CHAPTER 9

P.P.9.1 amplitude = **30**

phase = **–75°**

angular frequency (ω) = 4π = **12.57 rad/s**

period (T) $= \frac{2\pi}{\omega}$ = **0.5 s**

frequency (f) $= \frac{1}{T}$ = **2 Hz**

P.P.9.2 $i_1 = -4\sin(\omega t + 55°) = 4\cos(\omega t + 55° + 90°)$

$i_1 = 4\cos(\omega t + 145°)$, $\omega = 377$ rad/s

Compare this with

$i_2 = 5\cos(\omega t - 65°)$

indicates that the phase angle between i_1 and i_2 is

145° + 65° = 210°

Thus, **i_1 leads i_2 by 210°**

P.P.9.3 (a) (5 + j2)(-1 + j4) = -5 + j20 – j2 – 8 = -13 + j18

$5\angle 60°$ = 2.5 + j4.33

(5 + j2)(-1 + j4) – $5\angle 60°$ = -15.5 + j13.67

$[(5 + j2)(-1 + j4) - 5\angle 60]^*$ = **-15.5 – j13.67** = **$20.67\angle 221.41°$**

(b) $3\angle 40°$ = 2.298 + j1.928

10 + j5 + $3\angle 40°$ = 12.298 + j6.928 = $14.115\angle 29.39°$

–3 + j4 = $5\angle 126.87°$

$$\frac{10 + j5 + 3\angle 40°}{-3 + j4} = \frac{14.115\angle 29.39°}{5\angle 126.87°} = 2.823\angle -97.48°$$

$2.823\angle -97.48°$ = –0.3675 – j2.8

$10\angle 30°$ = 8.66 + j5

$$\frac{10 + j5 + 3\angle 40°}{-3 + j4} + 10\angle 30° + j5 = \mathbf{8.293 + j7.2}$$

P.P.9.4 (a) v = 7 cos(2t + 40°)

The phasor form is

V = $7\angle 40°$ V

(b) Since –sin(A) = cos(A + 90°),

$i = -4\sin(10t + 10°) = 4\cos(10t+10° + 90°)$
$i = 4\cos(10t + 100°)$

The phasor form is

$\mathbf{I} = \mathbf{4\angle 100°\ A}$

P.P.9.5 (a) Since $-1 = 1\angle\pm180°$ (we can use either sign)

$\mathbf{V} = -25\angle 40° = 25\angle(40°-180°) = 25\angle -140°$

The sinusoid is

$v(t) =$ **25 cos(ωt – 140°) V** or **25 cos(ωt+220°) V**

(b) $\mathbf{I} = j(12 - j5) = 5 + j12 = 13\angle 67.38°$

The sinusoid is

i(t) = 13 cos(ωt + 67.38°) A

P.P.9.6 Let $v(t) = -10\sin(\omega t-30°) + 20\cos(\omega t+45°)$
$= 10\cos(\omega t-30°+90°) + 20\cos(\omega t+45°)$

Taking the phasor of each term

$\mathbf{V} = 10\angle 60° + 20\angle 45°$
$\mathbf{V} = 5 + j8.66 + 14.142 + j14.142$
$\mathbf{V} = 19.142 + j22.8 = 29.77\angle 49.98°$

Converting **V** to the time domain

$v(t) =$ **29.77 cos(ωt + 49.98°) V**

P.P.9.7 Given that

$$2\frac{dv}{dt} + 5v + 10\int v\,dt = 50\cos(5t - 30°)$$

we take the phasor of each term to get

$$2j\omega\,\mathbf{V} + 5\,\mathbf{V} + \frac{10}{j\omega}\mathbf{V} = 50\angle -30°, \qquad \omega = 5$$

$$\mathbf{V}\,[j10 + 5 - j(10/5)] = \mathbf{V}\,(5 + j8) = 50\angle -30°$$

$$\mathbf{V} = \frac{50\angle -30°}{5 + j8} = \frac{50\angle -30°}{9.434\angle 58°}$$

