CHAPTER 11

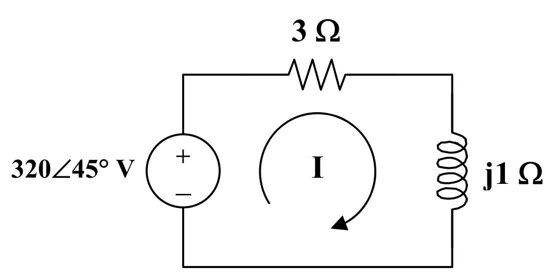
P.P.11.1 $i(t) = 33\sin(10t + 60°) = 33\cos(10t - 30°)$
$v(t) = 330\cos(10t + 20°)$

$$p(t) = v(t)\,i(t) = (330)(33)\cos(10t + 20°)\cos(10t - 30°)$$
$$p(t) = \frac{1}{2}\cdot 10890[\cos(20t + 20° - 30°) + \cos(20 - (-30°))]$$
$$\text{p(t)} = \mathbf{(3.5 + 5.445cos(20t–10^{\circ}))\ kW}$$
$$\text{P} = \frac{1}{2}\text{V}_\text{m}\text{I}_\text{m}\cos(\theta_\text{v} - \theta_\text{i}) = \mathbf{3.5\ kW}$$

P.P.11.2 $\mathbf{V} = \mathbf{I}\,\mathbf{Z} = 1320\angle 8°$

$$\text{P} = \frac{1}{2}\text{V}_\text{m}\text{I}_\text{m}\cos(\theta_\text{v} - \theta_\text{i})$$
$$P = \frac{1}{2}(1320)(33)\cos(8° - 30°) = \mathbf{20.19\ kW}$$

P.P.11.3



$$\mathbf{I} = \frac{320\angle 45°}{3 + j} = 101.19\angle 26.57°$$

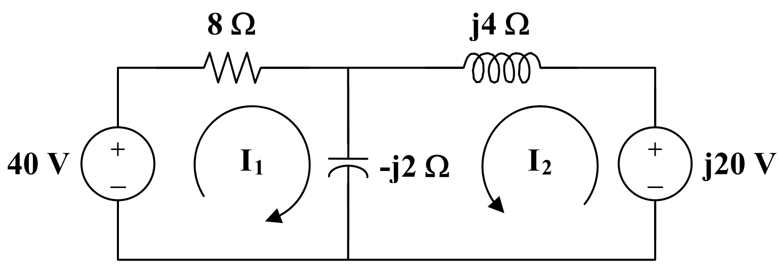
For the resistor,
$$\mathbf{I}_R = \mathbf{I} = 101.19\angle 26.57°$$
$$\mathbf{V}_R = 3\mathbf{I} = 303.6\angle 26.57°$$
$$P_R = \frac{1}{2}V_m I_m = \frac{1}{2}(303.6)(101.19) = \mathbf{15.361\ kW}$$

For the inductor,

$$\mathbf{I}_L = 101.19\angle 26.57°$$
$$\mathbf{V}_L = j\,\mathbf{I}_L = 101.19\angle(26.57° + 90°) = 101.19\angle 116.57°$$
$$P_L = \frac{1}{2}(101.19)^2 \cos(90°) = \mathbf{0\ W}$$

The average power supplied is
$$P = \frac{1}{2}(320)(101.19)\cos(45° - 26.57°) = \mathbf{15.361\ kW}$$

P.P.11.4 Consider the circuit below.



For mesh 1,
$$-40 + (8 - j2)\mathbf{I}_1 + (-j2)\mathbf{I}_2 = 0$$
$$(4 - j)\mathbf{I}_1 - j\mathbf{I}_2 = 20 \qquad (1)$$

For mesh 2,
$$-j20 + (j4 - j2)\mathbf{I}_2 + (-j2)\mathbf{I}_1 = 0$$
$$-j\mathbf{I}_1 + j\mathbf{I}_2 = j10 \qquad (2)$$

