CHAPTER 2

Transformers

Before we proceed with a study of electric machinery, it is desirable to discuss certain aspects of the theory of magnetically coupled circuits, with emphasis on transformer action. Although the static transformer is not an energy conversion device, it is an indispensable component in many energy conversion systems. A significant component of ac power systems, it makes possible electric generation at the most economical generator voltage, power transfer at the most economical transmission voltage, and power utilization at the most suitable voltage for the particular utilization device. The transformer is also widely used in low-power, low-current electronic and control circuits for performing such functions as matching the impedances of a source and its load for maximum power transfer, isolating one circuit from another, or isolating direct current while maintaining ac continuity between two circuits.

The transformer is one of the simpler devices comprising two or more electric circuits coupled by a common magnetic circuit. Its analysis involves many of the principles essential to the study of electric machinery. Thus, our study of the transformer will serve as a bridge between the introduction to magnetic-circuit analysis of Chapter 1 and the more detailed study of electric machinery to follow.

2.1 INTRODUCTION TO TRANSFORMERS

Essentially, a transformer consists of two or more windings coupled by mutual magnetic flux. If one of these windings, the *primary,* is connected to an alternating-voltage source, an alternating flux will be produced whose amplitude will depend on the primary voltage, the frequency of the applied voltage, and the number of turns. A portion of this flux, referred to as *mutual flux*, will link a second winding, the *secondary,*[1] and

[1] It is conventional to think of the "input" to the transformer as the primary and the "output" as the secondary. However, in many applications, power can flow either way and the concept of primary and secondary windings can become confusing. An alternate terminology, which refers to the windings as "high-voltage" and "low-voltage," is often used and eliminates this confusion.

will induce a voltage in it whose value will depend on the number of secondary turns as well as the magnitude of the mutual flux and the frequency. The *voltage ratio,* or *ratio of transformation,* between the two windings can be varied by proportioning the number of primary and secondary turns.

The essence of transformer action requires only the existence of time-varying mutual flux linking two windings. Such action can occur for two windings coupled through air. However, coupling between the windings can be made much more effective through the use of a core of iron or other ferromagnetic material because most of the flux will be confined to a definite, high-permeability path linking the windings. Such a transformer is commonly called an *iron-core transformer*. Most transformers are of this type. The following discussion is concerned almost wholly with iron-core transformers.

As discussed in Section 1.4, to reduce the losses caused by eddy currents in the core, the magnetic circuit in a transformer usually consists of a stack of thin laminations. Two common types of construction are shown schematically in Fig. 2.1. In the *core type* (Fig. 2.1a) the windings are wound around two legs of a rectangular magnetic core; in the *shell type* (Fig. 2.1b) the windings are wound around the center leg of a three-legged core. Silicon-steel laminations of thickness 0.014 in (0.55 mm) are commonly used for transformers operating at frequencies below a few hundred hertz. Silicon steel has the desirable properties of low cost, low core loss, and high permeability at high flux density. The cores of small transformers used in communication circuits at high frequencies and low energy levels are sometimes made of compressed powdered ferromagnetic alloys known as *ferrites*.

In each of these configurations, most of the flux is confined to the core and therefore links both windings. The windings also produce additional flux, known as

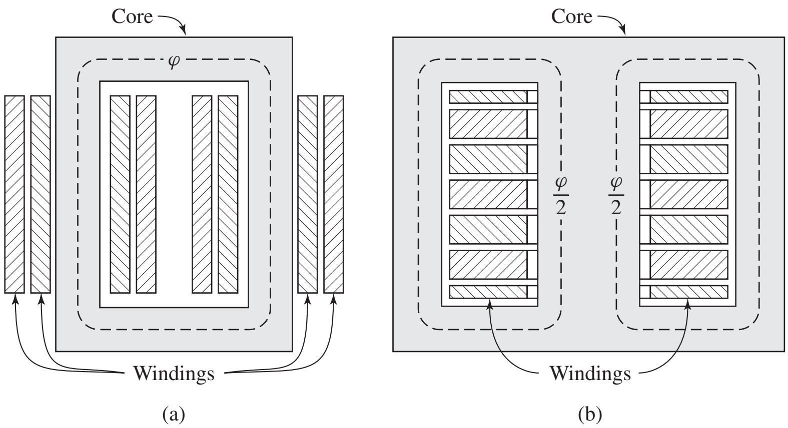

Figure 2.1 Schematic views of (a) core-type and (b) shell-type transformers.

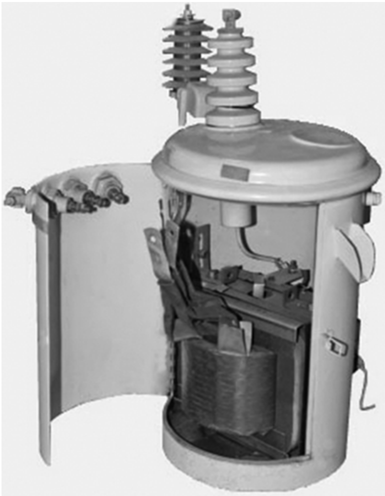

Figure 2.2 A self-protected distribution transformer typical of sizes 2 to 25 kVA, 7200:240/120 V. Only one high-voltage insulator and lightning arrester are needed because one side of the 7200-V line and one side of the primary are grounded.

leakage flux, which links one winding without linking the other. Although leakage flux is a small fraction of the total flux, it plays an important role in determining the behavior of the transformer. In practical transformers, leakage is reduced by subdividing the windings into sections placed as close together as possible. In the core-type construction, each winding consists of two sections, one section on each of the two legs of the core, the primary and secondary windings being concentric coils. In the shell-type construction, variations of the concentric-winding arrangement may be used or the windings may consist of a number of thin "pancake" coils assembled in a stack with primary and secondary coils interleaved.

Figure 2.2 shows the internal construction of a *distribution transformer* such as is used in public utility systems to provide the appropriate voltage for use by residential consumers. A large power transformer is shown in Fig. 2.3.

2.2 NO-LOAD CONDITIONS

Figure 2.4 shows in schematic form a transformer with its secondary circuit open and an alternating voltage v_1 applied to its primary terminals. To simplify the drawings, it is common on schematic diagrams of transformers to show the primary and secondary windings as if they were on separate legs of the core, as in Fig. 2.4, even though the windings are actually interleaved in practice. As discussed in Section 1.4, a small steady-state current i_φ, called the *exciting current,* flows in the primary and establishes

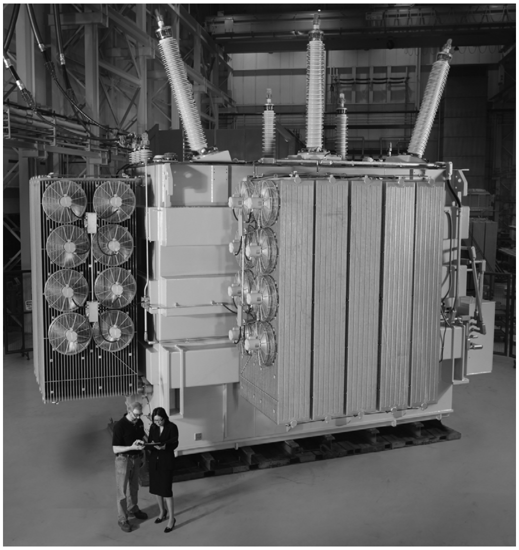

Figure 2.3 A 230 kV Y - 115 kV Y, 100/133/167 MVA Autotransformer. (*Photo courtesy of SPX Transformer Solutions, Inc.*)

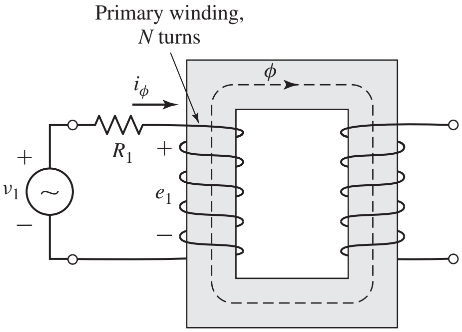

Figure 2.4 Transformer with open secondary.

an alternating flux in the magnetic circuit.[2] This flux induces an emf[3] e_1 in the primary equal to

$$e_1 = \frac{d\lambda_1}{dt} = N_1 \frac{d\varphi}{dt} \tag{2.1}$$

[2] In general, the exciting current corresponds to the net ampere-turns (mmf) acting to produce the flux in the magnetic circuit and it is not possible to distinguish whether it flows in the primary or secondary winding or partially in each winding.

[3] As discussed in Chapter 1, the term emf (electromotive force) is often used instead of induced voltage to represent that component of voltage due to a time-varying flux linkage.

where

$$\begin{aligned} \lambda_1 &= \text{flux linkage of the primary winding} \\ \varphi &= \text{flux in the core linking both windings} \\ N_1 &= \text{number of turns in the primary winding} \end{aligned}$$

The voltage e_1 is in volts when φ is in webers. This emf, together with the voltage drop in the *primary resistance* R_1 (shown schematically as a series resistance in Fig. 2.4), must balance the applied voltage v_1; thus

$$v_1 = R_1 i_\varphi + e_1 \tag{2.2}$$

Note that for the purposes of the current discussion, we are neglecting the effects of primary leakage flux, which will add an additional induced-emf term in Eq. 2.2. In typical transformers, this flux is a small percentage of the core flux, and it is quite justifiable to neglect it for our current purposes. It does, however, play an important role in the behavior of transformers and is discussed in some detail in Section 2.4.

In most large transformers, the no-load resistance drop is very small indeed, and the induced emf e_1 very nearly equals the applied voltage v_1. Furthermore, the waveforms of voltage and flux are very nearly sinusoidal. The analysis can then be greatly simplified, as we have shown in Section 1.4. Thus, if the instantaneous flux φ is

$$\varphi = \phi_{\max} \sin \omega t \tag{2.3}$$

the induced voltage e_1 is

$$e_1 = N_1 \frac{d\varphi}{dt} = \omega N_1 \phi_{\max} \cos \omega t \tag{2.4}$$

where $\phi_{\max}$ is the maximum value of the flux and $\omega = 2\pi f$, the frequency being f Hz. For the current and voltage reference directions shown in Fig. 2.4, the induced emf leads the flux by 90°. The rms value of the induced emf e_1 is

$$E_1 = \frac{2\pi}{\sqrt{2}} f N_1 \phi_{\max} = \sqrt{2}\, \pi f N_1 \phi_{\max} \tag{2.5}$$

As can be seen from Eq. 2.2, if the resistive voltage drop is negligible, the counter emf equals the applied voltage. Under these conditions, if a sinusoidal voltage is applied to a winding, a sinusoidally varying core flux must be established whose maximum value $\phi_{\max}$ satisfies the requirement that E_1 in Eq. 2.5 equal the rms value V_1 of the applied voltage; thus

$$\phi_{\max} = \frac{V_1}{\sqrt{2}\pi f N_1} \tag{2.6}$$

Under these conditions, the core flux is determined solely by the applied voltage, its frequency, and the number of turns in the winding. This important relation applies not only to transformers but also to any device operated with a sinusoidally-alternating impressed voltage, as long as the resistance and leakage-inductance voltage drops are negligible. The core flux is fixed by the applied voltage, and the required exciting current is determined by the magnetic properties of the core; the exciting current must adjust itself so as to produce the mmf required to create the flux demanded by Eq. 2.6.

The importance and utility of this concept cannot be over-emphasized. It is often extremely useful in the analysis of electric machines which are supplied from single or poly-phase voltage sources. To a first approximation, the winding resistance can often be neglected and, in spite of additional windings (for example the shorted windings on the rotor of induction machines as will be seen in Chapter 6), the flux in the machine will be determined by the applied voltage and the winding currents must adjust to produce the corresponding mmf.

Because of the nonlinear magnetic properties of iron, the waveform of the exciting current differs from the waveform of the flux; the exciting current for a sinusoidal flux waveform will not be sinusoidal. This effect is especially pronounced in closed magnetic circuits such as are found in transformers. In magnetic circuits where the reluctance is dominated by an air gap with its linear magnetic characteristic, such as is the case in many electric machines, the relationship between the net flux and the applied mmf is relatively linear and the exciting current will be much more sinusoidal.

In the case of a closed magnetic circuit, a curve of the exciting current as a function of time can be found graphically from the ac hysteresis loop, as is discussed in Section 1.4 and shown in Fig. 1.11. If the exciting current is analyzed by Fourier-series methods, it is found to consist of a fundamental component and a series of odd harmonics. The fundamental component can, in turn, be resolved into two components, one in phase with the counter emf and the other lagging the counter emf by 90°. The in-phase component supplies the power absorbed by hysteresis and eddy-current losses in the core. It is referred to as *core-loss component* of the exciting current. When the core-loss component is subtracted from the total exciting current, the remainder is called the *magnetizing current*. It comprises a fundamental component lagging the counter emf by 90°, together with all the harmonics. The principal harmonic is the third. For typical power transformers, the third harmonic is usually about 40 percent of the exciting current.

Except in problems concerned directly with the effects of harmonic currents, the peculiarities of the exciting-current waveform usually need not be taken into account, because the exciting current itself is small, especially in large transformers. For example, the exciting current of a typical power transformer is about 1 to 2 percent of full-load current. Consequently the effects of harmonics are usually swamped out by the sinusoidal-currents supplied to other linear elements in the circuit. The exciting current can then be represented by an equivalent sinusoidal current which has the same rms value and frequency and produces the same average power as the actual exciting current.

Such a representation is essential to the construction of a *phasor diagram,* which represents the phase relationship between the various voltages and currents in a system in vector form. Each signal is represented by a phasor whose length is proportional to the amplitude of the signal and whose angle is equal to the phase angle of that signal as measured with respect to a chosen reference signal. In Fig. 2.5, the phasors $\hat{E}_1$

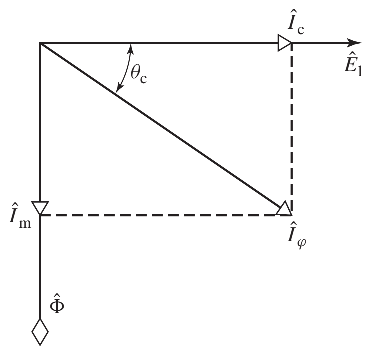

Figure 2.5 No-load phasor diagram.

and $\hat{\Phi}$ respectively, represent the complex amplitudes of the rms-induced emf and the flux. The phasor $\hat{I}_\varphi$ represents the complex amplitude of the rms equivalent sinusoidal exciting current. It lags the induced emf $\hat{E}_1$ by a phase angle θ_c. Also shown in the figure is the phasor $\hat{I}_c$, in phase with $\hat{E}_1$, which is the core-loss component of the exciting current, The component $\hat{I}_m$, in phase with the flux, represents an equivalent sine wave current having the same rms value as the magnetizing current.

The core loss P_{core}, equal to the product of the in-phase components of $\hat{E}_1$ and $\hat{I}_\varphi$, is given by

$$P_{core} = E_1 I_\varphi \cos\theta_c = E_1 I_c \tag{2.7}$$

Typical exciting volt-ampere and core-loss characteristics of high-quality silicon steel used for power and distribution transformer laminations are shown in Figs. 1.12 and 1.14.

EXAMPLE 2.1

In Example 1.8 the core loss and exciting voltamperes for the core of Fig. 1.15 at $B_{max} = 1.5$ T and 60 Hz were found to be

$$P_{core} = 16 \text{ W} \qquad (VI)_{rms} = 20 \text{ VA}$$

and the induced voltage was $V = 274/\sqrt{2} = 194$ V rms when the winding had 200 turns.

Find the power factor, the core-loss current I_c, and the magnetizing current I_m.

■ Solution

Power factor: $\cos\theta_c = \frac{16}{20} = 0.80$ (lag) thus $\theta_c = -36.9°$

Note that we know that the power factor is lagging because the system is inductive.

Exciting current: $I_\varphi = \frac{(VI)_{rms}}{V} = 0.10$ A rms

Core-loss component: $I_c = \frac{P_{core}}{V} = 0.082$ A rms

Magnetizing component: $I_m = I_\varphi \times \sin\theta_c = 0.060$ A rms

2.3 EFFECT OF SECONDARY CURRENT; IDEAL TRANSFORMER

As a first approximation to a quantitative theory, consider a transformer with a primary winding of N_1 turns and a secondary winding of N_2 turns, as shown schematically in Fig. 2.6. Notice that the secondary current is defined as positive out of the winding; thus positive secondary current produces an mmf in the opposite direction from that created by positive primary current. Let the properties of this transformer be idealized under the assumption that winding resistances are negligible, that all the flux is confined to the core and fully links both windings (i.e., leakage flux is assumed negligible), that there are no losses in the core, and that the permeability of the core is so high that only a negligible exciting mmf is required to establish the flux. These properties are closely approached but never actually attained in practical transformers. A hypothetical transformer having these properties is often called an *ideal transformer*.

Under the above assumptions, when a time-varying voltage v_1 is impressed on the primary terminals, a core flux φ must be established such that the counter emf e_1 equals the impressed voltage v_1. Thus

$$v_1 = e_1 = N_1 \frac{d\varphi}{dt} \tag{2.8}$$

The core flux also links the secondary and produces an induced emf e_2, and an equal secondary terminal voltage v_2, given by

$$v_2 = e_2 = N_2 \frac{d\varphi}{dt} \tag{2.9}$$

From the ratio of Eqs. 2.8 and 2.9,

$$\frac{v_1}{v_2} = \frac{N_1}{N_2} \tag{2.10}$$

Thus an ideal transformer transforms voltages in the direct ratio of the turns in its windings.

Now let a load which draws a current i_2 be connected to the secondary. The load current thus produces an mmf $N_2 i_2$ in the secondary. Since the impressed primary voltage sets the core flux as specified by Eq. 2.8, the core flux is unchanged by the presence of a load on the secondary. Furthermore, since the net exciting mmf acting

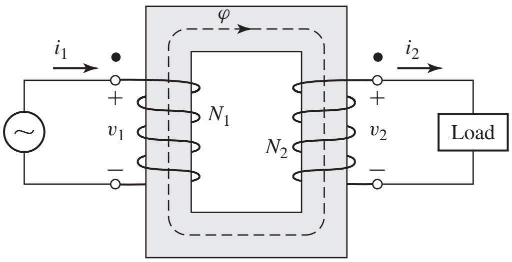

Figure 2.6 Ideal transformer and load.

on the core (equal to $N_1 i_1 - N_2 i_2$) must remain negligible, the primary and secondary currents must satisfy the relationship

$$N_1 i_1 - N_2 i_2 = 0 \tag{2.11}$$

From Eq. 2.11 we see that a compensating primary mmf must result to cancel that of the secondary. Hence

$$N_1 i_1 = N_2 i_2 \tag{2.12}$$

From this discussion, we see that the requirement that the core flux and hence the corresponding net mmf remain unchanged is the means by which the primary "knows" of the presence of load current in the secondary; any change in mmf flowing in the secondary as the result of a load must be accompanied by a corresponding change in the primary mmf. Note that for the reference directions shown in Fig. 2.6 the mmfs of i_1 and i_2 are in opposite directions and therefore compensate.

