodic, the maximum and minimum current, and hence the current ripple, can be easily calculated. Assigning time \( t = 0 \) to the time when switches S1 and S3 are first turned ON and the load current is minimum, the current during this time period will be given by

\[ i_L(t) = \frac{V_0}{R} + \left( (i_L)_{\text{min}} - \frac{V_0}{R} \right) e^{-\frac{t}{\tau}} \quad (0 \leq t \leq DT) \]  
(10.25)

where \( \tau = L/R \). The maximum load current \((i_L)_{\text{max}}\) is reached at time \( DT \)

\[ (i_L)_{\text{max}} = \frac{V_0}{R} + \left( (i_L)_{\text{max}} - \frac{V_0}{R} \right) e^{-\frac{DT}{\tau}} \]  
(10.26)

After switches S1 and S3 are turned OFF, the load voltage is \(-V_0\) and the current is given by

\[ i_L(t) = -\frac{V_0}{R} + \left( (i_L)_{\text{max}} + \frac{V_0}{R} \right) e^{-\frac{t}{\tau}} \quad (DT < t \leq T) \]  
(10.27)

Because the current is periodic with period \( T \), \( i_L(t) \) again will be equal to \((i_L)_{\text{min}}\) at time \( T \). Thus

\[ (i_L)_{\text{min}} = -\frac{V_0}{R} + \left( (i_L)_{\text{min}} + \frac{V_0}{R} \right) e^{-\frac{(T-DT)}{\tau}} \]  
(10.28)

Solving Eqs. 10.26 and 10.28 gives

\[ (i_L)_{\text{min}} = -\left( \frac{V_0}{R} \right) \frac{\left[ 1 - 2e^{-\frac{T(1-D)}{\tau}} + e^{-\frac{T}{\tau}} \right]}{\left( 1 - e^{-\frac{T}{\tau}} \right)} \]  
(10.29)

and

\[ (i_L)_{\text{max}} = \left( \frac{V_0}{R} \right) \frac{\left[ 1 - 2e^{-\frac{DT}{\tau}} + e^{-\frac{T}{\tau}} \right]}{\left( 1 - e^{-\frac{T}{\tau}} \right)} \]  
(10.30)

The current ripple \( \Delta i_L \) can be calculated as the difference between the maximum and minimum current.

\[ \Delta i_L = (i_L)_{\text{max}} - (i_L)_{\text{min}} \]  
(10.31)

In the limit that \( T \ll \tau \), this can be written as

\[ \Delta i_L \approx \left( \frac{2V_0}{R} \right) \left( \frac{T}{\tau} \right) D(1 - D) \]  
(10.32)
EXAMPLE 10.13

A PWM inverter such as that of Fig. 10.45 is operating from a dc voltage of 48 V and driving a load with \( L = 320 \text{ mH} \) and \( R = 3.7 \text{ } \Omega \). For a switching frequency of 1 kHz, calculate the average load current, the minimum and maximum current, and the current ripple for a duty cycle \( D = 0.8 \).

**Solution**

From Eq. 10.24, the average load current will equal

\[
(i_L)_{avg} = \frac{[2D - 1]V_0}{R} = \frac{0.6 \times 48}{3.7} = 7.78 \text{ A}
\]

For a frequency of 1 kHz, the period \( T = 1 \text{ msec} \). The time constant \( \tau = L/R = 86.5 \text{ msec} \). \((i_L)_{\text{min}}\) and \((i_L)_{\text{max}}\) can then be found from Eqs. 10.29 and 10.30 to be \((i_L)_{\text{min}} = 7.76 \text{ A}\) and \((i_L)_{\text{max}} = 7.81 \text{ A}\). Thus the current ripple, calculated from Eq. 10.31, is 0.05 A, which is equal to 0.6 percent of the average load current. Alternatively, using the fact that \( T/\tau = 0.012 \ll 1 \), the current ripple could have been calculated directly from Eq. 10.32

\[
\Delta i_L = \left( \frac{2V_0}{R} \right) \left( \frac{T}{\tau} \right) D(1 - D) = \left( \frac{2 \times 48}{3.7} \right) \left( \frac{1}{86.5} \right) \times 0.8 \times 0.2 = 0.05 \text{ A}
\]

**Practice Problem 10.11**

Calculate (a) the average current and (b) the current ripple for the PWM inverter of Example 10.13 if the switching frequency is reduced to 250 Hz.