In matrix form,
$$\begin{bmatrix} 4-j & -j \\ -j & j \end{bmatrix}\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 20 \\ j10 \end{bmatrix}$$

$$\Delta = 2 + j4, \quad \Delta_1 = -10 + j20, \quad \Delta_2 = 10 + j60$$

$$\mathbf{I}_1 = \frac{\Delta_1}{\Delta} = 5\angle 53.14° \quad \text{and} \quad \mathbf{I}_2 = \frac{\Delta_2}{\Delta} = 13.6\angle 17.11°$$

For the 40-V voltage source,
$$\mathbf{V}_s = 40\angle 0°$$
$$\mathbf{I}_1 = 5\angle 53.14°$$
$$P_s = \frac{-1}{2}(40)(5)\cos(-53.14°) = \mathbf{-60\ W}$$

For the j20-V voltage source,

$$\mathbf{V}_s = 20\angle 90°$$

$$\mathbf{I}_2 = 13.6\angle 17.11°$$

$$P_s = \frac{-1}{2}(20)(13.6)\cos(90° - 17.11°) = \mathbf{-40\ W}$$

For the resistor,

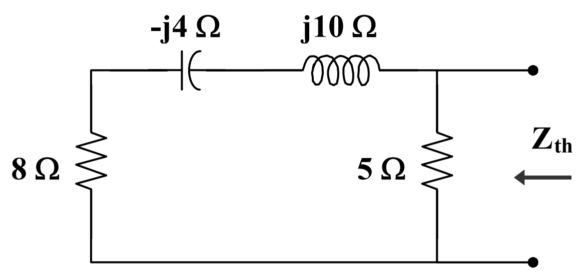
$$I = |\mathbf{I}_1| = 5$$

$$V = 8|\mathbf{I}_1| = 40$$

$$P = \frac{1}{2}(40)(5) = \mathbf{100\ W}$$

The average power absorbed by the inductor and capacitor is **zero watts**.

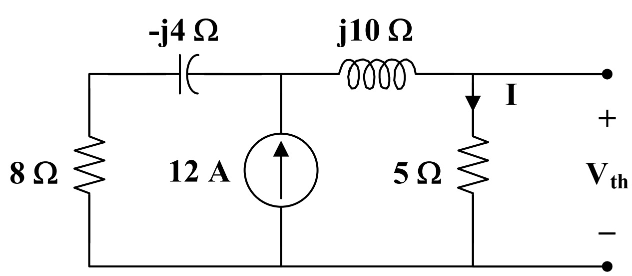
P.P.11.5 We first obtain the Thevenin equivalent circuit across $\mathbf{Z}_L$. $\mathbf{Z}_{Th}$ is obtained from the circuit in Fig. (a).



(a)

$$\mathbf{Z}_{Th} = 5 \,||\, (8 - j4 + j10) = \frac{(5)(8 + j6)}{13 + j6} = 3.415 + j0.7317$$

$\mathbf{V}_{Th}$ is obtained from the circuit in Fig. (b).



(b)

By current division,

$$\mathbf{I} = \frac{8 - j4}{8 - j4 + j10 + 5}(12)$$

$$\mathbf{V}_{Th} = 5\mathbf{I} = \frac{(60)(8 - j4)}{13 + j6} = 37.5\angle -51.34°$$

$\mathbf{Z}_L = (\mathbf{Z}_{Th})^* = [3.415 - j0.7317]\ \Omega$

$$P_{\max} = \frac{|\mathbf{V}_{Th}|^2}{8R_L} = \frac{(37.5)^2}{(8)(3.415)} = \mathbf{51.47\ W}$$

P.P.11.6 We first find $\mathbf{Z}_{Th}$ and $\mathbf{V}_{Th}$ across R_L.