From Eq. 2.12

$$\frac{i_1}{i_2} = \frac{N_2}{N_1} \tag{2.13}$$

Thus an ideal transformer transforms currents in the inverse ratio of the turns in its windings.

Also notice from Eqs. 2.10 and 2.13 that

$$v_1 i_1 = v_2 i_2 \tag{2.14}$$

i.e., the instantaneous power input to the primary equals the instantaneous power output from the secondary, a necessary condition because all dissipative and energy storage mechanisms in the transformer have been neglected.

An additional property of the ideal transformer can be seen by considering the case of a sinusoidal applied voltage and an impedance load. The circuit is shown in simplified form in Fig. 2.7a, in which the dot-marked terminals of the transformer correspond to the similarly marked terminals in Fig. 2.6. Because all the voltages and currents are sinusoidal, the voltages and currents are represented by their complex amplitudes. The dot markings indicate terminals of corresponding polarity; i.e., if

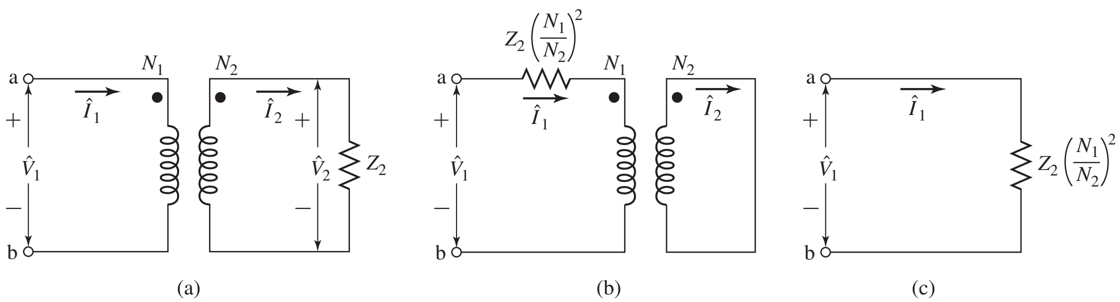

Figure 2.7 Three circuits which are identical at the terminal a-b when the transformer is ideal.

one follows through the primary and secondary windings of Fig. 2.6, beginning at their dot-marked terminals, one will find that both windings encircle the core in the same direction with respect to the flux. Therefore, if one compares the voltages of the two windings, the voltages from a dot-marked to an unmarked terminal will be of the same instantaneous polarity for primary and secondary. In other words, the voltages $\hat{V}_1$ and $\hat{V}_2$ in Fig. 2.7a are in phase. Also currents $\hat{I}_1$ and $\hat{I}_2$ are in phase as seen from Eq. 2.12. Note again that the polarity of $\hat{I}_1$ is defined as into the dotted terminal and the polarity of $\hat{I}_2$ is defined as out of the dotted terminal.

The circuits of Fig. 2.7 let us investigate the impedance transformation properties of the ideal transformer. In phasor form, Eqs. 2.10 and 2.13 can be expressed as

$$\hat{V}_1 = \frac{N_1}{N_2}\hat{V}_2 \quad \text{and} \quad \hat{V}_2 = \frac{N_2}{N_1}\hat{V}_1 \tag{2.15}$$

$$\hat{I}_1 = \frac{N_2}{N_1}\hat{I}_2 \quad \text{and} \quad \hat{I}_2 = \frac{N_1}{N_2}\hat{I}_1 \tag{2.16}$$

From these equations

$$\frac{\hat{V}_1}{\hat{I}_1} = \left(\frac{N_1}{N_2}\right)^2 \frac{\hat{V}_2}{\hat{I}_2} \tag{2.17}$$

We note that the load impedance Z_2 is related to the secondary voltages and currents as

$$Z_2 = \frac{\hat{V}_2}{\hat{I}_2} \tag{2.18}$$

where Z_2 is the complex impedance of the load. Thus, from Eqs. 2.17 and 2.18, we see that the impedance Z_1 seen at the terminals a-b is equal to

$$Z_1 = \frac{\hat{V}_1}{\hat{I}_1} = \left(\frac{N_1}{N_2}\right)^2 Z_2 \tag{2.19}$$

and consequently we see that from the primary terminals a-b, an impedance Z_2 in the secondary circuit can be replaced by an equivalent impedance Z_1 in the primary circuit satisfying the relationship

$$Z_1 = \left(\frac{N_1}{N_2}\right)^2 Z_2 \tag{2.20}$$

The three circuits of Fig. 2.7 are indistinguishable as far as their performance viewed from terminals a-b is concerned. Transferring an impedance from one side of a transformer to the other in this fashion is called *referring the impedance* to the other side; impedances transform as the square of the turns ratio. In a similar manner, voltages and currents can be *referred* to one side or the other by using Eqs. 2.15 and 2.16 to evaluate the equivalent voltage and current on that side.

To summarize, *in an ideal transformer, voltages are transformed by the direct ratio of turns, currents by the inverse ratio, impedances by square of the turns-ratio and power and voltamperes are unchanged.*

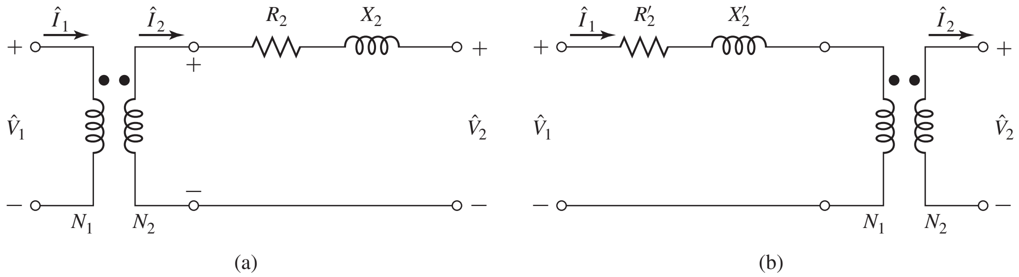

Figure 2.8 Equivalent circuits for Example 2.2. (a) Impedance in series with the secondary. (b) Impedance referred to the primary.

EXAMPLE 2.2

The equivalent circuit of Fig. 2.8a shows an ideal transformer with an impedance $R_2 + jX_2 = 1 + j4\ \Omega$ connected in series with the secondary. The turns ratio $N_1/N_2 = 5{:}1$. (a) Draw an equivalent circuit with the series impedance referred to the primary side. (b) For a primary voltage of 120 V rms and a short connected across the secondary terminals ($V_2 = 0$), calculate the primary current and the current flowing in the short.

■ Solution

a. The new equivalent is shown in Fig. 2.8b. The secondary impedance is referred to the primary by the turns ratio squared. Thus

$$R_2' + jX_2' = \left(\frac{N_1}{N_2}\right)^2 (R_2 + jX_2)$$
$$= 25 + j100\ \Omega$$

b. From Eq. 2.20, a short at terminals A-B will appear as a short at the primary of the ideal transformer in Fig. 2.8b since the zero voltage of the short is reflected by the turns ratio N_1/N_2 to the primary. Hence the primary current will be given by

$$\hat{I}_1 = \frac{\hat{V}_1}{R_2' + jX_2'} = \frac{120}{25 + j100} = 0.28 - j1.13\ \text{A rms}$$

corresponding to a magnitude of 1.16 A rms. From Eq. 2.13, the secondary current will equal $N_1/N_2 = 5$ times that of the current in the primary. Thus the current in the short will have a magnitude of $5(1.16) = 5.8$ A rms.

Practice Problem 2.1

Repeat part (*b*) of Example 2.2 for a series impedance $R_2 + jX_2 = 0.05 + j0.97\ \Omega$ and a turns ratio of 14:1.

Solution

The primary current is $0.03 - j0.63$ A rms, corresponding to a magnitude of 0.63 A rms. The current in the short will be 14 times larger and thus will be of magnitude 8.82 A rms.

2.4 TRANSFORMER REACTANCES AND EQUIVALENT CIRCUITS

The departures of an actual transformer from those of an ideal transformer must be included to a greater or lesser degree in most analyses of transformer performance. A more complete model must take into account the effects of winding resistances, leakage fluxes, and finite exciting current due to the finite (and indeed nonlinear) permeability of the core. In some cases, the capacitances of the windings also have important effects, notably in problems involving transformer behavior at frequencies above the audio range or during rapidly changing transient conditions such as those encountered in power system transformers as a result of voltage surges caused by lightning or switching transients. The analysis of these high-frequency problems is beyond the scope of the present treatment however, and accordingly capacitances of the windings will be neglected.

Two methods of analysis by which departures from the ideal can be taken into account are (1) an equivalent-circuit technique based on physical reasoning and (2) a mathematical approach based on the classical theory of magnetically coupled circuits. Both methods are in everyday use, and both have very close parallels in the theories of rotating machines. Because it offers an excellent example of the thought process involved in translating physical concepts to a quantitative theory, the equivalent-circuit technique is presented here.

To begin the development of a transformer equivalent circuit, we first consider the primary winding. The total flux linking the primary winding can be divided into two components: the resultant mutual flux, confined essentially to the iron core and produced by the combined mmfs of the primary and secondary currents, and the primary leakage flux, which links only the primary. These components are identified in the schematic transformer shown in Fig. 2.9, where for simplicity the primary and

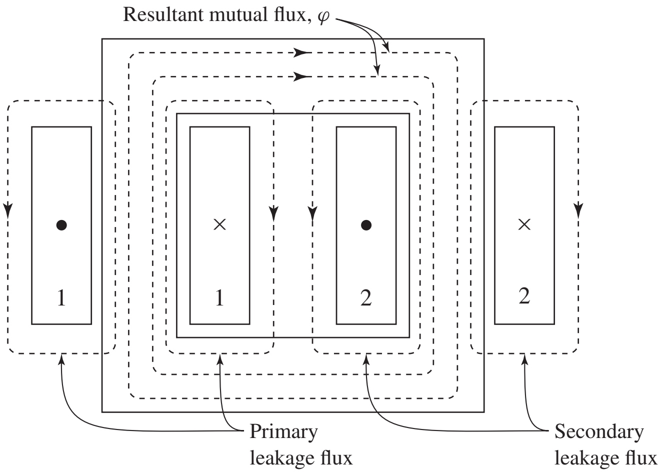

Figure 2.9 Schematic view of mutual and leakage fluxes in a transformer. The "X" and the dot indicate current directions in the various coils.

secondary windings are shown on opposite legs of the core. In an actual transformer with interleaved windings, the details of the flux distribution are more complicated, but the essential features remain the same.

The leakage flux induces voltage in the primary winding which adds to that produced by the mutual flux. Because the leakage path is largely in air, this flux and the voltage induced by it vary linearly with primary current $\hat{I}_1$. It can therefore be represented by a *primary leakage inductance* L_{l_1} (equal to the leakage-flux linkages with the primary per unit of primary current). The corresponding *primary leakage reactance* X_{l_1} is found as

$$X_{l_1} = 2\pi f L_{l_1} \tag{2.21}$$

In addition, there will be a voltage drop in the *primary resistance* R_1 (not shown in Fig. 2.9).

We now see that the primary terminal voltage $\hat{V}_1$ consists of three components: the $\hat{I}_1 R_1$ drop in the primary resistance, the $j\hat{I}_1 X_{l_1}$ drop arising from primary leakage flux, and the emf $\hat{E}_1$ induced in the primary by the resultant mutual flux. Fig. 2.10a shows an equivalent circuit for the primary winding which includes each of these voltages.

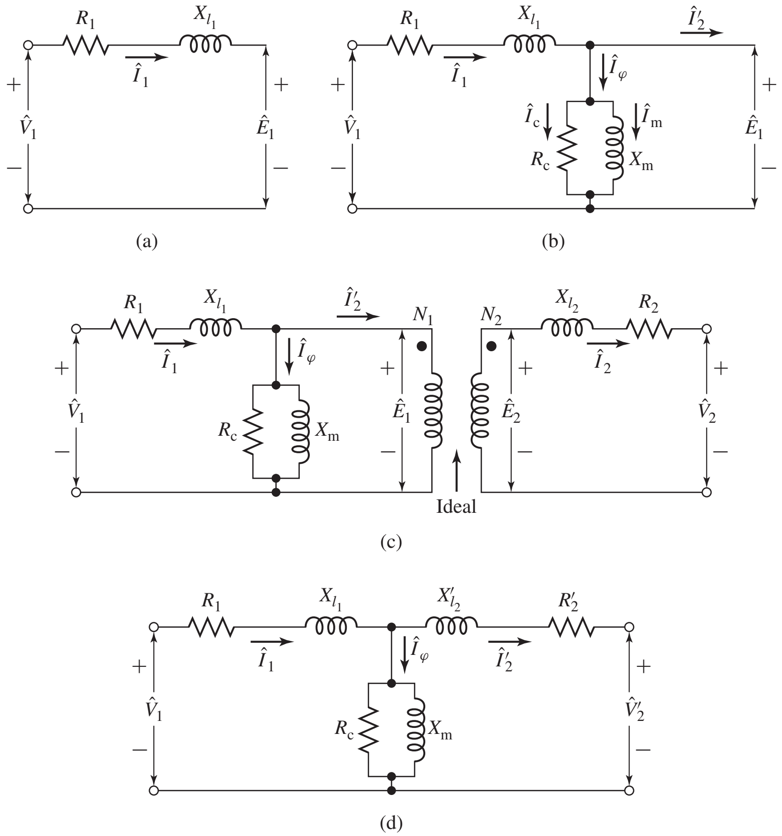

Figure 2.10 Steps in the development of the transformer equivalent circuit.

The resultant mutual flux links both the primary and secondary windings and is created by their combined mmfs. It is convenient to treat these mmfs by considering that the primary current must meet two requirements of the magnetic circuit: It must not only produce the mmf required to produce the resultant mutual flux, but it must also counteract the effect of the secondary mmf which acts to demagnetize the core. An alternative viewpoint is that the primary current must not only magnetize the core, it must also supply current to the load connected to the secondary. According to this picture, it is convenient to resolve the primary current into two components: an exciting component and a load component. The *exciting component* $\hat{I}_\varphi$ is defined as the additional primary current required to produce the resultant mutual flux. It is a nonsinusoidal current of the nature described in Section 2.2.[4] The *load component* $\hat{I}'_2$ is defined as the component current in the primary which would exactly counteract the mmf of secondary current $\hat{I}_2$.

Since it is the exciting component which produces the core flux, the net mmf must equal $N_1\hat{I}_\varphi$ and thus we see that

$$\begin{aligned} N_1\hat{I}_\varphi &= N_1\hat{I}_1 - N_2\hat{I}_2 \\ &= N_1(\hat{I}_\varphi + \hat{I}'_2) - N_2\hat{I}_2 \end{aligned} \tag{2.22}$$

and from Eq. 2.22 we see that

$$\hat{I}'_2 = \frac{N_2}{N_1}\hat{I}_2 \tag{2.23}$$

From Eq. 2.23, we see that the load component of the primary current equals the secondary current referred to the primary as in an ideal transformer.

The exciting current can be treated as an equivalent sinusoidal current $\hat{I}_\varphi$, in the manner described in Section 2.2, and can be resolved into a core-loss component $\hat{I}_c$ in phase with the emf $\hat{E}_1$ and a magnetizing component $\hat{I}_m$ lagging $\hat{E}_1$ by 90°. In the equivalent circuit of Fig. 2.10b the equivalent sinusoidal exciting current is accounted for by means of a shunt branch connected across $\hat{E}_1$, comprising a *core-loss resistance* R_c in parallel with a *magnetizing inductance* L_m whose reactance, known as the *magnetizing reactance,* is given by

$$X_m = 2\pi f L_m \tag{2.24}$$

In the equivalent circuit of Fig. 2.10b the power E_1^2/R_c accounts for the core loss due to the resultant mutual flux. R_c, also referred to as the *magnetizing resistance*, together with X_m forms the *excitation branch* of the equivalent circuit, and we will refer to the parallel combination of R_c and X_m as the *magnetizing impedance* Z_φ. When R_c is assumed constant, the core loss is thereby assumed to vary as E_1^2. Strictly speaking, the magnetizing reactance X_m varies with the saturation of the iron. However, X_m is often assumed constant and the magnetizing current is thereby assumed to be independent of frequency and directly proportional to the resultant mutual flux.

[4] In fact, the exciting current corresponds to the net mmf acting on the transformer core and cannot, in general, be considered to flow in the primary alone. However, for the purposes of this discussion, this distinction is not significant.

Both R_c and X_m are usually determined at rated voltage and frequency; they are then assumed to remain constant for the small departures from rated values associated with normal operation.

We will next add to our equivalent circuit a representation of the secondary winding. We begin by recognizing that the resultant mutual flux $\hat{\Phi}$ induces an emf $\hat{E}_2$ in the secondary, and since this flux links both windings, the induced-emf ratio must equal the winding turns ratio, i.e.,

$$\frac{\hat{E}_1}{\hat{E}_2} = \frac{N_1}{N_2} \tag{2.25}$$

just as in an ideal transformer. This voltage transformation and the current transformation of Eq. 2.23 can be accounted for by introducing an ideal transformer in the equivalent circuit, as in Fig. 2.10c. Just as is the case for the primary winding, the emf $\hat{E}_2$ is not the secondary terminal voltage because of the *secondary resistance* R_2 and because the secondary current $\hat{I}_2$ creates secondary leakage flux (see Fig. 2.9). The secondary terminal voltage $\hat{V}_2$ differs from the induced voltage $\hat{E}_2$ by the voltage drops due to secondary resistance R_2 and *secondary leakage reactance* X_{l_2} (corresponding to the *secondary leakage inductance* L_{l_2}) as in the portion of the complete transformer equivalent circuit (Fig. 2.10c) to the right of $\hat{E}_2$.