**Solution**

a. 7.78 A (unchanged from Example 10.13)

b. 0.19 A

Now let us consider the situation for which the duty cycle is varied with time, i.e., \( D = D(t) \). If \( D(t) \) varies slowly in comparison to the period \( T \) of the switching frequency, from Eq. 10.23, the average load voltage will be equal to

\[
(u_L)_{avg} = [2D(t) - 1]V_0 \quad (10.33)
\]

and the average load current will be

\[
(i_L)_{avg} = \frac{[2D(t) - 1]V_0}{R} \quad (10.34)
\]

Figure 10.47a illustrates a method for producing the variable duty cycle for this system. Here we see a saw-tooth waveform which varies between −1 and 1. Also shown is a reference waveform \( W_{ref}(t) \) which is constrained to lie within the range −1 and 1. The switches will be controlled in pairs. During the time that \( W_{ref}(t) \) is greater than the saw-tooth waveform, switches S1 and S3 will be ON and the load voltage will be \( V_0 \). Similarly, when \( W_{ref}(t) \) is less than the saw-tooth waveform, switches S2
Figure 10.47 (a) Method for producing a variable duty cycle from a reference waveform $W_{\text{ref}}(t)$. (b) Load voltage and average load voltage for $W_{\text{ref}}(t) = 0.9 \sin \omega t$.

and S4 will be ON and the load voltage will be $-V_0$. Thus,

$$D(t) = \frac{(1 + W_{\text{ref}}(t))}{2}$$

and thus

$$(v_L)_{\text{avg}} = \left[2 \left( \frac{1 + W_{\text{ref}}(t)}{2} \right) - 1 \right] V_0 = W_{\text{ref}}(t) V_0$$

Figure 10.47b shows the load voltage $v_L(t)$ for a sinusoidal reference waveform $W_{\text{ref}}(t) = 0.9 \sin \omega t$. The average voltage across the load, $(v_L)_{\text{avg}} = W_{\text{ref}}(t) V_0$, is also shown.
Inversion: Conversion of DC to AC

Figure 10.48 Three-phase inverter configurations. (a) Voltage-source. (b) Current-source.

Note that the H-bridge inverter configuration of Fig. 10.43 can be used to produce a PWM current-source inverter. In a fashion directly analogous to the derivation of Eq. 10.36, one can show that such an inverter would produce an average current of the form

\[(i_L)_{avg} = W_{ref}(t) I_0\]  

(10.37)

where \(I_0\) is the magnitude of the dc link current feeding the H-bridge. Note, however, that the sudden current swings between \(I_0\) and \(-I_0\) associated with such an inverter will produce large voltages should the load have any inductive component. As a result, practical inverters of this type require large capacitive filters to absorb the harmonic components of the PWM current and to protect the load against damage due to voltage-induced insulation failure.

### 10.3.3 Three-Phase Inverters

Although the single-phase motor drives of Section 10.3.2 demonstrate the important characteristics of inverters, most variable-frequency drives are three phase. Figures 10.48a and 10.48b show the basic configuration of three-phase motor inverters (voltage- and current-source respectively). Here we have shown the switches as ideal switches, recognizing that in a practical implementation, bidirectional capability will be achieved by a combination of a semiconductor switching device, such as an IGBT and a MOSFET, and a reverse-polarity diode.

These configurations can be used to produce both stepped waveforms (either voltage-source or current-source) as well as pulse-width-modulated waveforms. This will be illustrated in the following example.

**EXAMPLE 10.14**

The three-phase current-source inverter configuration of Fig. 10.48b is to be used to produce a three-phase stepped current waveform of the form shown in Fig 10.49. (a) Determine the switch sequence over the period \(0 \leq t \leq T\) and (b) calculate the fundamental, third, fifth, and seventh harmonics of the phase-\(a\) current waveform.
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Figure 10.49 Three-phase stepped current waveform for Example 10.14.