Let $\mathbf{Z}_1 = 80 + j60$

$$\mathbf{Z}_2 = 90 \parallel (-j30) = \frac{(90)(-j30)}{90 - j30} = 9(1 - j3)$$

$$\mathbf{Z}_{Th} = \mathbf{Z}_1 \parallel \mathbf{Z}_2 = \frac{(80 + j60)(9 - j27)}{80 + j60 + 9 - j27} = 17.181 - j24.57\ \Omega$$

$$\mathbf{V}_{Th} = \frac{\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}(120\angle 60°) = \frac{(9)(1 - j3)}{89 + j33}(120\angle 60°)$$

$$\mathbf{V}_{Th} = 35.98\angle -31.91°$$

$$R_L = |\mathbf{Z}_{Th}| = \mathbf{30\ \Omega}$$

The current through the load is

$$\mathbf{I} = \frac{\mathbf{V}_{Th}}{\mathbf{Z}_{Th} + R_L} = \frac{35.98\angle -31.91°}{47.181 - j24.57} = 0.6764\angle -4.4°$$

The maximum average power absorbed by R_L is

$$P_{\max} = \frac{1}{2}|\mathbf{I}|^2 R_L = \frac{1}{2}(0.6764)^2(30) = \mathbf{6.863\ W}$$

P.P.11.7 $i(t) = \begin{cases} 16t & 0 < t < 1 \\ 32 - 16t & 1 < t < 2 \end{cases}$ T = 2

$$I_{rms}^2 = \frac{1}{T}\int_0^T i^2\, dt = \frac{1}{2}\left[\int_0^1 (16t)^2\, dt + \int_1^2 (32 - 16t)^2\, dt\right]$$

$$I_{rms}^2 = \frac{256}{2}\left[\int_0^1 t^2\, dt + \int_1^2 (4 - 4t + t^2)\, dt\right]$$

$$I_{rms}^2 = 128\left[\frac{1}{3} + \left(4t - 2t^2 + \frac{t^3}{3}\right)\Big|_1^2\right] = \frac{256}{3}$$

$$I_{rms} = \sqrt{\frac{256}{3}} = \mathbf{9.238\ A}$$

$$P = I_{rms}^2 R = (9.238^2)(9) = \mathbf{768\ w}$$

P.P.11.8 T = π , $v(t) = 100\sin(t), \quad 0 < t < \pi$

$$V_{rms}^2 = \frac{1}{T}\int_0^T v^2\ dt = \frac{1}{\pi}\int_0^{\pi} (100\sin(t))^2\ dt$$

$$V_{rms}^2 = \frac{10^4}{\pi}\int_0^{\pi} \frac{1}{2}\left[1 - \cos(2t)\right] dt = 5000$$

$$V_{rms} = \mathbf{70.71\ V}$$

$$P = \frac{V_{rms}^2}{R} = \frac{5000}{6} = \mathbf{833.3\ W}$$

P.P.11.9 The load impedance is

$$\mathbf{Z} = 60 + j40 = 72.11\angle 33.7°\ \Omega$$

The power factor is

$$\text{pf} = \cos(33.7°) = \mathbf{0.8321\ lagging}$$

Since the load is inductive

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{320\angle 10°}{72.11\angle 33.7°} = 4.438\angle -23.69°\ A$$

The apparent power is

$$\mathbf{S} = \mathbf{V_{rms}}(\mathbf{I_{rms}})^* = 0.5(320)(4.438)\angle(10° - (-23.69°)) = \mathbf{710\angle 33.69°\ VA}$$

P.P.11.10 The total impedance as seen by the source is

$$\mathbf{Z} = 10 + j4 \,\|\, (8 - j6) = 10 + \frac{(j4)(8 - j6)}{8 - j2}$$

$$\mathbf{Z} = 12.69\angle 20.62°$$

The power factor is

$$\text{pf} = \cos(20.62°) = \mathbf{0.936 \quad (lagging)}$$

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{165\angle 0°}{12.69\angle 20.62°} = 13.002\angle -20.62°$$

The average power supplied by the source is equal to the power absorbed by the load.

$$P = I_{rms}^2 R = (13.002)^2 (11.88) = 1{,}062\ W = \mathbf{2.008\ kW}$$

or $$P = V_{rms} I_{rms}\, pf = (165)(13.002)(0.936) = \mathbf{2.008\ kW}$$

P.P.11.11

(a) $\mathbf{S} = \mathbf{V}_{rms}\mathbf{I}^*_{rms} = (110\angle 85°)(0.4\angle -15°)$