From the equivalent circuit of Fig. 2.10, the actual transformer therefore can be seen to be equivalent to an ideal transformer plus external impedances. By referring all quantities to the primary or secondary, the ideal transformer in Fig. 2.10c can be moved out to the right or left, respectively, of the equivalent circuit. This is almost invariably done, and the equivalent circuit is usually drawn as in Fig. 2.10d, with the ideal transformer not shown and all voltages, currents, and impedances referred to either the primary or secondary winding. Specifically, for Fig. 2.10d,

$$X'_{l_2} = \left(\frac{N_1}{N_2}\right)^2 X_{l_2} \tag{2.26}$$

$$R'_2 = \left(\frac{N_1}{N_2}\right)^2 R_2 \tag{2.27}$$

and

$$V'_2 = \frac{N_1}{N_2} V_2 \tag{2.28}$$

The circuit of Fig. 2.10d is called the *equivalent-T circuit* for a transformer.

In Fig. 2.10d, in which the secondary quantities are referred to the primary, the referred secondary values are indicated with primes, for example, X'_{l_2} and R'_2, to distinguish them from the actual values of Fig. 2.10c. In the discussion that follows we almost always deal with referred values, and the primes will be omitted. One must simply keep in mind the side of the transformers to which all quantities have been referred.

EXAMPLE 2.3

A 50-kVA 2400:240-V 60-Hz distribution transformer has a leakage impedance of $0.72 + j0.92\ \Omega$ in the high-voltage winding and $0.0070 + j0.0090\ \Omega$ in the low-voltage winding. At rated voltage and frequency, the impedance Z_φ of the shunt branch (equal to the impedance of

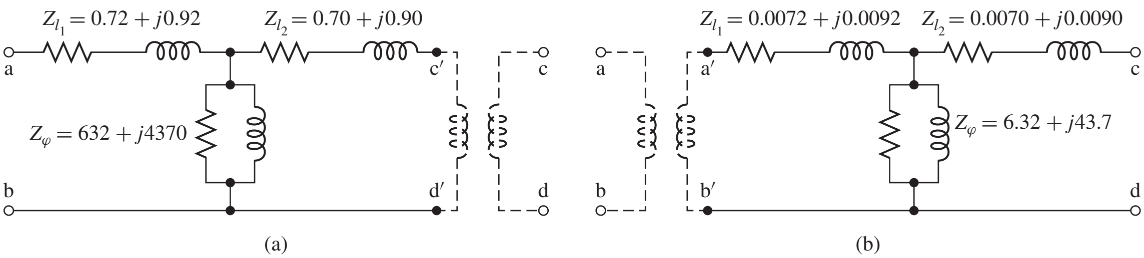

Figure 2.11 Equivalent circuits for transformer of Example 2.3 referred to (a) the high-voltage side and (b) the low-voltage side.

R_c and jX_m in parallel) accounting for the exciting current is $6.32 + j43.7\ \Omega$ when viewed from the low-voltage side. Draw the equivalent circuit referred to (a) the high-voltage side and (b) the low-voltage side, and label the impedances numerically.

■ Solution

The circuits are given in Fig. 2.11a and b, respectively, with the high-voltage side numbered 1 and the low-voltage side numbered 2. The voltages given on the nameplate of a power system transformer are based on the turns ratio and neglect the small leakage-impedance voltage drops under load. Since this is a 10-to-1 transformer, impedances are referred by multiplying or dividing by 100; for example, the value of an impedance referred to the high-voltage side is greater by a factor of 100 than its value referred to the low-voltage side.

The ideal transformer may be explicitly drawn, as shown dotted in Fig. 2.11, or it may be omitted in the diagram and remembered mentally, making the unprimed letters the terminals. If this is done, one must of course remember to refer all connected impedances and sources to be consistent with the omission of the ideal transformer.

Practice Problem 2.2

If 2,400 V rms is applied to the high-voltage side of the transformer of Example 2.3, calculate the magnitude of the current into the magnetizing impedance Z_φ in Figs. 2.11a and b respectively.

Solution

The current through Z_φ is 0.543 A rms when it is referred to the high-voltage side as in Fig. 2.11a and 5.43 A rms when it is referred to the low-voltage side.

2.5 ENGINEERING ASPECTS OF TRANSFORMER ANALYSIS

In engineering analyses involving the transformer as a circuit element, it is customary to adopt one of several approximate forms of the equivalent circuit of Fig. 2.10 rather than the full circuit. The approximations chosen in a particular case depend largely on physical reasoning based on orders of magnitude of the neglected quantities. The

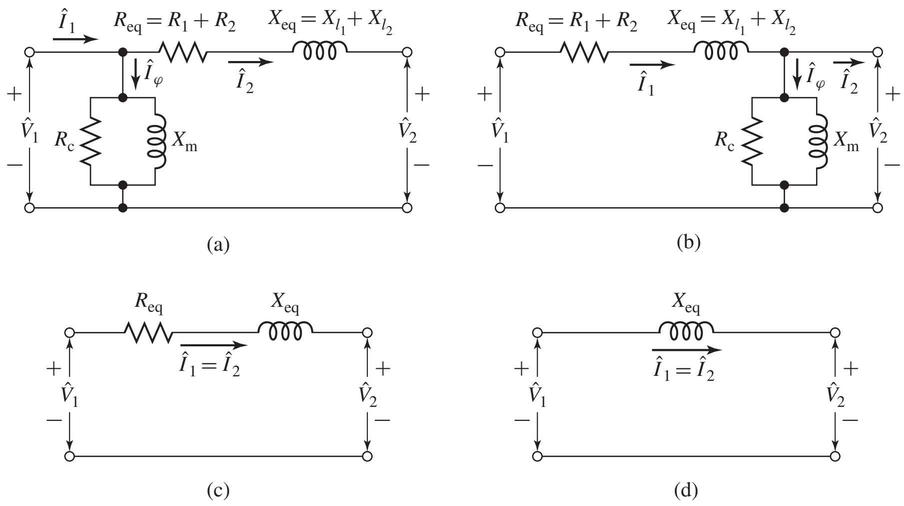

Figure 2.12 Approximate transformer equivalent circuits.

more common approximations are presented in this section. In addition, test methods are given for determining the transformer constants.

The approximate equivalent circuits commonly used for constant-frequency power transformer analyses are summarized for comparison in Fig. 2.12. All quantities in these circuits are referred to either the primary or the secondary, and the ideal transformer is not shown.

Computations can often be greatly simplified by moving the shunt branch representing the exciting current out from the middle of the T circuit to either the primary or the secondary terminals, as in Fig. 2.12a and b. These forms of the equivalent circuit are referred to as *cantilever circuits*. The series branch is the combined resistance and leakage reactance of the primary and secondary, referred to the same side. This impedance is sometimes called the *equivalent series impedance* and its components the *equivalent series resistance* R_{eq} and *equivalent series reactance* X_{eq}, as shown in Fig. 2.12a and b.

As compared to the equivalent-T circuit of Fig. 2.10d, the cantilever circuit is in error in that it neglects the voltage drop in the primary or secondary leakage impedance caused by the exciting current. Because the impedance of the exciting branch is typically quite large in large power transformers, the corresponding exciting current is quite small. This error is insignificant in most situations involving large transformers.

EXAMPLE 2.4

Consider the equivalent-T circuit of Fig. 2.11a of the 50-kVA 2400:240 V distribution transformer of Example 2.3 in which the impedances are referred to the high-voltage side. (a) Draw the cantilever equivalent circuit with the shunt branch at the high-voltage terminal. Calculate

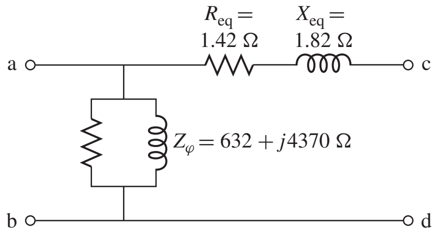

Figure 2.13 Cantilever equivalent circuit for Example 2.4.

and label R_{eq} and X_{eq}. (b) With the low-voltage terminal open-circuit and 2400 V applied to the high-voltage terminal, calculate the voltage at the low-voltage terminal as predicted by each equivalent circuit.

■ Solution

a. The cantilever equivalent circuit is shown in Fig. 2.13. R_{eq} and X_{eq} are found simply as the sum of the high- and low-voltage winding series impedances of Fig. 2.11a

$$R_{eq} = 0.72 + 0.70 = 1.42\ \Omega$$

$$X_{eq} = 0.92 + 0.90 = 1.82\ \Omega$$

b. For the equivalent-T circuit of Fig. 2.11a, the voltage at the terminal labeled c′-d′ will be given by

$$\hat{V}_{c'\text{-}d'} = 2400\left(\frac{Z_\varphi}{Z_\varphi + Z_{l_1}}\right) = 2399 + j0.3\ \text{V}$$

with an rms magnitude of 2399 V. Reflected to the low-voltage terminals by the low- to high-voltage turns ratio, this in turn corresponds to a voltage of 239.9 V.

Because the magnetizing impedance is connected directly across the high-voltage terminals in the cantilever equivalent circuit of Fig. 2.13, there will be no voltage drop across any series leakage impedance and the predicted secondary voltage will be 240 V. These two solutions differ by 0.025 percent, well within reasonable engineering accuracy and clearly justifying the use of the cantilever equivalent circuit for analysis of this transformer.

Further analytical simplification results from neglecting the exciting current entirely, as in Fig. 2.12c, in which the transformer is represented as an equivalent series impedance. If the transformer is large (several hundred kilovoltamperes or more), the equivalent resistance R_{eq} is small compared with the equivalent reactance X_{eq} and can frequently be neglected, giving the equivalent circuit of Fig. 2.12d. The circuits of Fig. 2.12c and d are sufficiently accurate for most ordinary power-system problems and are used in all but the most detailed analyses. Finally, in situations where the currents and voltages are determined almost wholly by components external to the transformer or when a high degree of accuracy is not required, the entire transformer impedance can be neglected and the transformer considered to be ideal, as in Section 2.3.

The circuits of Fig. 2.12 have the additional advantage that the total equivalent resistance R_{eq} and equivalent reactance X_{eq} can be found from a very simple test in which one terminal is short-circuited. On the other hand, the process of determining the individual leakage reactances X_{l_1} and X_{l_2} and a complete set of parameters for the equivalent-T circuit of Fig. 2.10c is more difficult. Example 2.4 illustrates that, due to the voltage drop across leakage impedances, the ratio of the measured voltages of a transformer will not be identically equal to the idealized voltage ratio which would be measured if the transformer were ideal. In fact, without some apriori knowledge of the turns ratio (based for example upon knowledge of the internal construction of the transformer), it is not possible to make a set of measurements which uniquely determine the turns ratio, the magnetizing inductance, and the individual leakage impedances.

It can be shown that, with respect to terminal measurements, neither the turns ratio, the magnetizing reactance, or the leakage reactances are unique characteristics of a transformer equivalent circuit. For example, the turns ratio can be chosen arbitrarily and for each choice of turns ratio, there will be a corresponding set of values for the leakage and magnetizing reactances which matches the measured characteristic. Each of the resultant equivalent circuits will have the same electrical terminal characteristics, a fact which has the fortunate consequence that any self-consistent set of empirically determined parameters will adequately represent the transformer.

EXAMPLE 2.5

The 50-kVA 2400:240-V transformer whose parameters are given in Example 2.3 is used to step down the voltage at the load end of a feeder whose impedance is $0.30 + j1.60\ \Omega$. The voltage V_s at the sending (primary) end of the feeder is 2400 V.

Find the voltage at the secondary terminals of the transformer when the load connected to its secondary draws rated current from the transformer and the power factor of the load is 0.80 lagging. Neglect the voltage drops in the transformer and feeder caused by the exciting current.

■ Solution

The equivalent circuit with all quantities referred to the high-voltage (primary) side of the transformer is shown in Fig. 2.14a, where the transformer is represented by its equivalent impedance, as in Fig. 2.12c. From Fig. 2.11a, the value of the equivalent impedance is $Z_{eq} = 1.42 + j1.82\ \Omega$ and the combined impedance of the feeder and transformer in series is $Z = 1.72 + j3.42\ \Omega$. From the transformer rating, the load current referred to the high-voltage side is $I = 50{,}000/2400 = 20.8$ A.

Note that the power factor is defined at the load side of the transformer and hence defines the phase angle θ between the load current $\hat{I}$ and the voltage $\hat{V}_2$ where

$$\theta = -\cos^{-1}(0.80) = -36.87^\circ$$

Thus

$$\hat{I} = 20.8\, e^{-j36.87^\circ}\ \text{A}$$

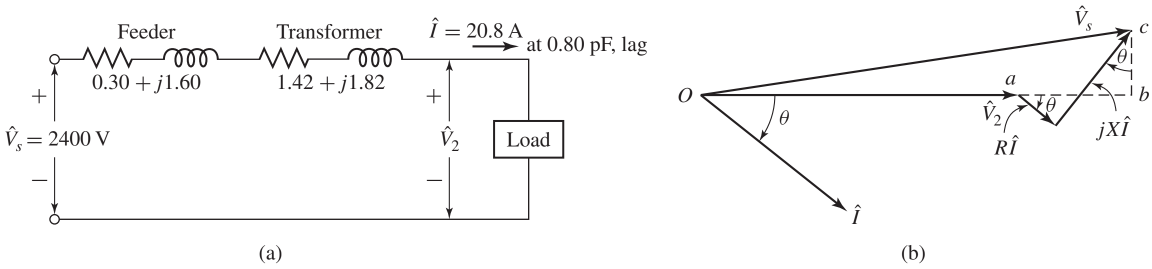

Figure 2.14 (a) Equivalent circuit and (b) phasor diagram for Example 2.5.

From the equivalent-circuit of Fig. 2.11 we see that

$$\hat{V}_2 = \hat{V}_s - Z\hat{I} = 2400 - (1.72 + j3.42) \times 20.8\, e^{-j36.87^\circ}$$
$$= 2329\, e^{-j0.87^\circ} \text{ V}$$

Although an algebraic solution of the complex equation is often the simplest and most direct way to obtain a solution, it is sometimes useful to solve these type of problems with the aid of a phasor diagram. We will illustrate this with a phasor diagram referred to the high-voltage side as shown in Fig. 2.14b. From the phasor diagram

$$Ob = \sqrt{V_s^2 - (bc)^2} \quad \text{and} \quad V_2 = Ob - ab$$

Note that

$$bc = IX\cos\theta - IR\sin\theta \quad ab = IR\cos\theta + IX\sin\theta$$

where R and X are the combined transformer and feeder resistance and reactance, respectively. Thus

$$bc = 20.8(3.42)(0.80) - 20.8(1.72)(0.60) = 35.5 \text{ V}$$

$$ab = 20.8(1.72)(0.80) + 20.8(3.42)(0.60) = 71.4 \text{ V}$$

Substitution of numerical values shows that $V_2 = 2329$ V, referred to the high-voltage side. The actual voltage at the secondary terminals is 2329/10, or

$$V_2 = 233 \text{ V}$$

Practice Problem 2.3

Repeat Example 2.5 for a load which draws rated current from the transformer with a power factor of 0.8 leading.

Solution

$$V_2 = 239 \text{ V}$$

Two very simple tests serve to determine the parameters of the equivalent circuits of Fig. 2.10 and 2.12. These consist of measuring the input voltage, current, and power at one side of the transformer, first with the second side short-circuited and then with

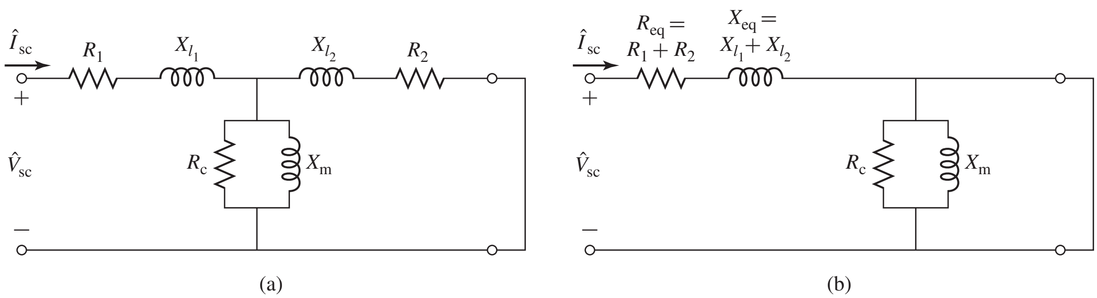

Figure 2.15 Equivalent circuit with short-circuited secondary. (a) Complete equivalent circuit. (b) Cantilever equivalent circuit with the exciting branch at the transformer secondary.

the second side open-circuited. Note that, following common practice, the transformer voltage ratio is used as the turns ratio when referring parameters from side to side for the purposes of parameter determination.

Short-Circuit Test The *short-circuit test* can be used to find the equivalent series impedance $R_{eq} + jX_{eq}$. Although the choice of winding to short-circuit is arbitrary, for the sake of this discussion we will consider the short circuit to be applied to the transformer secondary and voltage applied to primary. For convenience, the high-voltage side is usually taken as the primary in this test. Because the equivalent series impedance in a typical transformer is relatively small, typically an applied primary voltage on the order of 10 to 15 percent or less of the rated value will result in rated current.

Figure 2.15a shows the equivalent circuit with transformer secondary impedance referred to the primary side and a short circuit applied to the secondary. The short-circuit impedance Z_{sc} looking into the primary under these conditions is

$$Z_{sc} = R_1 + jX_{l_1} + \frac{Z_\varphi(R_2 + jX_{l_2})}{Z_\varphi + R_2 + jX_{l_2}} \tag{2.29}$$

Because the impedance Z_φ of the exciting branch is much larger than that of the secondary leakage impedance (which will be true unless the core is heavily saturated by excessive voltage applied to the primary; certainly not the case here), the short-circuit impedance can be approximated as

$$Z_{sc} \approx R_1 + jX_{l_1} + R_2 + jX_{l_2} = R_{eq} + jX_{eq} \tag{2.30}$$

Note that the approximation made here is equivalent to the approximation made in reducing the equivalent-T circuit to the cantilever equivalent. This can be seen from Fig. 2.15b; the impedance seen at the input of this equivalent circuit is clearly $Z_{sc} = Z_{eq} = R_{eq} + jX_{eq}$ since the exciting branch is directly shorted out by the short on the secondary.