Solution

a. By observing that switch S1 is ON when the phase-α current is positive, switch S4 is ON when it is negative, and so forth, the following table of switching operations can be produced.

<table>
<thead>
<tr>
<th></th>
<th>0–(T/6)</th>
<th>(T/6)–(T/3)</th>
<th>(T/3)–(T/2)</th>
<th>(T/2)–(2T/3)</th>
<th>(2T/3)–(5T/6)</th>
<th>(5T/6)–T</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>ON</td>
<td>OFF</td>
<td>OFF</td>
<td>OFF</td>
<td>OFF</td>
<td>ON</td>
</tr>
<tr>
<td>S2</td>
<td>OFF</td>
<td>ON</td>
<td>ON</td>
<td>OFF</td>
<td>OFF</td>
<td>OFF</td>
</tr>
<tr>
<td>S3</td>
<td>OFF</td>
<td>OFF</td>
<td>ON</td>
<td>ON</td>
<td>OFF</td>
<td>OFF</td>
</tr>
<tr>
<td>S4</td>
<td>OFF</td>
<td>OFF</td>
<td>ON</td>
<td>ON</td>
<td>OFF</td>
<td>OFF</td>
</tr>
<tr>
<td>S5</td>
<td>OFF</td>
<td>OFF</td>
<td>OFF</td>
<td>ON</td>
<td>ON</td>
<td>ON</td>
</tr>
<tr>
<td>S6</td>
<td>ON</td>
<td>ON</td>
<td>OFF</td>
<td>OFF</td>
<td>OFF</td>
<td>OFF</td>
</tr>
</tbody>
</table>

b. The amplitudes of the harmonic components of the phase current can be determined from Eqs. 10.21 and 10.22 by setting Δ1 = 1/3. Thus,

\[ I_{a1} = \left( \frac{2\sqrt{3}}{\pi} \right) I_0 \quad I_{a3} = 0 \]

\[ I_{a5} = -\left( \frac{2\sqrt{3}}{5\pi} \right) I_0 \quad I_{a7} = \left( \frac{2\sqrt{3}}{7\pi} \right) I_0 \]

10.4 SUMMARY

The goal of this chapter is relatively modest. Our focus has been to introduce some basic principles of power electronics and to illustrate how they can be applied to the design of various power-conditioning circuits that are commonly found in motor drives. Although the discussion in this chapter is neither complete nor extensive, it
10.4 Summary

is intended to provide the background required to support the various discussions of motor control which are presented in this book.

We began with a brief overview of a few of the available solid-state switching devices: diodes, SCRs, IGBTs and MOSFETs, and so on. We showed that, for the purposes of a preliminary analysis, it is quite sufficient to represent these devices as ideal switches. To emphasize the fact that they typically can pass only unidirectional current, we included ideal diodes in series with these switches. The simplest of these devices is the diode, which has only two terminals and is turned ON and OFF simply by the conditions of the external circuit. The remainder have a third terminal which can be used to turn the device ON and, in the case of transistors such as MOSFETS and IGBTs, OFF again.

A typical variable-frequency, variable-voltage motor-drive system can be considered to consist of three sections. The input section rectifies the power-frequency, fixed-voltage ac input and produces a dc voltage or current. The middle section filters the rectifier output, producing a relatively constant dc current or voltage, depending upon the type of drive under consideration. The output inverter section converts the dc to variable-frequency, variable-voltage ac voltages or currents which can be applied to the terminals of a motor.

The simplest inverters we investigated produce stepped voltage or current waveforms whose amplitude is equal to that of the dc source and whose frequency can be controlled by the timing of the inverter switches. To produce a variable-amplitude output waveform, it is necessary to apply additional control to the rectifier stage to vary the amplitude of the dc bus voltage or link current supplied to the inverter.

We also discussed pulse-width-modulated voltage-source inverters. In this type of inverter, the voltage to the load is switched between $V_0$ and $-V_0$ such that the average load voltage is determined by the duty cycle of the switching waveform. Loads whose time constant is long compared to the switching time of the inverter will act as filters, and the load current will then be determined by the average load voltage. Pulse-width modulated current-source inverters were also discussed briefly.