$\mathbf{S} = \mathbf{44\angle 70°\ VA}$

$S = |\mathbf{S}| = \mathbf{44\ VA}$

(b) $\mathbf{S} = 44\angle 70° = 15.05 + j41.35$

$P = \mathbf{15.05\ W}$, $Q = \mathbf{41.35\ VAR}$

(c) $pf = \cos(70°) = \mathbf{0.342}$ **(lagging)**

$$\mathbf{Z} = \frac{\mathbf{V}_{rms}}{\mathbf{I}_{rms}} = \frac{110\angle 85°}{0.4\angle -15°} = 275\angle 70°$$

$\mathbf{Z} = \mathbf{94.06 + j258.4\ \Omega}$

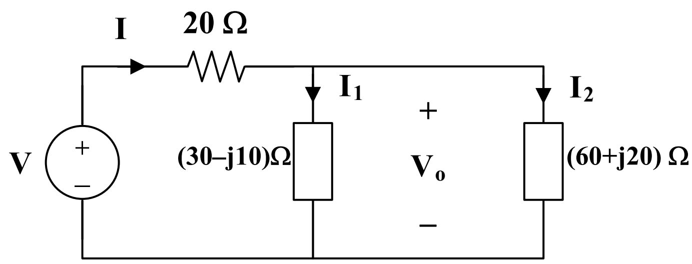
P.P.11.12

(a) If $\mathbf{Z} = 250\angle -75°$, $pf = \cos(-75°) = \mathbf{0.2588}$ **(leading)**

(b) $$Q = S\sin\theta \longrightarrow S = \frac{Q}{\sin\theta} = \frac{-100\ kVAR}{\sin(-75°)} = \mathbf{103.53\ kVA}$$

(c) $$S = \frac{V_{rms}^2}{|\mathbf{Z}|} \longrightarrow V_{rms} = \sqrt{S\cdot|\mathbf{Z}|} = \sqrt{(103530)(250)} = \mathbf{5.087\ kV}$$

P.P.11.13 Consider the circuit below.



Let $\mathbf{I}_2$ be the current through the 60-Ω resistor.

$$P = I_2^2 R \longrightarrow I_2^2 = \frac{P}{R} = \frac{240}{60} = 4$$

$\mathbf{I}_2 = 2$ (rms)

$$\mathbf{V}_o = \mathbf{I}_2\,(60 + j20) = 120 + j40$$

$$\mathbf{I}_1 = \frac{\mathbf{V}_o}{30 - j10} = 3.2 + j2.4$$

$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 = 5.2 + j2.4$$

$$\mathbf{V} = 20\mathbf{I} + \mathbf{V}_o = (104 + j48) + (120 + j40)$$

$$\mathbf{V} = 224 + j88 = \mathbf{240.7\angle 21.45^\circ\ V_{rms}}$$

For the 20-Ω resistor,

$$\mathbf{V} = 20\mathbf{I} = 204 + j48 = 114.54\angle 24.8^\circ$$

$$\mathbf{I} = 5.2 + j2.4 = 5.727\angle 24.8^\circ$$

$$\mathbf{S} = \mathbf{V}\,\mathbf{I}^* = (114.54\angle 24.8^\circ)(5.727\angle -24.8^\circ)$$

$$\mathbf{S} = \mathbf{656\ VA}$$

For the (30 – j10)-Ω impedance,

$$\mathbf{V}_o = 120 + j40 = 126.5\angle 18.43^\circ$$

$$\mathbf{I}_1 = 3.2 + j2.4 = 4\angle 36.87^\circ$$

$$\mathbf{S}_1 = \mathbf{V}_o\,\mathbf{I}_1^* = (126.5\angle 18.43^\circ)(4\angle -36.87^\circ)$$

$$\mathbf{S}_1 = 506\angle -18.44^\circ = \mathbf{[480 - j160]\ VA}$$

For the (60 + j20)-Ω impedance,

$$\mathbf{I}_2 = 2\angle 0^\circ$$

$$\mathbf{S}_2 = \mathbf{V}_o\,\mathbf{I}_2^* = (126.5\angle 18.43^\circ)(2\angle -0^\circ)$$

$$\mathbf{S}_2 = 253\angle 18.43^\circ = \mathbf{[240 + j80]\ VA}$$

The overall complex power supplied by the source is

$$\mathbf{S}_T = \mathbf{V}\,\mathbf{I}^* = (240.67\angle 21.45^\circ)(5.727\angle -24.8^\circ)$$