Typically the instrumentation used for this test will measure the rms magnitude of the applied voltage V_{sc}, the short-circuit current I_{sc}, and the power P_{sc}. Based upon these three measurements, the equivalent resistance and reactance (referred to the primary) can be found from

$$|Z_{\mathrm{eq}}| = |Z_{\mathrm{sc}}| = \frac{V_{\mathrm{sc}}}{I_{\mathrm{sc}}} \tag{2.31}$$

$$R_{\mathrm{eq}} = R_{\mathrm{sc}} = \frac{P_{\mathrm{sc}}}{I_{\mathrm{sc}}^2} \tag{2.32}$$

$$X_{\mathrm{eq}} = X_{\mathrm{sc}} = \sqrt{|Z_{\mathrm{sc}}|^2 - R_{\mathrm{sc}}^2} \tag{2.33}$$

where the notation | | indicates the magnitude of the enclosed complex quantity. The equivalent impedance can, of course, be referred from one side to the other in the usual manner.

Note that the short-circuit test does not provide sufficient information to determine the individual leakages impedances of the primary and secondary windings. On the occasions when the equivalent-T circuit in Fig. 2.10d must be used, approximate values of the individual primary and secondary resistances and leakage reactances can be obtained by assuming that $R_1 = R_2 = 0.5R_{\mathrm{eq}}$ and $X_{\mathrm{l}_1} = X_{\mathrm{l}_2} = 0.5X_{\mathrm{eq}}$ when all impedances are referred to the same side. Strictly speaking, of course, it is possible to measure R_1 and R_2 directly by a dc resistance measurement on each winding (and then referring one or the other to the other side of the ideal transformer). However, as has been discussed, no such simple test exists for the leakage reactances X_{l_1} and X_{l_2}.

Open-Circuit Test The *open-circuit test* is performed with the secondary open-circuited and a voltage impressed on the primary. Under this condition an exciting current of a few percent of full-load current (less on large transformers and more on smaller ones) is obtained. Typically, the test is conducted at rated voltage to insure that the core, and hence the magnetizing reactance, will be operating at a flux level close to that which will exist under normal operating conditions. If the transformer is to be used at other than its rated voltage, the test should be done at that voltage. For convenience, the low-voltage side is usually taken as the primary in this test. If the primary is chosen to be the opposite winding from that of the short-circuit test, one must of course be careful to refer the various measured impedances to the same side of the transformer in order to obtain a self-consistent set of parameter values.

Figure 2.16a shows the equivalent circuit with the transformer secondary impedance referred to the primary side and with the secondary open-circuited. The open-circuit impedance Z_{oc} looking into the primary under these conditions is

$$Z_{\mathrm{oc}} = R_1 + jX_{l_1} + Z_\varphi = R_1 + jX_{l_1} + \frac{R_{\mathrm{c}}\,(jX_{\mathrm{m}})}{R_{\mathrm{c}} + jX_{\mathrm{m}}} \tag{2.34}$$

Because the impedance of the exciting branch is quite large, the voltage drop in the primary leakage impedance caused by the exciting current is typically negligible, and the primary impressed voltage $\hat{V}_{\mathrm{oc}}$ very nearly equals the emf $\hat{E}_{\mathrm{oc}}$ induced by the resultant core flux. Similarly, the primary $I_{\mathrm{oc}}^2 R_1$ loss caused by the exciting current is negligible, so that the power input P_{oc} very nearly equals the core loss $E_{\mathrm{oc}}^2/R_{\mathrm{c}}$. As a result, it is common to ignore the primary leakage impedance and to approximate the

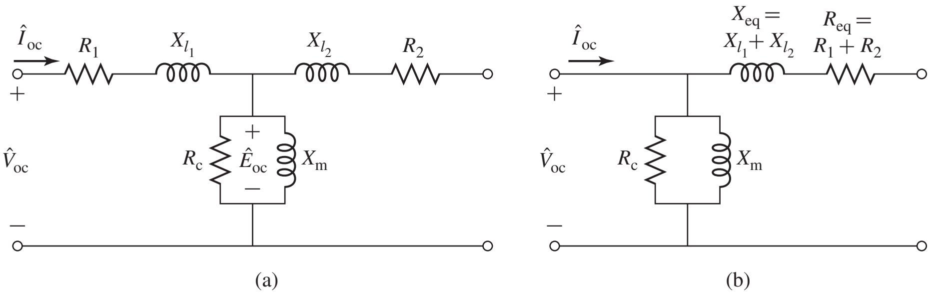

Figure 2.16 Equivalent circuit with open-circuited secondary. (a) Complete equivalent circuit. (b) Cantilever equivalent circuit with the exciting branch at the transformer primary.

open-circuit impedance as being equal to the magnetizing impedance

$$Z_{oc} \approx Z_{\varphi} = \frac{R_c(jX_m)}{R_c + jX_m} \tag{2.35}$$

Note that the approximation made here is equivalent to the approximation made in reducing the equivalent-T circuit to the cantilever equivalent circuit of Fig. 2.16b; the impedance seen at the input of this equivalent circuit is clearly Z_{φ} since no current will flow in the open-circuited secondary.

As with the short-circuit test, typically the instrumentation used for this test will measure the rms magnitude of the applied voltage, V_{oc}, the open-circuit current I_{oc}, and the power P_{oc}. Neglecting the primarily leakage impedance and based upon these three measurements, the magnetizing resistance and reactance (referred to the primary) can be found from

$$R_c = \frac{V_{oc}^2}{P_{oc}} \tag{2.36}$$

$$|Z_{\varphi}| = \frac{V_{oc}}{I_{oc}} \tag{2.37}$$

$$X_m = \frac{1}{\sqrt{(1/|Z_{\varphi}|)^2 - (1/R_c)^2}} \tag{2.38}$$

The values obtained are, of course, referred to the side used as the primary in this test.

The open-circuit test can be used to obtain the core loss for efficiency computations and to check the magnitude of the exciting current. Sometimes the voltage at the terminals of the open-circuited secondary is measured as a check on the turns ratio.

Note that, if desired, a slightly more accurate calculation of X_m and R_c can be found by retaining the measurements of R_1 and X_{l_1} obtained from the short-circuit test (referred to the proper side of the transformer) and basing the derivation on Eq. 2.34. However, such additional effort is rarely necessary for the purposes of engineering accuracy.

EXAMPLE 2.6

With the instruments located on the high-voltage side and with the low-voltage side short-circuited, the short-circuit test readings for the 50-kVA 2400:240-V transformer of Example 2.3 are 48 V, 20.8 A, and 617 W. An open-circuit test with the low-voltage side energized gives instrument readings on that side of 240 V, 5.41 A, and 186 W. Determine the efficiency and the voltage regulation of the transformer operating at full load, 0.80 power factor lagging.

■ Solution

From the short-circuit test, the magnitude of the equivalent impedance, the equivalent resistance, and the equivalent reactance of the transformer (referred to the high-voltage side as denoted by the subscript H) are

$$|Z_{\text{eq,H}}| = \frac{48}{20.8} = 2.31\ \Omega \qquad R_{\text{eq,H}} = \frac{617}{20.8^2} = 1.42\ \Omega$$

$$X_{\text{eq,H}} = \sqrt{2.31^2 - 1.42^2} = 1.82\ \Omega$$

Operation at full-load, 0.80 power factor lagging corresponds to a current of

$$I_{\text{H}} = \frac{50000}{2400} = 20.8\ \text{A}$$

and an output power

$$P_{\text{output}} = P_{\text{load}} = (0.8)50000 = 40000\ \text{W}$$

Note that the short-circuit test was conducted at rated current and hence the full-load I^2R loss will equal that of the short-circuit test. Similarly, the open-circuit test was conducted at rated voltage and hence the full-load core loss is equal to that of the open-circuit test. As a result, the total loss under this operating condition is equal to the sum of the winding loss

$$P_{\text{winding}} = I_{\text{H}}^2 R_{\text{eq,H}} = 20.8^2(1.42) = 617\ \text{W}$$

and the open-circuit core loss

$$P_{\text{core}} = 186\ \text{W}$$

Thus

$$P_{\text{loss}} = P_{\text{winding}} + P_{\text{core}} = 803\ \text{W}$$

and the power input to the transformer is

$$P_{\text{input}} = P_{\text{output}} + P_{\text{loss}} = 40803\ \text{W}$$

The *efficiency* of a power conversion device is defined as

$$\text{efficiency} = \frac{P_{\text{output}}}{P_{\text{input}}} = \frac{P_{\text{input}} - P_{\text{loss}}}{P_{\text{input}}} = 1 - \frac{P_{\text{loss}}}{P_{\text{input}}}$$

which can be expressed in percent by multiplying by 100 percent. Hence, for this operating condition

$$\text{efficiency} = 100\% \left(\frac{P_{\text{output}}}{P_{\text{input}}}\right) = 100\% \left(\frac{40000}{40000 + 803}\right) = 98.0\%$$

The *voltage regulation* of a transformer is defined as the change in secondary terminal voltage from no load to full load and is usually expressed as a percentage of the full-load value. In power systems applications, regulation is one figure of merit for a transformer; a low value indicates that load variations on the secondary of that transformer will not significantly affect the magnitude of the voltage being supplied to the load. It is calculated under the assumption that the primary voltage remains constant as the load is removed from the transformer secondary.

The equivalent circuit of Fig. 2.12c will be used with all quantities referred to the high-voltage side. The primary voltage is assumed to be adjusted so that the secondary terminal voltage has its rated value at full load, or $V_{2\mathrm{H}} = 2400$ V. For a load of rated value and 0.8 power factor lagging (corresponding to a power-factor angle $\theta = -\cos^{-1}(0.8) = -36.9^\circ$), the load current will be

$$\hat{I}_{\mathrm{H}} = \left(\frac{50 \times 10^3}{2400}\right) e^{-j36.9^\circ} = 20.8\, e^{-j36.9^\circ} = 16.6 - j12.5 \text{ A}$$

The required value of the primary voltage $V_{1\mathrm{H}}$ can be calculated as

$$\begin{aligned}\hat{V}_{1\mathrm{H}} &= \hat{V}_{2\mathrm{H}} + \hat{I}_{\mathrm{H}}(R_{\mathrm{eq,H}} + jX_{\mathrm{eq,H}}) \\ &= 2400 + (16.6 - j12.5)\,(1.42 + j1.82) \\ &= 2446\, e^{j0.29^\circ} \text{ V}\end{aligned}$$

The magnitude of $\hat{V}_{1\mathrm{H}}$ is 2446 V. If this voltage were held constant and the load removed, the secondary voltage on open circuit would rise to 2446 V referred to the high-voltage side. Then

$$\text{Regulation} = \left(\frac{2446 - 2400}{2400}\right) \times 100\% = 1.92\%$$

Practice Problem 2.4

Repeat the voltage-regulation calculation of Example 2.6 for a load of 50 kW (rated load, unity power factor).

Solution

$$\text{Regulation} = 1.24\%$$

2.6 AUTOTRANSFORMERS; MULTIWINDING TRANSFORMERS

The principles discussed in previous sections have been developed with specific reference to two-winding transformers. They are also applicable to transformers with other winding configurations. Aspects relating to autotransformers and multiwinding transformers are considered in this section.

2.6.1 Autotransformers

In Fig. 2.17a, a two-winding transformer is shown with N_1 and N_2 turns on the primary and secondary windings respectively. Substantially the same transformation effect on voltages, currents, and impedances can be obtained when these windings are connected as shown in Fig. 2.17b. However, note that in Fig. 2.17b, winding *bc* is

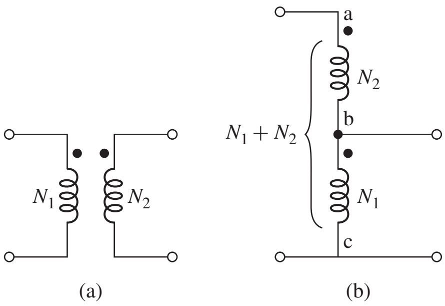

Figure 2.17 (a) Two-winding transformer. (b) Connection as an autotransformer.

common to both the primary and secondary circuits. This type of transformer is called an *autotransformer*. It is similar to a normal transformer connected in a special way, with the exception that the windings must be appropriately insulated for the operating operating voltage.

One important difference between the two-winding transformer and the autotransformer is that the windings of the two-winding transformer are electrically isolated whereas those of the autotransformer are connected directly together. Also, in the autotransformer connection, winding *ab* must be provided with extra insulation since it must be insulated against the full maximum voltage of the autotransformer. Autotransformers have lower leakage reactances, lower losses, and smaller exciting current and cost less than two-winding transformers when the voltage ratio does not differ too greatly from 1:1.

The following example illustrates the benefits of an autotransformer for those situations where electrical isolation between the primary and secondary windings is not an important consideration.

EXAMPLE 2.7

The 2400:240-V 50-kVA transformer of Example 2.6 is connected as an autotransformer, as shown in Fig. 2.18a, in which ab is the 240-V winding and bc is the 2400-V winding. (It is assumed that the 240-V winding has enough insulation to withstand a voltage of 2640 V to ground.)

a. Compute the voltage ratings V_H and V_X of the high- and low-voltage sides, respectively, for this autotransformer connection.
b. Compute the kVA rating as an autotransformer.
c. Data with respect to the losses are given in Example 2.6. Compute the full-load efficiency as an autotransformer operating with a rated load of 0.80 power factor lagging.

■ Solution

a. Since the 2400-V winding bc is connected to the low-voltage circuit, $V_L = 2400$ V. When $V_{bc} = 2400$ V, a voltage $V_{ab} = 240$ V in phase with V_{bc} will be induced in winding ab

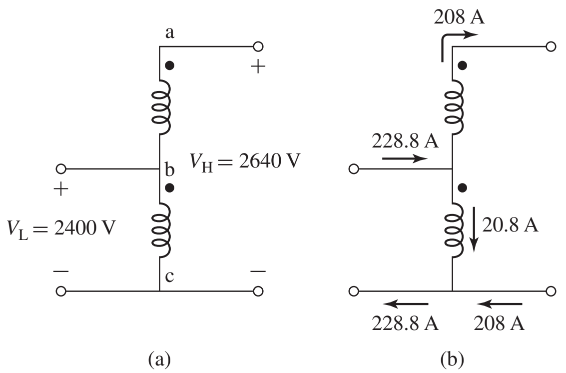

Figure 2.18 (a) Autotransformer connection for Example 2.7. (b) Currents under rated load.

(leakage-impedance voltage drops being neglected). The voltage of the high-voltage side therefore is

$$V_H = V_{ab} + V_{bc} = 2640 \text{ V}$$

b. From the rating of 50 kVA as a normal two-winding transformer, the rated current of the 240-V winding is 50,000/240 = 208 A. Since the high-voltage lead of the autotransformer is connected to the 240-V winding, the rated current I_H at the high-voltage side of the autotransformer is equal to the rated current of the 240-V winding or 208 A. The kVA rating as an autotransformer therefore is

$$\frac{V_H I_H}{1000} = \frac{2640 \times 208}{1000} = 550 \text{ kVA}$$

Note that, in this connection, the autotransformer has an equivalent turns ratio of 2640/2400. Thus the rated current at the low-voltage winding (the 2400-V winding in this connection) must be

$$I_L = \left(\frac{2640}{2400}\right) 208 \text{ A} = 229 \text{ A}$$

At first, this seems rather unsettling since the 2400-V winding of the transformer has a rated current of 50 kVA/2400 V = 20.8 A. Further puzzling is that fact that this transformer, whose rating as a normal two-winding transformer is 50 kVA, is capable of handling 550 kVA as an autotransformer.

The higher rating as an autotransformer is a consequence of the fact that not all the 550 kVA has to be transformed by electromagnetic induction. In fact, all that the transformer has to do is to boost a current of 208 A through a potential rise of 240 V, corresponding to a power transformation capacity of 50 kVA. This fact is perhaps best illustrated by Fig. 2.18b which shows the currents in the autotransformer under rated conditions. Note that the windings carry only their rated currents in spite of higher rating of the transformer.

c. When it is connected as an autotransformer with the currents and voltages shown in Fig. 2.18, the losses are the same as in Example 2.6, namely 803 W. But the output as an autotransformer at full load, 0.80 power factor is 0.80 × 550,000 = 440,000 W. The efficiency therefore is

$$\left(1 - \frac{803}{440{,}803}\right) 100\% = 99.82\%$$

The efficiency is so high because the losses are those corresponding to transforming only 50 kVA.

Practice Problem 2.5

A 450-kVA, 460-V:7.97-kV transformer has an efficiency of 97.8 percent when supplying a rated load of unity power factor. If it is connected as a 7.97:8.43-kV autotransformer, calculate its rated terminal currents, rated kVA, and efficiency when supplying a unity-power-factor load.

Solution

The rated current at the 8.43-kV terminal is 978 A, at the 7.97-kV terminal is 1034 A and the transformer rating is 8.25 MVA. Its efficiency supplying a rated, unity-power-factor load is 99.88 percent.

From Example 2.7, we see that when a transformer is connected as an autotransformer as shown in Fig. 2.17, the rated voltages of the autotransformer can be expressed in terms of those of the two-winding transformer as

Low-voltage:

$$V_{\mathrm{L}_{\mathrm{rated}}} = V_{1_{\mathrm{rated}}} \tag{2.39}$$

High-voltage:

$$V_{\mathrm{H}_{\mathrm{rated}}} = V_{1_{\mathrm{rated}}} + V_{2_{\mathrm{rated}}} = \left(\frac{N_1 + N_2}{N_1}\right) V_{\mathrm{L}_{\mathrm{rated}}} \tag{2.40}$$

The effective turns ratio of the autotransformer is thus $(N_1 + N_2)/N_1$. In addition, the power rating of the autotransformer is equal to $(N_1 + N_2)/N_2$ times that of the two-winding transformer, although the actual power processed by the transformer will not increase over that of the standard two-winding connection.

2.6.2 Multiwinding Transformers

Transformers having three or more windings, known as *multiwinding* or *multicircuit transformers,* are often used to interconnect three or more circuits which may have different voltages. For these purposes a multiwinding transformer costs less and is more efficient than an equivalent number of two-winding transformers. Transformers having a primary and multiple secondaries are frequently found in multiple-output dc power supplies for electronic applications. Distribution transformers used to supply power for domestic purposes usually have two 120-V secondaries connected in series.

Circuits for lighting and low-power applications are connected across each of the 120-V windings, while electric ranges, domestic hot-water heaters, clothes-dryers, and other high-power loads are supplied with 240-V power from the series-connected secondaries.