$$\mathbf{V} = 5.3\angle -88°$$

Converting **V** to the time domain

$v(t) =$ **5.3 cos(5t – 88°)V**

P.P.9.8 For the capacitor,

$\mathbf{V} = \mathbf{I}/(j\omega C)$, where $\mathbf{V} = 10\angle 30°$, $\omega = 100$
$\mathbf{I} = j\omega C\,\mathbf{V} = (j100)(50\text{x}10^{-6})(10\angle 30°)$
$\mathbf{I} = 50\angle 120°$ mA
$i(t) =$ **50 cos(100t + 120°) mA**

P.P.9.9 $\mathbf{V}_s = 20\angle 30°$, $\omega = 10$

$\mathbf{Z} = 4 + j\omega L = 4 + j2$

$$\mathbf{I} = \mathbf{V}_s / \mathbf{Z} = \frac{20\angle 30°}{4 + j2} = \frac{20\angle 30°(4 - j2)}{16 + 4} = 4.472\angle 3.43°$$

$$\mathbf{V} = j\omega L\, \mathbf{I} = j2\, \mathbf{I} = (2\angle 90°)(4.472\angle 3.43°) = 8.944\angle 93.43°$$

Therefore, $v(t) = \mathbf{8.944 \sin(10t + 93.43°)\ V}$

$i(t) = \mathbf{4.472 \sin(10t + 3.43°)\ A}$

P.P.9.10

Let $\mathbf{Z}_1$ = impedance of the 1-mF capacitor in series with the 100-Ω resistor

$\mathbf{Z}_2$ = impedance of the 1-mF capacitor

$\mathbf{Z}_3$ = impedance of the 8-H inductor in series with the 200-Ω resistor

$$\mathbf{Z}_1 = 100 + \frac{1}{j\omega C} = 100 + \frac{1}{j(10)(1\times 10^{-3})} = 80 - j100$$

$$\mathbf{Z}_2 = \frac{1}{j\omega C} = \frac{1}{j(10)(1\times 10^{-3})} = -j100$$

$$\mathbf{Z}_3 = 200 + j\omega L = 200 + j(10)(8) = 200 + j80$$

$$\mathbf{Z}_{in} = \mathbf{Z}_1 + \mathbf{Z}_2 \parallel \mathbf{Z}_3 = \mathbf{Z}_1 + \mathbf{Z}_2\mathbf{Z}_3 / (\mathbf{Z}_2 + \mathbf{Z}_3)$$

$$\mathbf{Z}_{in} = 100 - j100 + \frac{-j100x(200 + j80)}{-j100 + 200 + j80}$$

$$\mathbf{Z}_{in} = 100 - j100 + 49.52 - j95.04$$

$$\mathbf{Z}_{in} = \mathbf{[149.52 - j195]\ \Omega}$$

P.P.9.11 In the frequency domain,

the voltage source is $\mathbf{V}_s = 20\angle 100°$

the 0.5-H inductor is $j\omega L = j(10)(0.5) = j5$

the $\frac{1}{20}$-F capacitor is $\frac{1}{j\omega C} = \frac{1}{j(10)(1/20)} = -j2$

Let $\mathbf{Z}_1$ = impedance of the 0.5-H inductor in parallel with the 10-Ω resistor

and $\mathbf{Z}_2$ = impedance of the (1/20)-F capacitor

$$\mathbf{Z}_1 = 10 \parallel j5 = \frac{(10)(j5)}{10 + j5} = 2 + j4 \qquad \text{and} \qquad \mathbf{Z}_2 = -j2$$