The reader should approach the presentation here with great caution. It is important to recognize that a complete treatment of power electronics and motor drives is typically the topic of a multiple-course sequence of study. Although the basic principles discussed here apply to a wide range of motor drives, there are many details which must be included in the design of practical motor drives. Drive circuitry to turn ON the “switches” (gate drives for SCRs, MOSFETs, IGBTs, etc.) must be carefully designed to provide sufficient drive to fully turn on the devices and to provide the proper switching sequences. The typical inverter includes a controller and a protection system which is quite elaborate. Typically, the design of a specific drive is dominated by the current and voltage ratings of available switches devices. This is especially true in the case of high-power drive systems in which switches must be connected in series and/or parallel to achieve the desired power rating. The reader is referred to references in the bibliography for a much more complete discussion of power electronics and inverter systems than has been presented here.

Motor drives based upon the configurations discussed here can be used to control motor speed and motor torque. In the case of ac machines, the application of
power-electronic based motor drives has resulted in performance that was previously available only with dc machines and has led to widespread use of these machines in most applications.

10.5 BIBLIOGRAPHY

This chapter is intended to serve as an introduction to the discipline of power electronics. For readers who wish to study this topic in more depth, this bibliography lists a few of the many textbooks which have been written on this subject.


10.6 PROBLEMS

10.1 Consider the half-wave rectifier circuit of Fig. 10.3a. The circuit is driven by a triangular voltage source \( v_s(t) \) of amplitude \( V_0 = 9 \) V as shown in Fig. 10.50. Assuming the diode to be ideal and for a resistor \( R = 1.5 \) k\( \Omega \):

a. Plot the resistor voltage \( v_R(t) \).

b. Calculate the rms value of the resistor voltage.

c. Calculate the time-averaged power dissipation in the resistor.

10.2 Repeat Problem 10.1 assuming the diode to have a fixed 0.6 V voltage drop when it is ON but to be otherwise ideal. In addition, calculate the time-averaged power dissipation in the diode.

10.3 Consider the half-wave SCR rectifier circuit of Fig. 10.6 supplied from the triangular voltage source of Fig. 10.50. Assuming the SCR to be ideal, calculate the rms resistor voltage as a function of the firing-delay time \( t_d \) (0 \( \leq t_d \leq T/2 \)).

10.4 Consider the rectifier system of Example 10.5. Write a MATLAB script to plot the ripple voltage as a function of filter capacitance as the filter
10.5 Consider the full-wave rectifier system of Fig. 10.16 with $R = 500 \, \Omega$ and $C = 200 \, \mu F$. Assume each diode to have a constant voltage drop of 0.7 V when it is ON but to be otherwise ideal. For a 220 V rms, 50 Hz sinusoidal source, write a MATLAB script to calculate
   a. the peak voltage across the load resistor.
   b. the magnitude of the ripple voltage.
   c. the time-averaged power supplied to the load resistor.
   d. the time-averaged power dissipation in the diode bridge.

10.6 Consider the half-wave rectifier system of Fig. 10.51. The voltage source is $v_s(t) = V_0 \sin \omega t$ where $V_0 = 15 \, \text{V}$, and the frequency is 100 Hz. For $L = 1 \, \text{mH}$ and $R = 1 \, \Omega$, plot the inductor current $i_L(t)$ for the first 1-1/2 cycles of the applied waveform assuming the switch closes at time $t = 0$.