Similarly, a large distribution system may be supplied through a three-phase bank of multiwinding transformers from two or more transmission systems having different voltages. In addition, the three-phase transformer banks used to interconnect two transmission systems of different voltages often have a third, or tertiary, set of windings to provide voltage for auxiliary power purposes in substations or to supply a local distribution system. Static capacitors or synchronous condensers may be connected to the tertiary windings for power factor correction or voltage regulation. Sometimes Δ-connected tertiary windings are put on three-phase banks to provide a low-impedance path for third harmonic components of the exciting current to reduce third-harmonic components of the neutral voltage.

Some of the issues arising in the use of multiwinding transformers are associated with the effects of leakage impedances on voltage regulation, short-circuit currents, and division of load among circuits. These problems can be solved by an equivalent-circuit technique similar to that used in dealing with two-circuit transformers.

The equivalent circuits of multiwinding transformers are more complicated than in the two-winding case because they must take into account the leakage impedances associated with each pair of windings. Typically, in these equivalent circuits, all quantities are referred to a common base, either by use of the appropriate turns ratios as referring factors or by expressing all quantities in per unit. The exciting current usually is neglected.

2.7 TRANSFORMERS IN THREE-PHASE CIRCUITS

Three single-phase transformers can be connected to form a *three-phase transformer bank* in any of the four ways shown in Fig. 2.19. In all four parts of this figure, the windings at the left are the primaries, those at the right are the secondaries, and each primary winding in one transformer corresponds to the secondary winding drawn parallel to it. Also shown are the voltages and currents resulting from balanced impressed primary line-to-line voltages V and line currents I when the ratio of primary-to-secondary turns $N_1/N_2 = a$ and ideal transformers are assumed.[5] Note that the rated voltage and current at the primary and secondary of the three-phase transformer bank depend upon the connection used but that the rated kVA of the three-phase bank is three times that of the individual single-phase transformers, regardless of the connection.

The Y-Δ connection is commonly used in stepping down from a high voltage to a medium or low voltage. One reason is that a neutral is thereby provided for grounding on the high-voltage side, a procedure which can be shown to be desirable in many cases. Conversely, the Δ-Y connection is commonly used for stepping up to

[5] The relationship between three-phase and single-phase quantities is discussed in Appendix A.

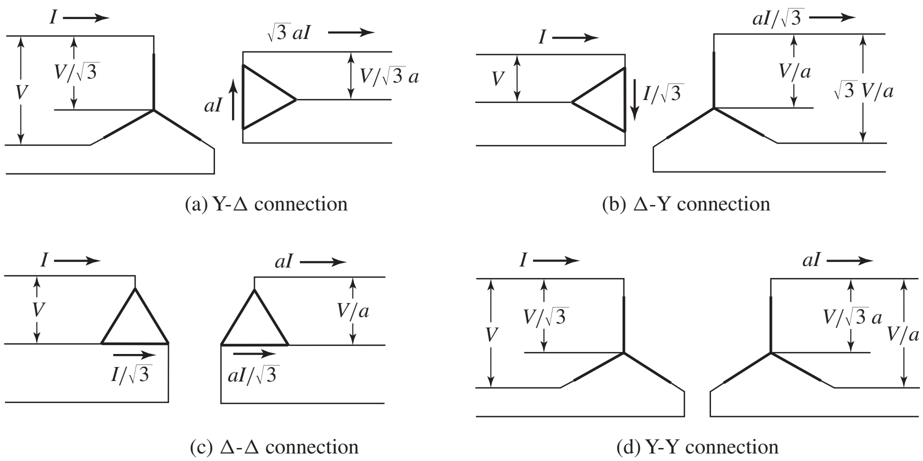

Figure 2.19 Common three-phase transformer connections; the transformer windings are indicated by the heavy lines.

a high voltage. The Δ-Δ connection has the advantage that one transformer can be removed for repair or maintenance while the remaining two continue to function as a three-phase bank with the rating reduced to 58 percent of that of the original bank; this is known as the *open-delta,* or V, connection. The Y-Y connection is seldom used because of difficulties with exciting-current phenomena.[6]

Instead of three single-phase transformers, a three-phase bank may consist of one *three-phase transformer* having all six windings on a common multi-legged core and contained in a single tank. Advantages of three-phase transformers over connections of three single-phase transformers are that they cost less, weigh less, require less floor space, and have somewhat higher efficiency. A photograph of the internal parts of a three-phase transformer is shown in Fig. 2.20.

Circuit computations involving three-phase transformer banks under balanced conditions can be made by dealing with only one of the transformers or phases and recognizing that conditions are the same in the other two phases except for the phase displacements associated with a three-phase system. It is usually convenient to carry out the computations on a single-phase (per-phase-Y, line-to-neutral) basis, since transformer impedances can then be added directly in series with transmission line impedances. The impedances of transmission lines can be referred from one side of the transformer bank to the other by use of the square of the ideal line-to-line voltage ratio of the bank. In dealing with Y-Δ or Δ-Y banks, all quantities can be referred to the Y-connected side. In dealing with Δ-Δ banks in series with transmission lines, it is convenient to replace the Δ-connected impedances of the transformers by equivalent Y-connected impedances. It can be shown that a balanced Δ-connected circuit of

[6] Because there is no neutral connection to carry harmonics of the exciting current, harmonic voltages are produced which significantly distort the transformer voltages.

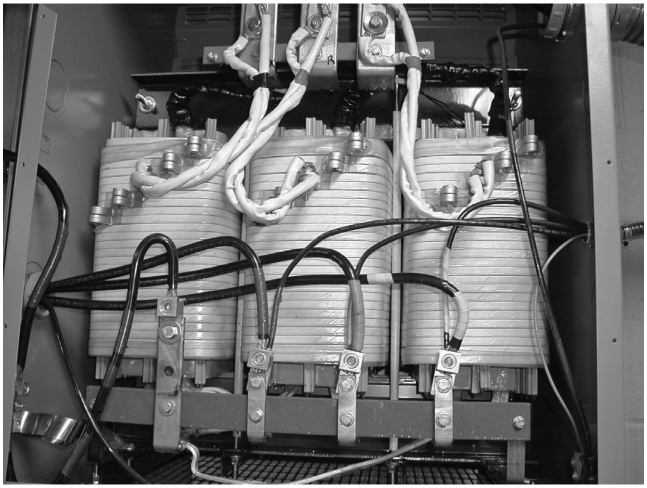

Figure 2.20 Internal view of a three-phase, 480V-Y/208V-Δ, 112-kVA transformer.

Z_Δ Ω/phase is equivalent to a balanced Y-connected circuit of Z_Y Ω/phase if

$$Z_Y = \frac{1}{3} Z_\Delta \tag{2.41}$$

EXAMPLE 2.8

Three single-phase, 50-kVA 2400:240-V transformers, each identical with that of Example 2.6, are connected Y-Δ in a three-phase 150-kVA bank to step down the voltage at the load end of a feeder whose impedance is $0.15 + j1.00$ Ω/phase. The voltage at the sending end of the feeder is 4160 V line-to-line. On their secondary sides, the transformers supply a balanced three-phase load through a feeder whose impedance is $0.0005 + j0.0020$ Ω/phase. Find the line-to-line voltage at the load when the load draws rated current from the transformers at a power factor of 0.80 lagging.

■ Solution

For the given connection, the rated line-line voltage at the high-voltage terminals of the three-phase transformer bank will $\sqrt{3}\, 2400 \approx 4160$ V. Thus, the transformer bank will have an rated turns ratio of 4160/240. The computations can be made on a single-phase basis by referring everything to the high-voltage, Y-connected side of the transformer bank. The voltage at the sending end of the feeder is equivalent to a source voltage V_s of

$$V_s = \frac{4160}{\sqrt{3}} \approx 2400 \text{ V line-to-neutral}$$

From the transformer rating, the rated current on the high-voltage side is 20.8 A/phase Y. The low-voltage feeder impedance referred to the high voltage side by means of the square of the rated turns ratio

$$Z_{\mathrm{lv,H}} = \left(\frac{4160}{240}\right)^2 (0.0005 + j0.0020) = 0.15 + j0.60\ \Omega$$

and the combined series impedance of the high- and low-voltage feeders referred to the high-voltage side is thus

$$Z_{\text{feeder,H}} = 0.30 + j1.60\ \Omega/\text{phase Y}$$

Because the transformer bank is Y-connected on its high-voltage side, its equivalent single-phase series impedance is equal to the single-phase series impedance of each single-phase transformer as referred to its high-voltage side. This impedance was originally calculated in Example 2.4 as

$$Z_{\text{eq,H}} = 1.42 + j1.82\ \Omega/\text{phase Y}$$

Due to the choice of values selected for this example, the single-phase equivalent circuit for the complete system is identical to that of Example 2.5, as can been seen with specific reference to Fig. 2.14a. In fact, the solution on a per-phase basis is exactly the same as the solution to Example 2.5, whence the load voltage referred to the high-voltage side is 2329 V to neutral. The actual line-neutral load voltage can then be calculated by referring this value to the low-voltage side of the transformer bank as

$$V_{\text{load}} = 2329\left(\frac{240}{4160}\right) = 134\ \text{V line-to-neutral}$$

which can be expressed as a line-to-line voltage by multiplying by $\sqrt{3}$

$$V_{\text{load}} = 134\sqrt{3} = 233\ \text{V line-to-line}$$

Note that this line-line voltage is equal to the line-neutral load voltage calculated in Example 2.5 because in this case the transformers are delta connected on their low-voltage side and hence the line-line voltage on the low-voltage side is equal to the low-voltage terminal voltage of the transformers.

Practice Problem 2.6

Repeat Example 2.8 with the transformers connected Y-Y and all other aspects of the problem statement remaining unchanged.

Solution

405 V line-line

EXAMPLE 2.9

The three transformers of Example 2.8 are reconnected Δ-Δ and supplied with power through a 2400-V (line-to-line) three-phase feeder whose reactance is 0.80 Ω/phase as shown in Fig. 2.21. At its sending end, the feeder is connected to the secondary terminals of a three-phase Y-Δ-connected transformer whose rating is 500 kVA, 24 kV:2400 V (line-to-line). The equivalent series impedance of the sending-end transformer is $0.17 + j0.92$ Ω/phase referred to the 2400-V side. The voltage applied to the primary terminals of the sending-end transformer is 24.0 kV line-to-line.

A three-phase short circuit occurs at the 240-V terminals of the receiving-end transformers. Compute the steady-state short-circuit current in the 2400-V feeder phase wires, in the primary and secondary windings of the receiving-end transformers, and at the 240-V terminals.

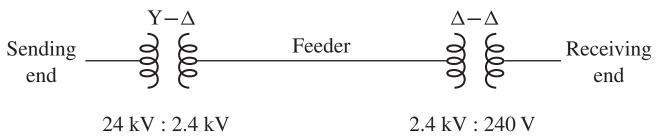

Figure 2.21 One-line diagram for Example 2.9 .

■ Solution

The computations will be made on an equivalent line-to-neutral basis with all quantities referred to the 2400-V feeder. The source voltage then is

$$\frac{2400}{\sqrt{3}} = 1385 \text{ V line-to-neutral}$$

From Eq. 2.41, the single-phase-equivalent series impedance of the Δ-Δ transformer seen at its 2400-V side is

$$Z_{\mathrm{eq}} = R_{\mathrm{eq}} + jX_{\mathrm{eq}} = \frac{1.42 + j1.82}{3} = 0.47 + j0.61 \ \Omega/\text{phase}$$

The total series impedance to the short circuit is then the sum of this impedance, that of sending-end transformer and the reactance of the feeder

$$Z_{\mathrm{tot}} = (0.47 + j0.61) + (0.17 + j0.92) + j0.80 = 0.64 + j2.33 \ \Omega/\text{phase}$$

which has a magnitude of

$$|Z_{\mathrm{tot}}| = 2.42 \ \Omega/\text{phase}$$

The magnitude of the phase current in the 2400-V feeder can now simply be calculated as the line-neutral voltage divided by the magnitude of the series impedance

$$\text{Current in 2400-V feeder} = \frac{1385}{2.42} = 572 \text{ A}$$

and, as is shown in Fig. 2.19c, the winding current in the 2400-V winding of the receiving-end transformer is equal to the phase current divided by $\sqrt{3}$ or

$$\text{Current in 2400-V windings} = \frac{572}{\sqrt{3}} = 330 \text{ A}$$

while the current in the 240-V windings is 10 times this value

$$\text{Current in 240-V windings} = 10 \times 330 = 3300 \text{ A}$$

Finally, again with reference to Fig. 2.19c, the phase current at the 240-V terminals into the short circuit is given by

$$\text{Current at the 240-V terminals} = 3300\sqrt{3} = 5720 \text{ A}$$

Note of course that this same result could have been computed simply by recognizing that the turns ratio of the Δ-Δ transformer bank is equal to 10:1 and hence, under balanced-three-phase conditions, the phase current on the low voltage side will be 10 times that on the high-voltage side.

Practice Problem 2.7

Repeat Example 2.9 under the condition that the three transformers are connected Δ-Y instead of Δ-Δ such that the short low-voltage side of the three-phase transformer is rated 416 V line-to-line.

Solution

Current in 2400-V feeder = 572 A

Current in 2400-V windings = 330 A

Current in 416-V windings = 3300 A

Current at the 416-V terminals = 3300A

2.8 VOLTAGE AND CURRENT TRANSFORMERS

Transformers are often used in instrumentation applications to match the magnitude of a voltage or current to the range of a meter or other instrumentation. For example, most 60-Hz power-systems' instrumentation is based upon voltages in the range of 0-120 V rms and currents in the range of 0–5 A rms. Since power system voltages range up to 765-kV line-to-line and currents can be 10s of kA, some method of supplying an accurate, low-level representation of these signals to the instrumentation is required.

One common technique is through the use of specialized transformers known as *potential transformers* or *PTs* and *current transformers* or *CTs*. If constructed with a turns ratio of N_1:N_2, an ideal potential transformer would have a secondary voltage equal in magnitude to N_2/N_1 times that of the primary and identical in phase. Similarly, an ideal current transformer would have a secondary output current equal to N_1/N_2 times the current input to the primary, again identical in phase. In other words, potential and current transformers (also referred to as *instrumentation transformers*) are designed to approximate ideal transformers as closely as is practically possible.

The equivalent circuit of Fig. 2.22 shows a transformer loaded with an impedance $Z_\text{b} = R_\text{b} + jX_\text{b}$ at its secondary. For the sake of this discussion, the core-loss resistance R_c has been neglected; if desired, the analysis presented here can be easily expanded to include its effect. Following conventional terminology, the load on an instrumentation transformer is frequently referred to as the *burden* on that transformer, hence the subscript ′b′. To simplify our discussion, we have chosen to refer all the secondary quantities to the primary side of the ideal transformer.

Consider first a potential transformer. Ideally it should accurately measure voltage while appearing as an open circuit to the system under measurement, i.e., drawing negligible current and power. Thus, its load impedance should be "large" in a sense we will now quantify.

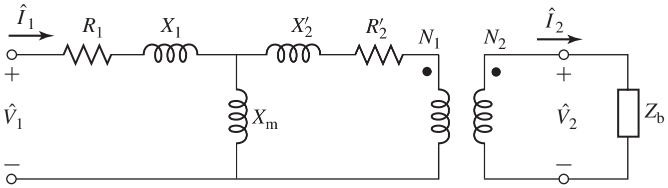

Figure 2.22 Equivalent circuit for an instrumentation transformer.

First, let us assume that the transformer secondary is open-circuited (i.e., $|Z_b| = \infty$). In this case we can write that

$$\frac{\hat{V}_2}{\hat{V}_1} = \left(\frac{N_2}{N_1}\right) \frac{jX_m}{R_1 + j(X_1 + X_m)} \tag{2.42}$$

From this equation, we see that a potential transformer with an open-circuited secondary has an inherent error (in both magnitude and phase) due to the voltage drop of the magnetizing current through the primary resistance and leakage reactance. To the extent that the primary resistance and leakage reactance can be made small compared to the magnetizing reactance, this inherent error can be made quite small.

The situation is worsened by the presence of a finite burden. Including the effect of the burden impedance, Eq. 2.42 becomes

$$\frac{\hat{V}_2}{\hat{V}_1} = \left(\frac{N_2}{N_1}\right) \frac{Z_{eq}Z_b'}{(R_1 + jX_1)(Z_{eq} + Z_b' + R_2' + jX_2')} \tag{2.43}$$

where

$$Z_{eq} = \frac{jX_m(R_1 + jX_1)}{R_1 + j(X_m + X_1)} \tag{2.44}$$

and

$$Z_b' = \left(\frac{N_1}{N_2}\right)^2 Z_b \tag{2.45}$$

is the burden impedance referred to the transformer primary.

From these equations, it can be seen that the characteristics of an accurate potential transformer include a large magnetizing reactance (more accurately, a large magnetizing impedance since the effects of core loss, although neglected in the analysis presented here, must also be minimized) and relatively small winding resistances and leakage reactances. Finally, as will be seen in Example 2.10, the burden impedance must be kept above a minimum value to avoid introducing excessive errors in the magnitude and phase angle of the measured voltage.

EXAMPLE 2.10

A 2400:120-V, 60-Hz potential transformer has the following parameter values (referred to the 2400-V winding):

$$X_1 = 143\ \Omega \qquad X_2' = 164\ \Omega \qquad X_m = 163\ \text{k}\Omega$$

$$R_1 = 128\ \Omega \qquad R_2' = 141\ \Omega$$

(a) Assuming a 2400-V input, which ideally should produce a voltage of 120 V at the low-voltage winding, calculate the magnitude and relative phase-angle errors of the secondary voltage if the secondary winding is open-circuited. (b) Assuming the burden impedance to be purely resistive ($Z_b = R_b$), calculate the minimum resistance (maximum burden) that can be applied to the secondary such that the magnitude error is less than 0.5 percent. (c) Repeat part (b) but find the minimum resistance such that the phase-angle error is less than 1.0 degree.

■ Solution

a. This problem is most easily solved using MATLAB.† From Eq. 2.42 with $\hat{V}_1 = 2400$ V, the following MATLAB script gives

$$\hat{V}_2 = 119.90 \angle 0.045^\circ \text{ V}$$

which corresponds to a magnitude error of less than 0.1% and a phase angle error of 0.045°.