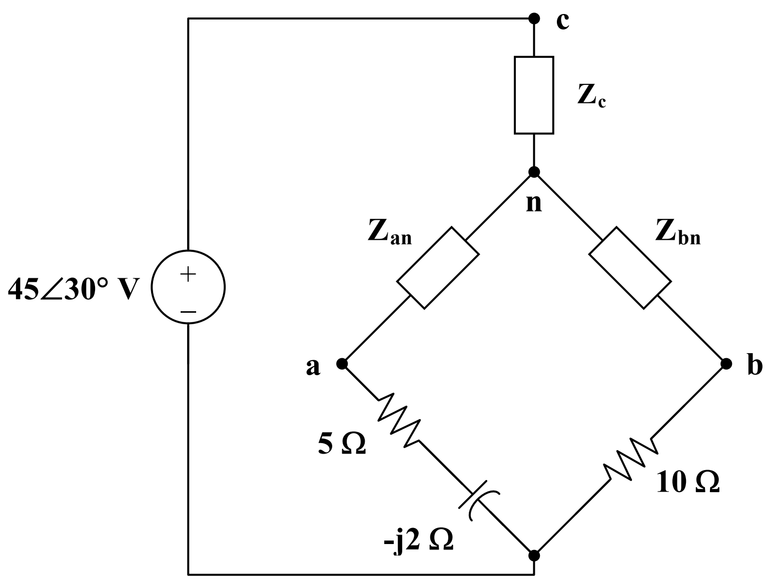
$$\mathbf{V}_o = \mathbf{Z}_2 / (\mathbf{Z}_1 + \mathbf{Z}_2)\, \mathbf{V}_s$$

$$\mathbf{V}_o = \frac{-j2}{2 + j4 - j2}(50\angle 30°) = \frac{-j(50\angle 30°)}{1 + j} = \frac{50\angle(30° - 90°)}{\sqrt{2}\angle 45°}$$

$$\mathbf{V}_o = 35.36\angle -105°$$

$v_o(t) = \mathbf{35.36 \cos(10t - 105°)\ V}$

P.P.9.12 We need to find the equivalent impedance via a delta-to-wye transformation as shown below.



$$\mathbf{Z}_{an} = \frac{j4(8+j5)}{j4+8+j5-j3} = \frac{4(-5+j8)}{8+j6} = 0.32 + j3.76$$

$$\mathbf{Z}_{bn} = \frac{-j3(8+j5)}{8+j6} = \frac{3(5-j8)(8-j6)}{100} = -0.24 - j2.82$$

$$\mathbf{Z}_{cn} = \frac{j4(-j3)}{8+j6} = \frac{12(8-j6)}{100} = 0.96 - j0.72$$

The total impedance from the source terminals is

$$\mathbf{Z} = \mathbf{Z}_{cn} + (\mathbf{Z}_{an} + 5 - j2) \parallel (\mathbf{Z}_{bn} + 10)$$

$$\mathbf{Z} = \mathbf{Z}_{cn} + (5.32 + j1.76) \parallel (9.76 - j2.82)$$

$$\mathbf{Z} = \mathbf{Z}_{cn} + \frac{(5.32 + j1.76)(9.76 - j2.82)}{(5.32 + j1.76) + (9.76 - j2.82)}$$