10.7 Repeat Problem 10.6, using MATLAB to plot the inductor current for the first 10 cycles following the switch closing at time $t = 0$. (Hint: This problem can be easily solved, using simple Euler integration to solve for the current.)
10.8 Consider the half-wave rectifier system of Fig. 10.52 as \( L \) becomes sufficiently large such that \( \omega (L/R) \gg 1 \), where \( \omega \) is the supply frequency. In this case, the inductor current will be essentially constant. For \( R = 5 \, \Omega \) and \( v_s(t) = V_0 \sin \omega t \) where \( V_0 = 45 \, \text{V} \) and \( \omega = 100\pi \, \text{rad/sec} \). Assume the diodes to be ideal.

a. Calculate the average (dc) value \( V_{dc} \) of the voltage \( v_s'(t) \) across the series resistor/inductor combination.

b. Using the fact that, in the steady state, there will be zero average voltage across the inductor, calculate the dc inductor current \( I_{dc} \).

c. Plot the instantaneous inductor voltage \( v(t) \) over one cycle of the supply voltage.

d. Plot the instantaneous source current \( i_s(t) \).

10.9 Consider the half-wave, phase-controlled rectifier system of Fig. 10.53. This is essentially the same circuit as that of Problem 10.8 with the exception that diode D1 of Fig. 10.52 has been replaced by an SCR, which you can consider to be ideal. Let \( R = 5 \, \Omega \) and \( v_s(t) = V_0 \sin \omega t \), where \( V_0 = 45 \, \text{V} \) and \( \omega = 100\pi \, \text{rad/sec} \). Assume that the inductor \( L \) is sufficiently large such that \( \omega (L/R) \gg 1 \) and that the SCR is triggered ON at time \( t_d \) (\( 0 \leq t_d \leq \pi/\omega \)).

a. Find an expression for the average (dc) value \( V_{dc} \) of the voltage \( v_s'(t) \) across the series resistor/inductor combination as a function of the delay time \( t_d \).
problems

10.6 Problems

Figure 10.54 Full-wave, phase-controlled rectifier system for Problem 10.10.

b. Using the fact that, in the steady state, there will be zero average voltage across the inductor, find an expression for the dc inductor current $I_{dc}$, again as a function of the delay time $t_d$.

c. Plot $I_{dc}$ as a function of $t_d$ for $(0 \leq t_d \leq \pi/\omega)$.

10.10 The half-wave, phase-controlled rectifier system of Problem 10.9 and Fig. 10.53 is to be replaced by the full-wave, phase-controlled system of Fig. 10.54. SCR T1 will be triggered ON at time $t_d$ ($0 \leq t_d \leq \pi/\omega$), and SCR T4 will be triggered exactly one half cycle later.

a. Find an expression for the average (dc) value $V_{dc}$ of the voltage $v'_s(t)$ across the series resistor/inductor combination as a function of the delay time $t_d$.

b. Using the fact that, in the steady state, there will be zero average voltage across the inductor, find an expression for the dc inductor current $I_{dc}$, again as a function of the delay time $t_d$.

c. Plot $I_{dc}$ as a function of $t_d$ for $(0 \leq t_d \leq \pi/\omega)$.

d. Plot the source current $i_s(t)$ for one cycle of the source voltage for $t_d = 3$ msec.

10.11 The full-wave, phase-controlled rectifier of Fig. 10.55 is supplying a highly inductive load such that the load current can be assumed to be purely dc, as represented by the current source $I_{dc}$ in the figure. The source voltage is a

Figure 10.55 Full-wave, phase-controlled rectifier for Problem 10.11.
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sinusoid, \( v_s(t) = V_0 \sin \omega t \). As shown in Fig. 10.31, SCRs T1 and T3 are triggered together at delay angle \( \alpha_d \) (0 \( \leq \alpha_d \leq \pi \)), and SCRs T2 and T4 are triggered exactly one-half cycle later.

a. For \( \alpha_d = \pi/4 \):
   (i) Sketch the load voltage \( v_s'(t) \).
   (ii) Calculate the average (dc) value \( V_{dc} \) of \( v_s'(t) \).
   (iii) Calculate the time-averaged power supplied to the load.

b. Repeat part (a) for \( \alpha_d = 3\pi/4 \).

10.12 A full-wave diode rectifier is fed from a 50-Hz, 220-V rms source whose series inductance is 12 mH. It drives a load with a resistance 8.4 \( \Omega \) which is sufficiently inductive that the load current can be considered to be essentially dc.

a. Calculate the dc load current \( I_{dc} \) and the commutation time \( t_c \).

b. Compare the dc current of part (a) with the dc current which would result if the commutating inductance could be eliminated from the system.