Here is the MATLAB script:

```
clc
clear

%PT parameters
R1 = 128;
X1 = 143;
Xm = 163e3;
N1 = 2400;
N2 = 120;
N = N1/N2;

%Primary voltage
V1 = 2400;

%Secondary voltage
V2 = V1*(N2/N1)*(j*Xm/(R1+ j*(X1+Xm)));
magV2 = abs(V2);
phaseV2 = 180*angle(V2)/pi;

fprintf('\nMagnitude of V2 = %g [V]',magV2)
fprintf('\n    and angle = %g [degrees]\n\n',phaseV2)
```

b. Here, again, it is relatively straight forward to write a MATLAB script to implement Eq. 2.43 and to calculate the percentage error in the magnitude of voltage $\hat{V}_2$ as compared to the 120 Volts that would be measured if the PT were ideal. The resistive burden R_b can be initialized to a large value and then reduced until the magnitude error reaches

† MATLAB is a registered trademark of The MathWorks, Inc.

0.5 percent. The result of such an analysis would show that the minimum resistance is 162.5 Ω, corresponding to a magnitude error of 0.50 percent and a phase angle of 0.22°. (Note that this appears as a resistance of 65 kΩ when referred to the primary.)

c. The MATLAB script of part (*b*) can be modified to search for the minimum resistive burden that will keep the phase angle error less than 1.0 degrees. The result would show that the minimum resistance is 41.4 Ω, corresponding to a phase angle of 1.00° and a magnitude error of 1.70 percent.

Practice Problem 2.8

Using MATLAB, repeat parts (*b*) and (*c*) of Example 2.10 assuming the burden impedance is purely reactive ($Z_\mathrm{b} = jX_\mathrm{b}$) and finding the corresponding minimum impedance X_b in each case.

Solution

The minimum burden reactance which results in a secondary voltage magnitude within 0.5 percent of the expected 120 V is $X_\mathrm{b} = 185.4\ \Omega$, for which the phase angle is 0.25°. The minimum burden reactance which results in a secondary voltage phase-angle of within 1.0° of that of the primary voltage is $X_\mathrm{b} = 39.5\ \Omega$, for which the voltage-magnitude error is 2.0 percent.

Consider next a current transformer. An ideal current transformer would accurately measure current while appearing as a short circuit to the system under measurement, i.e., developing negligible voltage drop and drawing negligible power. Thus, its load impedance should be "small" in a sense we will now quantify.

Let us begin with the assumption that the transformer secondary is short-circuited (i.e., $|Z_\mathrm{b}| = 0$). In this case we can write that

$$\frac{\hat{I}_2}{\hat{I}_1} = \left(\frac{N_1}{N_2}\right) \frac{jX_\mathrm{m}}{R_2' + j(X_2' + X_\mathrm{m})} \tag{2.46}$$

Based upon an argument similar to that used in the discussion of a potential transformer, Eq. 2.46 shows that a current transformer with a shorted secondary has an inherent error (in both magnitude and phase) due to the fact that some of the primary current is shunted through the magnetizing reactance and does not reach the secondary. To the extent that the magnetizing reactance can be made large in comparison to the secondary resistance and leakage reactance, this error can be made quite small.

A finite burden appears in series with the secondary impedance and increases the error. Including the effect of the burden impedance, Eq. 2.46 becomes

$$\frac{\hat{I}_2}{\hat{I}_1} = \left(\frac{N_1}{N_2}\right) \frac{jX_\mathrm{m}}{Z_\mathrm{b}' + R_2' + j(X_2' + X_\mathrm{m})} \tag{2.47}$$

From these equations, it can be seen that an accurate current transformer should have a large magnetizing impedance and relatively small winding resistances and leakage reactances. In addition, as is seen in Example 2.11, the burden impedance on a current transformer must be kept below a maximum value to avoid introducing excessive additional magnitude and phase errors in the measured current.

EXAMPLE 2.11

A 800:5-A, 60-Hz current transformer has the following parameter values (referred to the 800-A winding):

$$X_1 = 44.8\ \mu\Omega \qquad X_2' = 54.3\ \mu\Omega \qquad X_\mathrm{m} = 17.7\ \mathrm{m}\Omega$$

$$R_1 = 10.3\ \mu\Omega \qquad R_2' = 9.6\ \mu\Omega$$

Assuming that the high-current winding is carrying a current of 800 amperes, calculate the magnitude and relative phase of the current in the low-current winding if the load impedance is purely resistive with $R_\mathrm{b} = 2.5\ \Omega$.

■ Solution

The secondary current can be found from Eq. 2.47 by setting $\hat{I}_1 = 800$ A and $R_\mathrm{b}' = (N_1/N_2)^2 R_\mathrm{b} =$ 0.097 mΩ. The following MATLAB script gives

$$\hat{I}_2 = 4.98\ \angle 0.346^\circ\ \mathrm{A}$$

Here is the MATLAB script:

```
clc
clear

%CT parameters
R_2p = 9.6e-6;
X_2p = 54.3e-6;
X_m = 17.7e-3;

N_1 = 5;
N_2 = 800;
N = N_1/N_2;

%Load impedance
R_b = 2.5;
X_b = 0;
Z_bp = N\^{}2*(R_b + j * X_b);

% Primary current
I1 = 800;

%Secondary current
I2 = I1*N*j*X_m/(Z_bp + R_2p + j*(X_2p + X_m));

magI2 = abs(I2);
phaseI2 = 180*angle(I2)/pi;

fprintf('\nSecondary current magnitude = %g [A]',magI2)
fprintf('\n    and phase angle = \%g [degrees]\n$\n',phaseI2)
```

Practice Problem 2.9

For the current transformer of Example 2.11, find the maximum purely reactive burden $Z_b = jX_b$ such that, for 800 A flowing in the transformer primary, the secondary current will be greater than 4.95 A (i.e., there will be at most a 1.0 percent error in current magnitude).

Solution

X_b must be less than 3.19 Ω

2.9 THE PER-UNIT SYSTEM

Electric power systems typically consist of the interconnection of a large number of generators, transformers, transmission lines and loads (a large fraction of which include electric motors). The characteristics of these components vary over a large range; with voltages ranging from hundreds of volts to hundreds of kilovolts and power ratings ranging from kilowatts to hundreds of megawatts. Power-system analyses, and indeed analyses of individual power-system components are often carried out in *per-unit* form, i.e., with all pertinent quantities expressed as decimal fractions of appropriately chosen *base values*. All the usual computations are then carried out in these per-unit values instead of the familiar volts, amperes, ohms, and so on.

There are a number of advantages to the use of the per-unit system. One is that, when expressed in per-unit based upon their rating, the parameter values of machines and transformers typically fall in a reasonably narrow numerical range. This both permits a quick "sanity check" of parameter values as well enables "back-of-the envelope" estimates of parameter values which are otherwise not available. A second advantage is that when transformer equivalent-circuit parameters are converted to their per-unit values, the ideal transformer turns ratio becomes 1:1 and hence the ideal transformer can be eliminated from the equivalent circuit. This greatly simplifies analyses since it eliminates the need to refer impedances to one side or the other of transformers.

Quantities such as voltage V, current I, power P, reactive power Q, voltamperes VA, resistance R, reactance X, impedance Z, conductance G, susceptance B, and admittance Y can be translated to and from per-unit form as follows:

$$\text{Quantity in per unit} = \frac{\text{Actual quantity}}{\text{Base value of quantity}} \tag{2.48}$$

where "Actual quantity" refers to the value in volts, amperes, ohms, and so on. To a certain extent, base values can be chosen arbitrarily, but certain relations between them must be observed for the normal electrical laws to hold in the per-unit system. Thus, for a single-phase system, the power base (total, real and reactive power) is related to the base voltage and base current as

$$VA_{\text{base}}\ (P_{\text{base}}, Q_{\text{base}}) = V_{\text{base}} \times I_{\text{base}} \tag{2.49}$$

and the impedance base (complex, real and reactive) is related to the base voltage and base current as

$$Z_{\text{base}}\ (R_{\text{base}}, X_{\text{base}}) = \frac{V_{\text{base}}}{I_{\text{base}}} \tag{2.50}$$

The net result is that *only two independent base quantities can be chosen arbitrarily;* the remaining quantities are determined by the relationships of Eqs. 2.49 and 2.50. In typical usage, values of VA_{base} and V_{base} are chosen first; values of I_{base} and all other quantities in Eqs. 2.49 and 2.50 are then uniquely established.

The value of VA_{base} must be the same over the entire system under analysis. As can be seen with reference to the equivalent circuit of Fig. 2.10c, if the base voltages of the primary and secondary are chosen to be in the ratio of the turns of the ideal transformer, the per-unit ideal transformer will have a unity turns ratio and hence can be eliminated. Usually the rated or nominal voltages of the respective sides are chosen as the base values. Although, as we have seen, transformer equivalent-circuit parameters values vary by the square of the turns ratio as they are reflected from one side of the transformer to the other, the per-unit impedances will be the same independent of the side of the transformer from which they are initially calculated. This is consistent with the unity-turns-ratio per-unit ideal transformer and is automatically accounted for by using Eqs. 2.49 and 2.50 to determine the per-unit values.

If these rules are followed, the procedure for performing system analyses in per-unit can be summarized as follows:

1. Select a *VA* base and a base voltage at some point in the system.
2. Convert all quantities to per unit on the chosen *VA* base and with a voltage base that transforms as the turns ratio of any transformer which is encountered as one moves through the system.
3. Perform a standard electrical analysis on the resultant electric circuit with all quantities in per unit.
4. When the analysis is completed, all quantities can be converted back to real units (e.g., volts, amperes, watts, etc.) by multiplying their per-unit values by their corresponding base values.

When only one electric device, such as a transformer, is involved, the device's own rating is generally used for the volt-ampere base. When their parameters are expressed in per-unit on their rating as a base, the characteristics of power and distribution transformers do not vary much over a wide range of ratings. For example, the exciting current is often between 0.02 and 0.06 per unit (2 percent to 6 percent of rated current) or less on the largest transformers, the equivalent resistance is usually between 0.005 and 0.02 per unit (the smaller values applying to large transformers), and the equivalent reactance is usually between 0.05 and 0.10 per unit (with the larger values applying to large high-voltage transformers as required to limit short-circuit currents). Similarly, the per-unit values of synchronous- and induction-machine parameters fall within a relatively narrow range. The reason for this is that the physics behind each type of device is the same and, in a crude sense, they can each be considered to be simply scaled versions of the same basic device. As a result, when normalized to their own rating, the effect of the scaling is eliminated and the result is a set of per-unit parameter values which is quite similar over the whole size range of that device.

Often, manufacturers supply device parameters in per unit on the device base. When several devices are involved, however, an arbitrary choice of volt-ampere base must usually be made, and that value must then be used for the overall system. As a result, when performing a system analysis, it may be necessary to convert the supplied per-unit parameter values to per-unit values on the base chosen for the analysis. The following relations can be used to convert per-unit (pu) values from one base to another:

$$(P, Q, VA)_{\text{pu on base 2}} = (P, Q, VA)_{\text{pu on base 1}} \left[\frac{VA_{\text{base 1}}}{VA_{\text{base 2}}}\right] \tag{2.51}$$

$$(R, X, Z)_{\text{pu on base 2}} = (R, X, Z)_{\text{pu on base 1}} \left[\frac{(V_{\text{base 1}})^2 VA_{\text{base 2}}}{(V_{\text{base 2}})^2 VA_{\text{base 1}}}\right] \tag{2.52}$$

$$V_{\text{pu on base 2}} = V_{\text{pu on base 1}} \left[\frac{V_{\text{base 1}}}{V_{\text{base 2}}}\right] \tag{2.53}$$

$$I_{\text{pu on base 2}} = I_{\text{pu on base 1}} \left[\frac{V_{\text{base 2}} VA_{\text{base 1}}}{V_{\text{base 1}} VA_{\text{base 2}}}\right] \tag{2.54}$$

EXAMPLE 2.12

The equivalent circuit for a 100-MVA, 7.97-kV:79.7-kV transformer is shown in Fig. 2.23a. The equivalent-circuit parameters are:

$$X_L = 0.040\ \Omega \qquad X_H = 3.75\ \Omega \qquad X_m = 114\ \Omega$$

$$R_L = 0.76\ \text{m}\Omega \qquad R_H = 0.085\ \Omega$$

Note that the magnetizing inductance has been referred to the low-voltage side of the equivalent circuit. Convert the equivalent circuit parameters to per unit using the transformer rating as base.

■ Solution

The base quantities for the transformer are:

Low-voltage side:

$$VA_{\text{base}} = 100\ \text{MVA} \qquad V_{\text{base}} = 7.97\ \text{kV}$$

and from Eqs. 2.49 and 2.50

$$R_{\text{base}} = X_{\text{base}} = \frac{V_{\text{base}}^2}{VA_{\text{base}}} = 0.635\ \Omega$$

High-voltage side:

$$VA_{\text{base}} = 100\ \text{MVA} \qquad V_{\text{base}} = 79.7\ \text{kV}$$

and from Eqs. 2.49 and 2.50

$$R_{\text{base}} = X_{\text{base}} = \frac{V_{\text{base}}^2}{VA_{\text{base}}} = 63.5\ \Omega$$

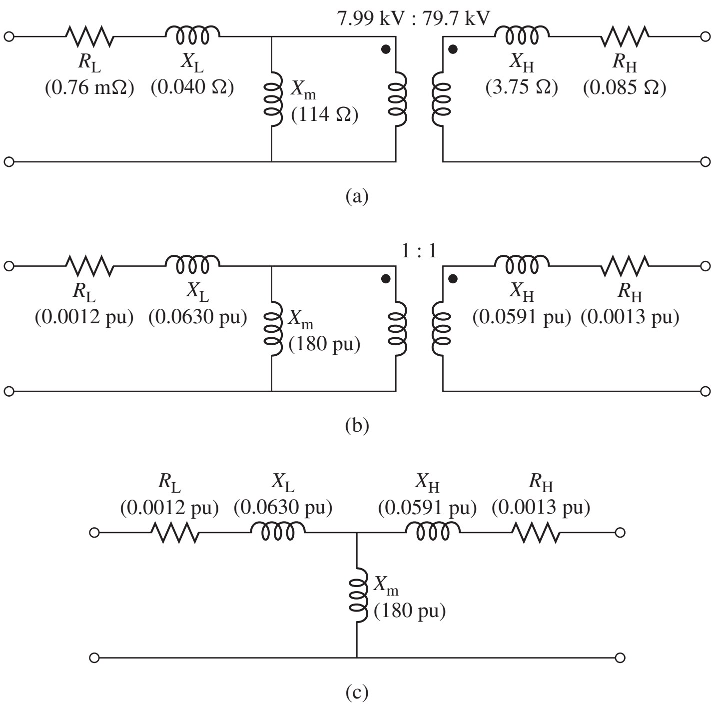

Figure 2.23 Transformer equivalent circuits for Example 2.12. (a) Equivalent circuit in actual units. (b) Per-unit equivalent circuit with 1:1 ideal transformer. (c) Per-unit equivalent circuit following elimination of the ideal transformer.

The per-unit values of the transformer parameters can now be calculated by division by their corresponding base quantities.

$$X_{\mathrm{L}} = \frac{0.040}{0.635} = 0.0630 \text{ per unit}$$

$$X_{\mathrm{H}} = \frac{3.75}{63.5} = 0.0591 \text{ per unit}$$

$$X_{\mathrm{m}} = \frac{114}{0.635} = 180 \text{ per unit}$$

$$R_{\mathrm{L}} = \frac{7.6 \times 10^{-4}}{0.635} = 0.0012 \text{ per unit}$$

$$R_{\mathrm{H}} = \frac{0.085}{63.5} = 0.0013 \text{ per unit}$$

Finally, the voltages representing the turns ratio of the ideal transformer must each be divided by the base voltage on that side of the transformer. Thus the turns ratio of 7.97-kV:79.7-kV becomes in per unit

$$\text{Per-unit turns ratio} = \left(\frac{7.97\ \text{kV}}{7.97\ \text{kV}}\right) : \left(\frac{79.7\ \text{kV}}{79.7\ \text{kV}}\right) = 1 : 1$$

The resultant per-unit equivalent circuit is shown in Fig. 2.23b. Because it has unity turns ratio, there is no need to keep the ideal transformer and hence this equivalent circuit can be reduced to the form of Fig. 2.23c.

EXAMPLE 2.13

The exciting current measured on the low-voltage side of a 50-kVA, 2400:240-V transformer is 5.41 A. Its equivalent impedance referred to the high-voltage side is $1.42 + j1.82\ \Omega$. Using the transformer rating as the base, express in per unit on the low- and high-voltage sides (a) the exciting current and (b) the equivalent impedance.

■ Solution

The base values of voltages and currents are

$$V_{\text{base,H}} = 2400\ \text{V} \quad V_{\text{base,L}} = 240\ \text{V} \quad I_{\text{base,H}} = 20.8\ \text{A} \quad I_{\text{base,L}} = 208\ \text{A}$$

where subscripts H and L indicate the high- and low-voltage sides, respectively.

From Eq. 2.50

$$Z_{\text{base,H}} = \frac{2400}{20.8} = 115.2\ \Omega \quad Z_{\text{base,L}} = \frac{240}{208} = 1.152\ \Omega$$

a. From Eq. 2.48, the exciting current in per unit referred to the low-voltage side can be calculated as:

$$I_{\varphi,\text{L}} = \frac{5.41}{208} = 0.0260 \text{ per unit}$$

The exciting current referred to the high-voltage side is 0.541 A. Its per-unit value is

$$I_{\varphi,\text{H}} = \frac{0.541}{20.8} = 0.0260 \text{ per unit}$$

Note that, as expected, the per-unit values are the same referred to either side, corresponding to a unity turns ratio for the ideal transformer in the per-unit transformer. This is a direct consequence of the choice of base voltages in the ratio of the transformer turns ratio and the choice of a constant volt-ampere base.

b. From Eq. 2.48 and the value for Z_{base}

$$Z_{\text{eq,H}} = \frac{1.42 + j1.82}{115.2} = 0.0123 + j0.0158 \text{ per unit}$$

The equivalent impedance referred to the low-voltage side is $0.0142 + j0.0182\Omega$. Its per-unit value is

$$Z_{\text{eq,L}} = \frac{0.142 + 0.0182}{1.152} = 0.0123 + j0.0158 \text{ per unit}$$

The per-unit values referred to the high- and low-voltage sides are the same, the transformer turns ratio being accounted for in per unit by the base values. Note again that this is consistent with a unity turns ratio of the ideal transformer in the per-unit transformer equivalent circuit.

Practice Problem 2.10

A 15-kVA 120:460-V transformer has an equivalent series impedance of $0.018 + j0.042$ per unit. Calculate the equivalent series impedance in ohms (a) referred to the low-voltage side and (b) referred to the high-voltage side.