$$\mathbf{Z} = 0.96 - j0.72 + 3.744 + j0.4074$$

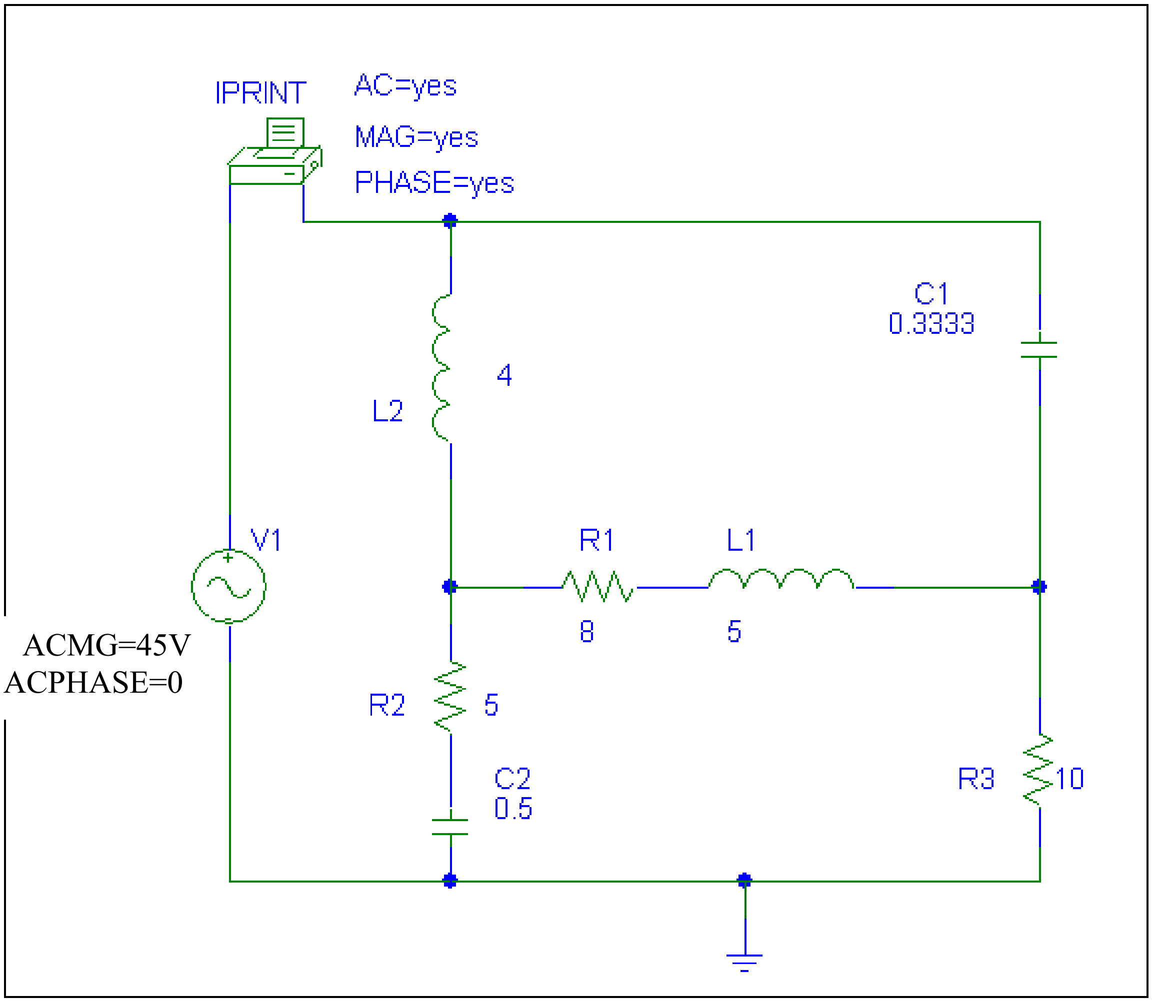
$$\mathbf{Z} = 4.704 - j0.3126 = 4.714\angle{-3.802^\circ}$$

Therefore,

$$\mathbf{I} = \mathbf{V} / \mathbf{Z} = \frac{45\angle 30^\circ}{4.714\angle -3.802^\circ}$$

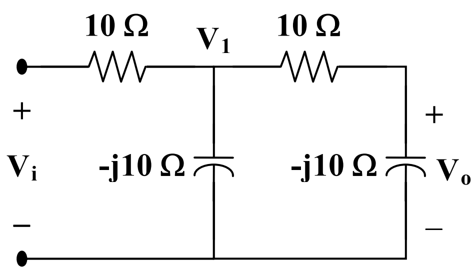
$$\mathbf{I} = \mathbf{9.546\ \angle 33.8^\circ\ A}$$

Let us now check this using PSpice. The solution produces the magnitude of I = 9.946E+00, and the phase angle = 33.803E+00, which agrees with the above answer.