10.13 A 1-kW, 85-V, permanent-magnet dc motor is to be driven from a full-wave, phase-controlled bridge such as is shown in Fig. 10.56. When operating at its rated voltage, the dc-motor has a no-load speed of 1725 r/min and an armature resistance \( R_a = 0.82 \Omega \). A large inductor (\( L = 580 \) mH) with resistance \( R_L = 0.39 \Omega \) has been inserted in series with the output of the rectifier bridge to reduce the ripple current applied to the motor. The source voltage is a 115-V rms, 60-Hz sinusoid.

With the motor operating at a speed of 1650 r/min, the motor current is measured to be 7.6 A.

a. Calculate the motor input power.

b. Calculate the firing delay angle \( \alpha_d \) of the SCR bridge.

10.14 Consider the dc-motor drive system of Problem 10.13. To limit the starting current of the dc motor to twice its rated value, a controller will be used to adjust the initial firing-delay angle of the SCR bridge. Calculate the required firing-delay angle \( \alpha_d \).

**Figure 10.56** Dc motor driven from a full-wave, phase-controlled rectifier.
Problem 10.13.
10.15 A three-phase diode bridge is supplied by a three-phase autotransformer such that the line-to-line input voltage to the bridge can be varied from zero to 230 V. The output of the bridge is connected to the shunt field winding of a dc motor. The resistance of this winding is 158 Ω. The autotransformer is adjusted to produce a field current of 1.75 A. Calculate the rms output voltage of the autotransformer.

10.16 A dc-motor shunt field winding of resistance 210 Ω is to be supplied from a 220-V rms, 50-Hz, three-phase source through a three-phase, phase-controlled rectifier. Calculate the delay angle α_d which will result in a field current of 1.1 A.

10.17 A superconducting magnet has an inductance of 4.9 H, a resistance of 3.6 mΩ, and a rated operating current of 80 A. It will be supplied from a 15-V rms, three-phase source through a three-phase, phase-controlled bridge. It is desired to “charge” the magnet at a constant rate to achieve rated current in 25 seconds.

a. Calculate the firing-delay angle α_d required to achieve this objective.

b. Calculate the firing-delay angle required to maintain a constant current of 80 A.

10.18 A voltage-source H-bridge inverter is used to produce the stepped waveform v(t) shown in Fig. 10.57. For V_0 = 50 V, T = 10 msec and D = 0.3:

a. Using Fourier analysis, find the amplitude of the fundamental time-harmonic component of v(t).

b. Use the MATLAB ‘fft()’ function to find the amplitudes of the first 10 time harmonics of v(t).

10.19 Consider the stepped voltage waveform of Problem 10.18 and Fig. 10.57.

a. Using Fourier analysis, find the value of D (0 ≤ D ≤ 0.5) such that the amplitude of the third-harmonic component of the voltage waveform is zero.

---

**Figure 10.57** Stepped voltage waveform for Problem 10.18.
b. Use the MATLAB ‘fft()’ function to find the amplitudes of the first 10 time harmonics of the resulting waveform.

10.20 Consider Example 10.12 in which a current-source inverter is driving a load consisting of a sinusoidal voltage. The inverter is controlled to produce the stepped current waveform shown in Fig. 10.58.

a. Create a table showing the switching sequence required to produce the specified waveform and the time period during which each switch is either ON or OFF.

b. Express the fundamental component of the current waveform in the form

\[ i_1(t) = I_1 \cos(\omega t + \phi_1) \]

where \( I_1 \) and \( \phi_1 \) are functions of \( I_0 \), \( D \) and the delay angle \( \alpha_d \).

c. Derive an expression for the time-averaged power delivered to the voltage source \( v_L(t) = V_a \cos(\omega t) \).

10.21 A PWM inverter such as that of Fig. 10.45 is operating from a dc voltage of 75 V and driving a load with \( L = 53 \) mH and \( R = 1.7 \) \( \Omega \). For a switching frequency of 1500 Hz, calculate the average load current, the minimum and maximum current, and the current ripple for a duty cycle \( D = 0.7 \).