Solution

$$Z_{\mathrm{eq,L}} = 0.017 + j0.040\ \Omega \quad \text{and} \quad Z_{\mathrm{eq,H}} = 0.25 + j0.60\ \Omega$$

When they are applied to the analysis of three-phase systems, the base values for the per-unit system are chosen so that the relations for a balanced three-phase system hold between them:

$$(P_{\mathrm{base}},\, Q_{\mathrm{base}},\, VA_{\mathrm{base}})_{3\mathrm{-phase}} = 3VA_{\mathrm{base,\ per\ phase}} \tag{2.55}$$

In dealing with three-phase systems, $VA_{\mathrm{base,\ 3-phase}}$, the three-phase volt-ampere base, and $V_{\mathrm{base,\ 3-phase}} = V_{\mathrm{base,\ l-l}}$, the line-to-line voltage base are usually chosen first. The base values for the phase (line-to-neutral) voltage then follows as

$$V_{\mathrm{base,\ l-n}} = \frac{1}{\sqrt{3}} V_{\mathrm{base,\ l-l}} \tag{2.56}$$

Note that the base current for three-phase systems is equal to the phase current, which is the same as the base current for a single-phase (per-phase) analysis. Hence

$$I_{\mathrm{base,\ 3-phase}} = I_{\mathrm{base,\ per\ phase}} = \frac{VA_{\mathrm{base,\ 3-phase}}}{\sqrt{3}\, V_{\mathrm{base,\ 3-phase}}} \tag{2.57}$$

Finally, the three-phase base impedance is chosen to the be the single-phase base impedance. Thus

$$\begin{aligned} Z_{\mathrm{base,\ 3-phase}} &= Z_{\mathrm{base,\ per\ phase}} \\ &= \frac{V_{\mathrm{base,\ l-n}}}{I_{\mathrm{base,\ per\ phase}}} \\ &= \frac{V_{\mathrm{base,\ 3-phase}}}{\sqrt{3} I_{\mathrm{base,\ 3-phase}}} \\ &= \frac{(V_{\mathrm{base,\ 3-phase}})^2}{VA_{\mathrm{base,\ 3-phase}}} \end{aligned} \tag{2.58}$$

The equations for conversion from base to base, Eqs. 2.51 through 2.54, apply equally to three-phase base conversion. Note that the factors of $\sqrt{3}$ and 3 relating Δ to Y quantities of volts, amperes, and ohms in a balanced three-phase system are automatically taken care of in per unit by the base values. Three-phase problems can thus be solved in per unit as if they were single-phase problems and the details of transformer (Y vs Δ on the primary and secondary of the transformer) and impedance (Y vs Δ) connections disappear, except in translating volt, ampere, and ohm values into and out of the per-unit system.

EXAMPLE 2.14

Rework Example 2.9 in per unit, specifically calculating the short-circuit phase currents which will flow in the feeder and at the 240-V terminals of the receiving-end transformer bank. Perform the calculations in per unit on the three-phase, 150-kVA, rated-voltage base of the receiving-end transformer.

■ Solution

We start by converting all the impedances to per unit. The impedance of the 500-kVA, 24 kV:2400 V sending end transformer is $0.17 + j0.92\ \Omega$/phase as referred to the 2400-V side. From Eq. 2.58, the base impedance corresponding to a 2400-V, 150-kVA base is

$$Z_{\text{base}} = \frac{2400^2}{150 \times 10^3} = 38.4\ \Omega$$

From Example 2.9, the total series impedance is equal to $Z_{\text{tot}} = 0.64 + j2.33\ \Omega$/phase and thus in per unit it is equal to

$$Z_{\text{tot}} = \frac{0.64 + j2.33}{38.4} = 0.0167 + j0.0607 \text{ per unit}$$

which is of magnitude

$$|Z_{\text{tot}}| = 0.0629 \text{ per unit}$$

The voltage applied to the high-voltage side of the sending-end transformer is $V_{\text{s}} = 24.0\ \text{kV} = 1.0$ per unit on a rated-voltage base and hence the short-circuit current will equal

$$I_{\text{sc}} = \frac{V_{\text{s}}}{|Z_{\text{tot}}|} = \frac{1.0}{0.0629} = 15.9 \text{ per unit}$$

To calculate the phase currents in amperes, it is simply necessary to multiply the per-unit short-circuit current by the appropriate base current. Thus, at the 2400-V feeder the base current is

$$I_{\text{base, 2400-V}} = \frac{150 \times 10^3}{\sqrt{3}\,2400} = 36.1\ \text{A}$$

and hence the feeder current will be

$$I_{\text{feeder}} = 15.9 \times 36.1 = 574\ \text{A}$$

The base current at the 240-V secondary of the receiving-end transformers is

$$I_{\text{base, 240-V}} = \frac{150 \times 10^3}{\sqrt{3}\,240} = 361\ \text{A}$$

and hence the short-circuit current is

$$I_{\text{240-V secondary}} = 15.9 \times 361 = 5.74\ \text{kA}$$

As expected, these values are equivalent within numerical accuracy to those calculated in Example 2.9.

Practice Problem 2.11

Calculate the magnitude of the short-circuit current in the feeder of Example 2.9 if the 2400-V feeder is replaced by a feeder with an impedance of $0.07 + j0.68\ \Omega$/phase. Perform this calculation on the 500-kVA, rated-voltage base of the sending-end transformer and express your solution both in per unit and in amperes per phase.

Solution

Short-circuit current = 5.20 per unit = 636 A

EXAMPLE 2.15

A three-phase load is supplied from a 2.4-kV:460-V, 250-kVA transformer whose equivalent series impedance is $0.026 + j0.12$ per unit on its own base. The load voltage is observed to be 438 V line-line, and it is drawing 95 kW at unity power factor. Calculate the voltage at the high-voltage side of the transformer. Perform the calculations on a 460-V, 100-kVA base.

■ Solution

The 460-V side base impedance for the transformer is

$$Z_{\text{base, transformer}} = \frac{460^2}{250 \times 10^3} = 0.846\ \Omega$$

while that based upon a 100-kVA base is

$$Z_{\text{base, 100-kVA}} = \frac{460^2}{100 \times 10^3} = 2.12\ \Omega$$

Thus, from Eq. 2.52 the per-unit transformer impedance on a 100-kVA base is

$$Z_{\text{transformer}} = (0.026 + j0.12)\left(\frac{0.864}{2.12}\right) = 0.0106 + j.0489 \text{ per unit}$$

The per-unit load voltage is

$$\hat{V}_{\text{load}} = \frac{438}{460} = 0.952\ \angle 0^\circ \text{ per unit}$$

where the load voltage has been chosen as the reference for phase-angle calculations.

The per-unit load power is

$$P_{\text{load}} = \frac{95}{100} = 0.95 \text{ per unit}$$

and hence the per-unit load current, which is in phase with the load voltage because the load is operating at unity power factor, is

$$\hat{I}_{\text{load}} = \frac{P_{\text{load}}}{V_{\text{load}}} = \frac{0.95}{0.952} = 0.998\ \angle 0^\circ \text{ per unit}$$

Thus we can now calculate the high-side voltage of the transformer

$$\begin{aligned}
\hat{V}_{\text{H}} &= \hat{V}_{\text{load}} + \hat{I}_{\text{load}}\, Z_{\text{transformer}} \\
&= 0.952 + 0.998 \times (0.0106 + j0.0489) \\
&= 0.963 + j0.0488 = 0.964\ \angle 29.0^\circ \text{ per unit}
\end{aligned}$$

Thus the high-side voltage is equal to 0.964×2400 V = 2313 V (line-line).

Practice Problem 2.12

Repeat Example 2.15 if the 250-kV three-phase transformer is replaced by a 150-kV transformer also rated at 2.4-kV:460-V and whose equivalent series impedance is $0.038 + j0.135$ per unit on its own base. Perform the calculations on a 460-V, 100-kVA base.

Solution

$$\text{High-side voltage } = 0.982 \text{ per unit } = 2357 \text{ V (line-line)}$$

2.10 SUMMARY

Although not an electromechanical device, the transformer is a common and indispensable component of ac systems where it is used to transform voltages, currents, and impedances to appropriate levels for optimal use. For the purposes of our study of electromechanical systems, transformers serve as valuable examples of the analysis techniques which will be employed. They offer us opportunities to investigate the properties of magnetic circuits, including the concepts of mmf, magnetizing current, and magnetizing, mutual, and leakage fluxes and their associated inductances.

In both transformers and rotating machines, a magnetic field is created by the combined action of the currents in the windings. In an iron-core transformer, most of this flux is confined to the core and links all the windings. This resultant mutual flux induces voltages in the windings proportional to their number of turns and is responsible for the voltage-changing property of a transformer. In rotating machines, the situation is similar, although there is an air gap which separates the rotating and stationary components of the machine. Directly analogous to the manner in which transformer core flux links the various windings on a transformer core, the mutual flux in rotating machines crosses the air gap, linking the windings on the rotor and stator. As in a transformer, the mutual flux induces voltages in these windings proportional to the number of turns and the time rate of change of the flux.

A significant difference between transformers and rotating machines is that in rotating machines there is relative motion between the windings on the rotor and stator. This relative motion produces an additional component of the time rate of change of the various winding flux linkages. As will be discussed in Chapter 3, the resultant voltage component, known as the *speed voltage,* is characteristic of the process of electromechanical energy conversion. In a static transformer, however, the time variation of flux linkages is caused simply by the time variation of winding currents; no mechanical motion is involved, and no electromechanical energy conversion takes place.

The resultant core flux in a transformer induces a counter emf in the primary which, together with the primary resistance and leakage-reactance voltage drops, must balance the applied voltage. Since the resistance and leakage-reactance voltage drops usually are small, the counter emf must approximately equal the applied voltage and the core flux must adjust itself accordingly. Exactly similar phenomena must take place in the armature windings of an ac motor; the resultant air-gap flux wave must adjust itself to generate a counter emf approximately equal to the applied voltage. In both transformers and rotating machines, the net mmf produced by all of the currents must accordingly adjust itself to create the resultant flux required by this voltage balance.

In a transformer, the secondary current is determined by the voltage induced in the secondary, the secondary leakage impedance, and the electric load. As we will see, in an induction motor, the secondary (rotor) current is determined by the voltage induced in the secondary, the secondary leakage impedance, and the mechanical load on its shaft. Essentially the same phenomena take place in the primary winding of the transformer and in the armature (stator) windings of induction and synchronous motors. In all three the story remains the same; the primary, or armature, current must adjust itself so that the combined mmf of all currents creates the flux required by the applied voltage and as a result, a change in the load current will result in a corresponding change in the primary current.

In addition to the useful mutual fluxes, in both transformers and rotating machines there are leakage fluxes which link individual windings without linking others. Although the detailed picture of the leakage fluxes in rotating machines is more complicated than that in transformers, their effects are essentially the same. In both, leakage fluxes produce leakage-reactance voltage drops in the windings and typically reduce the mutual flux below the level which would otherwise be produced by the applied voltage. In both, the reluctances of the leakage-flux paths are dominated by that of a path through air, and hence the leakage fluxes are nearly linearly proportional to the currents producing them. Leakage reactances therefore are often assumed to be constant, independent of the degree of saturation of the main magnetic circuit.

Further examples of the basic similarities between transformers and rotating machines can be cited. Except for friction and windage, the losses in transformers and rotating machines are essentially the same. Tests for determining the losses and equivalent circuit parameters are similar: an open-circuit, or no-load, test gives information regarding the excitation requirements and core losses (along with friction and windage losses in rotating machines), while a short-circuit test together with dc resistance measurements gives information regarding leakage reactances and winding resistances. Modeling of the effects of magnetic saturation is another example: In both transformers and ac rotating machines, the leakage reactances are usually assumed to be unaffected by saturation, and the saturation of the main magnetic circuit is assumed to be determined by the resultant mutual or air-gap flux.

2.11 CHAPTER 2 VARIABLES

λ	Flux linkages [Wb]
ω	Angular frequency [rad/sec]
φ, ϕ_{max}	Magnetic flux [Wb]
$\hat{\Phi}$	Magnetic flux, complex amplitude [Wb]
θ	Phase angle [rad]
B_{max}	Peak flux density [T]

e	Electromotive force (emf), induced voltage [V]
E	Voltage [V]
$\hat{E}$	EMF, voltage, complex amplitude [V]
f	Frequency [Hz]
i, I	Current [A]
i_{φ}	Exciting current [A]
$\hat{I}$	Current, complex amplitude [A]
$\hat{I}_{\mathrm{c}}$	Core-loss component of exciting current, complex amplitude [A]
$\hat{I}_{\mathrm{m}}$	Magnetizing current, complex amplitude [A]
$\hat{I}_{\varphi}$	Exciting current, complex amplitude [A]
L	Inductance [H]
N	Number of turns
Q	Reactive power [VAR]
R	Resistance [Ω]
R_{base}	Base resistance [Ω]
t	Time [sec]
v, V	Voltage [V]
V_{base}	Base voltage [V]
$\hat{V}$	Voltage, complex amplitude [V]
$VA =$	Voltamperes [VA]
X	Reactance [Ω]
Z	Impedance [Ω]
Z_{Δ}	Delta-equivalent line-neutral impedance [Ω/phase]
Z_{φ}	Exciting impedance [Ω]
Z_{Y}	Y-equivalent line-neutral impedance [Ω/phase]

Subscripts:

ϕ	Exciting
b	Burden
base	Base quantity
c	Core
eq	Equivalent
H	High-voltage side
l	Leakage
l-n	Line-to-neutral
L	Low-voltage side
m	Magnetizing
max	Maximum
oc	Open circuit
pu	Per unit
rms	Root mean square
s	Sending
sc	Short circuit
tot	Total

2.12 PROBLEMS

2.1 A transformer is made up of a 1150-turn primary coil and an open-circuited 80-turn secondary coil wound around a closed core of cross-sectional area 56 cm^2. The core material can be considered to saturate when the rms applied flux density reaches 1.45 T. What maximum 60-Hz rms primary voltage is possible without reaching this saturation level? What is the corresponding secondary voltage? How are these values modified if the applied frequency is lowered to 50 Hz?

2.2 A magnetic circuit with a cross-sectional area of 20 cm^2 is to be operated at 60 Hz from a 115-V rms supply. Calculate the number of turns such that the peak core magnetic flux density is 1.6 T.

2.3 A transformer is to be used to transform the impedance of a 75-Ω resistor to an impedance of 300 Ω. Calculate the required turns ratio, assuming the transformer to be ideal.

2.4 A 150 Ω resistor is connected to the secondary of a transformer with a turns ratio of 1:4 (primary to secondary). A 12 V rms, 1 kHz voltage source is connected to the primary. (a) Assuming the transformer to be ideal, calculate the primary current and the resistor voltage and power. (b) Repeat this calculation assuming that the transformer has a leakage inductance of 340 μH as referred to the primary.

2.5 A load consisting of a 5 Ω resistor in series with a 2.5 mH inductor is connected to the low-voltage winding of a 20:120 V transformer. A 110 V rms, 50-Hz supply is connected to the high-voltage winding. Assuming the transformer to be ideal, calculate the rms load current and the rms current which will be drawn from the supply.

2.6 A source which can be represented by a 12 V rms voltage source in series with a resistance of 1.5 kΩ is connected to a 75-Ω load resistance through an ideal transformer. Calculate the value of turns ratio for which maximum power is supplied to the load and the corresponding load power? Using MATLAB, plot the the power in milliwatts supplied to the load as a function of the transformer ratio, covering ratios from 1.0 to 10.0.

2.7 Repeat the calculations of Problem 2.6 with the source resistance replaced by a 1.5 kΩ inductive reactance.

2.8 A single-phase 60-Hz transformer has a nameplate voltage rating of 7.97 kV:120 V based on its known winding turns ratio. The manufacturer calculates that the primary (7.97-kV) leakage inductance is 193 mH and the primary magnetizing inductance is 167 H. For an applied primary voltage of 7970 V at 60 Hz, calculate the resultant open-circuit secondary voltage.

2.9 The manufacturer calculates that the transformer of Problem 2.8 has a secondary leakage inductance of 44 μH.

a. Calculate the magnetizing inductance as referred to the secondary side.

b. A voltage of 120 V, 60 Hz is applied to the secondary. Calculate (i) the resultant open-circuit primary voltage and (ii) the secondary current which would result if the primary were short-circuited.

2.10 A 230-V:6.6-kV, 50-Hz, 45 kVA transformer has a magnetizing reactance (as measured from the 230-V terminals) of 46.2 Ω. The 230-V winding has a leakage reactance of 27.8 mΩ and the 6.6-kV winding has a leakage reactance of 25.3 Ω.

a. With the secondary open-circuited and 230 V applied to the primary (230-V) winding, calculate the primary current and the secondary voltage.

b. With the secondary short-circuited, calculate the primary voltage which will result in rated current in the primary winding. Calculate the corresponding current in the secondary winding.

2.11 The transformer of Problem 2.10 is to be used on a 60-Hz system.

a. Calculate the magnetizing reactance referred to the low-voltage winding and the leakage reactance of each winding.

b. With 240 V applied to the low-voltage (primary) winding and with the secondary winding open-circuited, calculate the primary-winding current and the secondary voltage.

2.12 A 460-V:2400-V transformer has a series leakage reactance of 39.3 Ω as referred to the high-voltage side. A load connected to the low voltage side is observed to draw 42 kW at unity power factor and the voltage is measured to be 447 V. Calculate the corresponding voltage and power factor as measured at the high-voltage terminals.

2.13 The 460-V:2400-V transformer of Problem 2.12 is to be operated from a 50-Hz source. A unity-power-factor load connected to the low-voltage side is observed to draw 34.5 kW, unity-power-factor load at a voltage of 362 V. Calculate the voltage applied to the transformer high-voltage winding.

2.14 The resistances and leakage reactances of a 40-kVA 60-Hz 7.97-kV-V:240-V single-phase distribution transformer are

$$R_1 = 41.6\ \Omega \quad R_2 = 37.2\ \text{m}\Omega$$

$$X_{l_1} = 42.1\ \Omega \quad X_{l_2} = 39.8\ \text{m}\Omega$$

where subscript 1 denotes the 7.97-kV winding and subscript 2 denotes the 240-V winding. Each quantity is referred to its own side of the transformer.

a. Draw the equivalent circuit referred to (i) the high- and (ii) the low-voltage sides. Label the impedances numerically.

b. Consider the transformer to deliver its rated kVA to a load on the low-voltage side with 240 V across the load. (i) Find the high-side terminal voltage for a load power factor of 0.87 power factor lagging. (ii) Find the high-side terminal voltage for a load power factor of 0.87 power factor leading.

c. Consider a rated-kVA load connected at the low-voltage terminals. Assuming the load voltage to remain constant at 240 V, use MATLAB to plot the high-side terminal voltage as a function of the power-factor angle as the load power factor varies from 0.6 leading through unity power factor to 0.6 lagging.