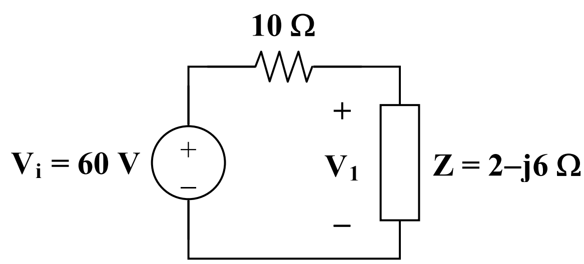


P.P.9.13 To show that the circuit in Fig. (a) meets the requirement, consider the equivalent circuit in Fig. (b).

$$\mathbf{Z} = \text{-j}10 \parallel (10 - \text{j}10) = \frac{\text{-j}10(10 - \text{j}10)}{10 - \text{j}20} = \frac{\text{-j}(10 - \text{j}10)}{1 - \text{j}2} = 2 - \text{j}6\ \Omega$$



(a)



(b)

$$\mathbf{V}_1 = \frac{2 - j6}{10 + 2 - j6}(60) = \frac{60}{3}(1 - j)$$

$$\mathbf{V}_o = \frac{-j10}{10 - j10}\mathbf{V}_1 = \left(\frac{-j}{1 - j}\right)\left(\frac{60}{3}\right)(1 - j) = -j20$$

$$\mathbf{V}_o = 20\angle -90^\circ$$

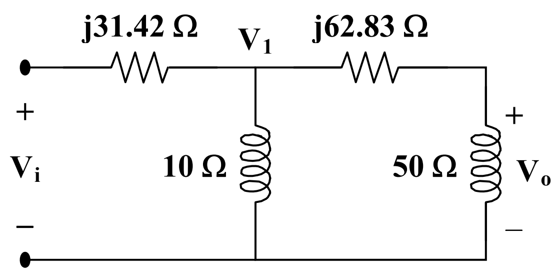
This implies that the RC circuit provides a 90° lagging phase shift.
The output voltage is = **20 V**

P.P.9.14

the 1-mH inductor is $j\omega L = j(2\pi)(5 \times 10^3)(1 \times 10^{-3}) = j31.42$

the 2-mH inductor is $j\omega L = j(2\pi)(5 \times 10^3)(2 \times 10^{-3}) = j62.83$

Consider the circuit shown below.



$$\mathbf{Z} = 10 \parallel (50 + j62.83) = \frac{(10)(50 + j62.83)}{60 + j62.83}$$

$$\mathbf{Z} = 9.205 + j0.833 = 9.243\angle 5.17^\circ$$

$$\mathbf{V}_1 = \mathbf{Z} / (\mathbf{Z} + j31.42)\,\mathbf{V}_i = \frac{9.243\angle 5.17^\circ}{9.205 + j32.253}(10)$$

$$= [(9.243\angle 5.17^\circ)/(33.54\angle 74.07^\circ)]10 = 2.756\angle -68.9^\circ$$

$$\mathbf{V}_o = \frac{50}{50 + j62.83}\mathbf{V}_1 = \frac{50(2.756\angle -68.9^\circ)}{80.297\angle 51.49^\circ} = 1.7161\angle -120.39^\circ$$

Therefore,

magnitude = **1.7161 V**
phase = **120.39°**
phase shift is **lagging**

P.P.9.15 $\mathbf{Z}_x = (\mathbf{Z}_3 / \mathbf{Z}_1)\,\mathbf{Z}_2$

$\mathbf{Z}_3 = 12\ \text{k}\Omega$

$\mathbf{Z}_1 = 4.8\ \text{k}\Omega$

$\mathbf{Z}_2 = 10 + j\omega L = 10 + j(2\pi)(6\times10^{6})(0.25\times10^{-6}) = 10 + j9.425$

$$\mathbf{Z}_x = \frac{12k}{4.8k}(10 + j9.425) = 25 + j23.5625\ \Omega$$

$R_x = 25, \quad X_x = 23.5625 = \omega L_x$

$$L_x = \frac{X_x}{2\pi f} = \frac{23.5625}{2\pi(6\times10^{6})} = 0.625\ \mu\text{H}$$

i.e. **a 25-Ω resistor in series with a 0.625-μH inductor**.