2.15 Repeat the calculations of Problem 2.14 for a 75-kVA, 50-Hz, 3.81-kV:230-V single-phase distribution transformer whose resistances and leakage reactances are

$$R_1 = 4.85\ \Omega \quad R_2 = 16.2\ \text{m}\Omega$$

$$X_{l_1} = 4.13\ \Omega \quad X_{l_2} = 16.9\ \text{m}\Omega$$

where subscript 1 denotes the 3.81-kV winding and subscript 2 denotes the 230-V winding. Each quantity is referred to its own side of the transformer. The load in parts (b) and (c) should be assumed to be operating at a voltage of 230 V.

2.16 A single-phase load is supplied through a 35-kV feeder whose impedance is $90 + j320\ \Omega$ and a 35-kV: 2400-V transformer whose equivalent series impedance is $0.21 + j1.33\ \Omega$ referred to its low-voltage side. The load is 135 kW at 0.78 leading power factor and 2385 V.

a. Compute the voltage at the high-voltage terminals of the transformer.

b. Compute the voltage at the sending end of the feeder.

c. Compute the power and reactive power input at the sending end of the feeder.

2.17 Write a MATLAB script to (a) repeat the calculations of Problem 2.16 for power factors of 0.78 leading, unity and 0.78 lagging assuming the load power remains constant at 135 kW and the load voltage remains constant at 2385 V. (b) Use your MATLAB script to plot (versus power factor angle) the sending-end voltage required to maintain a load voltage of 2385 V as the power factor varies from 0.7 leading through unity to 0.7 lagging.

2.18 Repeat Example 2.6 with the transformer operating at full load and unity power factor.

2.19 A 450-kVA 50-Hz single-phase transformer with a 11-kV primary winding draws 0.33 A and 2700 W at no load, rated voltage and frequency. Another transformer has a core with all its linear dimensions $\sqrt{2}$ times as large as the corresponding dimensions of the first transformer. The core material and lamination thickness are the same in both transformers. (a) If the primary windings of both transformers have the same number of turns, what impressed primary voltage will result in the same flux density in the core. (b) With the primary excited by the voltage found in part (a), calculate the primary current and power.

2.20 The nameplate on a 25-MVA, 60-Hz single-phase transformer indicates that it has a voltage rating of 8.0-kV:78-kV. A short-circuit test from the high-voltage side (low-voltage winding short circuited) gives readings of 4.53 kV, 321 A, and 77.5 kW. An open-circuit test is conducted from the low-voltage side and the corresponding instrument readings are 8.0 kV, 39.6 A, and 86.2 kW.

a. Calculate the equivalent series impedance of the transformer as referred to the high-voltage terminals.

b. Calculate the equivalent series impedance of the transformer as referred to the low-voltage terminals.

c. Making appropriate approximations, draw a T equivalent circuit for the transformer.

2.21 Perform the calculations of Problem 2.20 for a 175-kVA, 50-Hz single-phase transformer with a voltage rating of 3.8-kV:6.4-kV. An open-circuit test is conducted from the low-voltage side and the corresponding instrument readings are 3.8 kV, 0.58 A, and 603 W. Similarly, a short-circuit test from the high-voltage side (low-voltage winding short-circuited) gives readings of 372 V, 27.3 A, and 543 W.

2.22 A voltage of 7.96 kV is applied to the low-voltage winding of a 7.96 kV:39.8 kV, 60 Hz, 10 MVA single-phase transformer with the high-voltage winding open-circuited and the resultant current is 17.3 A and power is 48.0 kW. The low-voltage winding is then short-circuited and a voltage of 1.92 kV applied to the high-voltage winding results in a current of current of 252 A and a power of 60.3 kW.

a. Calculate the parameters of the cantilever equivalent circuits of Figs. 2.12a and b as referred to the transformer high-voltage winding.

b. Calculate the cantilever equivalent-circuit parameters as referred to the transformer low-voltage winding.

c. With the transformer carrying rated load and rated voltage at its low-voltage terminal, calculate the power dissipated in the transformer.

2.23 The following data were obtained on a 2.5 MVA, 50-Hz, 19.1-kV:3.81-kV single-phase transformer tested at 50 Hz:

	Voltage, V	Current, A	Power, kW
LV winding with HV terminals open-circuited	3810	9.86	8.14
HV winding with LV terminals short-circuited	920	141	10.3

a. Calculate the parameters of the cantilever equivalent circuits of Figs. 2.12a and b as referred to the transformer high-voltage winding.

b. Calculate the cantilever equivalent-circuit parameters as referred to the transformer low-voltage winding.

c. With the transformer carrying rated load and rated voltage at its low-voltage terminal, calculate the power dissipated in the transformer.

2.24 Write a MATLAB script to calculate the parameters for the cantilever transformer equivalent circuits of Figs. 2.12a and b with the parameters referred to the high-voltage winding based upon the following test data:

- Voltage, current and power from an open-circuit test conducted from the low-voltage winding (high-voltage winding open-circuited).
- Voltage, current and power from a short-circuit test conducted from the low-voltage winding (high-voltage winding short-circuited).

Test your script on the measurements made on the transformer of Problem 2.22.

2.25 The high-voltage winding of the transformer of Problem 2.22 is replaced by an otherwise identical winding of twice the number of turns with wire of half the cross-sectional area.

a. Calculate the rated voltage and power of this modified transformer.

b. With the high-voltage winding open-circuited and with rated voltage applied to the low-voltage winding, calculate the current and power supplied to the low-voltage winding.

c. With the low-voltage winding short-circuited, calculate the voltage applied to the high-voltage winding that will result in a short-circuit power dissipation of 60.3 kW.

d. Calculate the cantilever-equivalent-circuit parameters of this transformer referred to (i) the low-voltage side and (ii) the high-voltage side.

2.26 (a) Determine the efficiency and voltage regulation of the transformer of Problem 2.20 if it is supplying rated load (unity power factor) at rated voltage at its low-voltage terminals. (b) Repeat part (a), assuming the load to be at 0.9 power factor leading.

2.27 Assume the transformer of Problem 2.23 to be operating at rated voltage and with a load that draws rated current at its low-voltage terminals. Write a MATLAB script to plot (a) the efficiency and (b) the voltage regulation of the transformer as the as a function of the load power-factor as the power factor varies from 0.75 lagging through unity through 0.75 leading.

2.28 The following data were obtained for a 25-kVA, 60-Hz, 2400:240-V distribution transformer tested at 60 Hz:

	Voltage, V	Current, A	Power, W
LV winding with HV terminals open-circuited	240	1.37	139
HV winding with LV terminals short-circuited	67.8	10.1	174

a. Compute the transformer efficiency when the tranformer is operating at rated terminal voltage with an 0.85 power-factor (lagging) load at its secondary terminal that draws full-load current.

b. The transformer is observed to be operating with rated voltage at both its primary and secondary terminals and supplying a load at its secondary terminals which draws rated current. Calculate the power factor of the load. (HINT: Use MATLAB to search for the solution).

2.29 A 150-kVA, 240-V:7970-V, 60-Hz single-phase distribution transformer has the following parameters referred to the high-voltage side:

$$R_1 = 2.81\ \Omega \quad X_1 = 21.8\ \Omega$$

$$R_2 = 2.24\ \Omega \quad X_2 = 20.3\ \Omega$$

$$R_c = 127\ \text{k}\Omega \quad X_m = 58.3\ \text{k}\Omega$$

Assume that the transformer is supplying its rated kVA at its low-voltage terminals. Write a MATLAB script to determine the efficiency and regulation of the transformer for any specified load power-factor (leading or lagging). You may use reasonable engineering approximations to simplify your analysis. Use your MATLAB script to determine the efficiency and regulation for a load power-factor of 0.92 leading.

2.30 A 45-kVA, 120-V:280-V single-phase transformer is to be connected as a 280-V:400-V autotransformer. Determine the voltage ratings of the high- and low-voltage windings for this connection and the kVA rating of the autotransformer connection.

2.31 A 120:480-V, 10-kVA single-phase transformer is to be used as an autotransformer to supply a 480-V circuit from a 600-V source. When it is tested as a two-winding transformer at rated load, unity power factor, its efficiency is 0.982.

a. Make a diagram of connections as an autotransformer.

b. Determine its kVA rating as an autotransformer.

c. Find its efficiency as an autotransformer when operating with a load of rated kVA and 0.93 power factor leading and 480 V connected to the low-voltage winding.

2.32 Consider the 8-kV:78-kV, 25-MVA transformer of Problem 2.20 connected as a 78-kV:86-kV autotransformer.

a. Determine the voltage ratings of the high-and low-voltage windings for this connection and the MVA rating of the autotransformer connection.

b. Calculate the efficiency of the transformer in this connection when it is supplying its rated load at unity power factor.

2.33 Write a MATLAB script whose inputs are the rating (voltage and kVA) and rated-load, unity-power-factor efficiency of a single-phase transformer and whose output is the transformer rating and rated-load, unity-power-factor efficiency when connected as an autotransformer. Exercise your program on the autotransformer of Problem 2.32.

2.34 The high-voltage terminals of a three-phase transformer bank of three single-phase transformers are supplied from a three-wire, three-phase 13.8-kV (line-to-line) system. The low-voltage terminals are to be connected to a three-wire, three-phase substation load drawing up to 4500 kVA at 2300 V line to line. Specify the required voltage, current, and kVA ratings of each transformer (both high- and low-voltage windings) for the following connections:

	High-voltage Windings	Low-voltage Windings
a.	Y	Δ
b.	Δ	Y
c.	Y	Y
d.	Δ	Δ

2.35 Three 75-MVA single-phase transformers, rated at 39.8-kV:133-kV, are to be connected in a three-phase bank. Each transformer has a series impedance of $0.97 + j11.3\Omega$ referred to its 133-kV winding.

a. If the transformers are connected Y-Y, calculate (i) the voltage and power rating of the three-phase connection, (ii) the equivalent impedance as referred to its low-voltage terminals, and (iii) the equivalent impedance as referred to its high-voltage terminals.

b. Repeat part (a) if the transformer is connected Y on its low-voltage side and Δ on its high-voltage side.

2.36 Repeat the calculations of Problem 2.35 for three 225-kVA, 277-V:7.97-kV transformers whose series impedances is $3.1 + j21.5$ mΩ referred to its low-voltage winding.

2.37 Repeat Example 2.8 for a load drawing rated current from the transformers at unity power factor.

2.38 A three-phase Y-Y transformer is rated at 25 MVA, 13.8-kV:69-kV and has a single-phase equivalent series impedance $62 + j388$ mΩ referred to the low-voltage winding.

a. A three-phase short circuit is applied to the low-voltage winding. Calculate the voltage applied to the high-voltage winding which will result in rated current into the short circuit.

b. The short circuit is removed and a three-phase load is connected to the low-voltage winding. With rated voltage applied to the high-voltage winding, the input power to the transformer is observed to be 18 MW at 0.75 power-factor lagging. Calculate the line-line terminal voltage at the load.

2.39 A three-phase Y-Δ transformer is rated 225-kV:24-kV, 400 MVA and has a single-phase equivalent series reactance of 6.08 Ω as referred to its high-voltage terminals. The transformer is supplying a load of 375 MVA at 0.89 power factor leading at a voltage of 24 kV (line to line) on its low-voltage side. It is supplied from a feeder whose impedance is $0.17 + j2.2$ Ω connected to its high-voltage terminals. For these conditions, calculate (a) the line-to-line voltage at the high-voltage terminals of the transformer and (b) the line-to-line voltage at the sending end of the feeder.

2.40 Assume the apparent power of the load in the system of Problem 2.39 to remain constant at 375 MVA. Write a MATLAB script to calculate the line-to-line voltage which must be applied to the sending end of the feeder to maintain the load voltage at 24 kV line-to-line as a function of the load power factor. Plot the sending-end voltage as a function of power factor angle for power factors in range from 0.3 lagging to unity to 0.3 leading.

2.41 A Δ-Y connected bank of three identical 150-kVA, 2400-V:120-V, 60-Hz transformers is supplied at its high-voltage terminals through a feeder whose impedance is $6.4 + j154$ mΩ per phase. The voltage at the sending end of the feeder is held constant at 2400 V line to line. The results of a single-phase short-circuit test on one of the transformers with its low-voltage terminals short-circuited are

$$V_{\mathrm{H}} = 131\ \mathrm{V} \quad I_{\mathrm{H}} = 62.5\ \mathrm{A} \quad P = 1335\ \mathrm{W}$$

a. Calculate the series impedance of this three-phase transformer bank as referred to its high-voltage terminal.

b. Determine the line-to-line voltage supplied to the feeder when the transformer bank delivers rated current at rated voltage to a balanced three-phase unity power factor load at its low-voltage terminal.

2.42 A 13.8-kV:120-V 60-Hz potential transformer has the following parameters as seen from the high-voltage (primary) winding:

$$X_1 = 6.88\ \mathrm{k\Omega} \quad X_2' = 7.59\ \mathrm{k\Omega} \quad X_{\mathrm{m}} = 6.13\ \mathrm{M\Omega}$$

$$R_1 = 5.51\ \mathrm{k\Omega} \quad R_2' = 6.41\ \mathrm{k\Omega}$$

a. Assuming that the secondary is open-circuited and that the primary is connected to a 13.8-kV source, calculate the magnitude and phase angle (with respect to the high-voltage source) of the voltage at the secondary terminals.

b. Calculate the magnitude and phase angle of the secondary voltage if a 750 Ω resistive load is connected to the secondary terminals.

c. Repeat part (b) if the burden is changed to a 750 Ω reactance.

2.43 For the potential transformer of Problem 2.42, find the maximum reactive burden (minimum reactance) which can be applied at the secondary terminals such that the voltage magnitude error does not exceed 0.75 percent.

2.44 Consider the potential transformer of Problem 2.42 with connected to a 13.8 kV source.

a. Use MATLAB to plot the percentage error in voltage magnitude as a function of the magnitude of the burden impedance (i) for a resistive burden of $100\ \Omega \leq R_{\mathrm{b}} \leq 2000\ \Omega$ and (ii) for a reactive burden of $100\ \Omega \leq X_{\mathrm{b}} \leq 2000\ \Omega$. Plot these curves on the same axis.

b. Next plot the phase error in degrees as a function of the magnitude of the burden impedance (i) for a resistive burden of $100\ \Omega \leq R_{\mathrm{b}} \leq 2000\ \Omega$ and (ii) for a reactive burden of $100\ \Omega \leq X_{\mathrm{b}} \leq 2000\ \Omega$. Again, plot these curves on the same axis.

2.45 A 150-A:5-A, 60-Hz current transformer has the following parameters as seen from the 150-A (primary) winding:

$$X_1 = 1.70\ \mathrm{m\Omega} \quad X_2' = 1.84\ \mathrm{m\Omega} \quad X_{\mathrm{m}} = 1728\ \mathrm{m\Omega}$$

$$R_1 = 306\ \mu\Omega \quad R_2' = 291\ \mu\Omega$$

a. Assuming a current of 150 A in the primary and that the secondary is short-circuited, find the magnitude and phase angle of the secondary current.

b. Repeat the calculation of part (a) if the CT is shorted through a 0.1-mΩ burden.

2.46 Consider the current transformer of Problem 2.45.

a. Use MATLAB to plot the percentage error in current magnitude as a function of the magnitude of the burden impedance (i) for a resistive burden of 50 $\mu\Omega \leq R_b \leq 200\ \mu\Omega$ and (ii) for a reactive burden of $50\ \mu\Omega \leq X_b \leq 200\ \mu\Omega$. Plot these curves on the same axis.

b. Next plot the phase error in degrees as a function of the magnitude of the burden impedance (i) for a resistive burden of $50\ \mu\Omega \leq R_b \leq 200\ \mu\Omega$ and (ii) for a reactive burden of burden of $50\ \mu\Omega \leq X_b \leq 200\ \mu\Omega$. Again, plot these curves on the same axis.

2.47 A 15-kV:175-kV, 225-MVA, 60-Hz single-phase transformer has primary and secondary impedances of $0.0029 + j0.023$ per unit each. The magnetizing impedance is $j172$ per unit. All quantities are in per unit on the transformer base. Calculate the primary and secondary resistances and reactances and the magnetizing reactance in ohms (referred to the low-voltage side).

2.48 Calculate the per-unit parameters for a cantilever equivalent circuit for the transformer of Problem 2.20.

2.49 Calculate the per-unit parameters for a cantilever equivalent circuit for the transformer of Problem 2.23.

2.50 The nameplate on a 7.97-kV:266-V, 25-kVA single-phase transformer indicates that it has a series reactance of 7.5 percent (0.075 per unit).

a. Calculate the series reactance in ohms as referred to (i) the low-voltage terminal and (ii) the high-voltage terminal.

b. If three of these transformers are connected in a three-phase Y-Y connection, calculate (i) the three-phase voltage and power rating, (ii) the per unit impedance of the transformer bank, (iii) the series reactance in ohms as referred to the high-voltage terminal and (iv) the series reactance in ohms as referred to the low-voltage terminal.

c. Repeat part (b) if the three transformers are connected in Y on their HV side and Δ on their low-voltage side.

2.51 a. Consider the Y-Y transformer connection of Problem 2.50, part (b). If a line-line voltage of 500 V is applied to the high-voltage terminals and the three low-voltage terminals are short-circuited, calculate the magnitude of the phase current in per unit and in amperes on (i) the high-voltage side and (ii) the low-voltage side.

b. Repeat this calculation for the Y-Δ connection of Problem 2.50, part (c).

2.52 A three-phase generator step-up transformer is rated 26-kV:345-kV, 850 MVA and has a series impedance of $0.0025 + j0.057$ per unit on this base. It is connected to a 26-kV 800-MVA generator, which can be represented as a voltage source in series with a reactance of $j1.28$ per unit on the generator base.

a. Convert the per unit generator reactance to the step-up transformer base.
b. The system is supplying 750 MW at 345 kV and 0.90 power factor leading to the system at the transformer terminals. Draw a phasor diagram for this condition, using the transformer high-side voltage as the reference phasor.
c. Calculate the generator terminal voltage and internal voltage behind its reactance in kV for the conditions of part (b). Find the generator output power in MW and the power factor.