$\mathbf{S_T}$ = 1378.3∠–3.35° = **[1376–j80] VA**

P.P.11.14

For load 1,

$$P_1 = 2000\ , \qquad pf = 0.75 = \cos\theta_1 \longrightarrow \theta_1 = -41.41°$$

$$P_1 = S_1 \cos\theta_1 \longrightarrow S_1 = \frac{P_1}{\cos\theta_1} = 2666.67$$

$$Q_1 = S_1 \sin\theta_1 = -176.85$$

$$\mathbf{S}_1 = P_1 + jQ_1 = 2000 - j1763.85 \quad \text{(leading)}$$

For load 2,

$$P_2 = 4000\ , \quad pf = 0.95 = \cos\theta_2 \longrightarrow \theta_2 = 18.19°$$

$$S_2 = \frac{P_2}{\cos\theta_2} = 4210.53$$

$$Q_2 = S_2 \sin\theta_2 = 1314.4$$

$$\mathbf{S}_2 = P_2 + jQ_2 = 4000 + j1314.4 \quad \text{(lagging)}$$

The total complex power is

$$\mathbf{S = S_1 + S_2 = [6–j0.4495]\ kVA}$$

$$pf = \frac{P}{|\mathbf{S}|} = \frac{6000}{6016.18} = \mathbf{0.9972} \quad \textbf{(leading)}$$

P.P.11.15 $pf = 0.85 = \cos\theta \longrightarrow \theta = 31.79°$

$$Q = S\sin\theta \longrightarrow S = \frac{Q}{\sin\theta} = \frac{140}{\sin(31.79°)} = 265.8\ \text{kVA}$$

$$P = S\cos\theta = 225.93\ \text{kW}$$

For $pf = 1 = \cos\theta_1 \longrightarrow \theta_1 = 0°$

Since P remains the same,

$$P = P_1 = S_1 \cos\theta_1 \longrightarrow S_1 = \frac{P_1}{\cos\theta_1} = 225.93$$

$$Q_1 = S_1 \sin\theta_1 = 0$$

The difference between the new Q_1 and the old Q is Q_c.

$$Q_c = 140\ \text{kVAR} = \omega C V_{rms}^2$$

$$C = \frac{140 \times 10^3}{(2\pi)(60)(110)^2} = \mathbf{30.69\ mF}$$

P.P.11.16 The wattmeter measures the average power from the source.

Let $\mathbf{Z}_1 = 4 - j2$

$$\mathbf{Z}_2 = 12 \parallel j9 = \frac{(12)(j9)}{12 + j9} = 4.32 + j5.76$$

$$\mathbf{Z} = \mathbf{Z}_1 + \mathbf{Z}_2 = 8.32 + j3.76 = 9.13\angle 24.32°$$

$$\mathbf{S} = \mathbf{V}\,\mathbf{I}^* = \frac{|\mathbf{V}|^2}{\mathbf{Z}^*} = \frac{(120)^2}{9.13\angle -24.32°} = 1577.2\angle 24.32°\ VA$$

$$P = |\mathbf{S}|\cos\theta = \mathbf{1.437\ kW}$$

P.P.11.17 Demand charge $= \$5 \times 32{,}000 = \$160{,}000$
Energy charge for the first $50{,}000$ kWh $= \$0.08 \times 50{,}000 = \$4{,}000$
The remaining energy $= 500{,}000 - 50{,}000 = 450{,}000$ kWh
Charge for this bill $= \$0.05 \times 450{,}000 = \$22{,}500$
Total bill $= \$160{,}000 + \$4{,}000 + \$22{,}500 =$ **$186,500**

P.P.11.18 Energy consumed $= 800\ \text{kW} \times 20 \times 26 = 416{,}000$ kWh

The power factor of 0.88 exceeds 0.85 by 3×0.01. Hence, there is a power factor credit which amounts to an energy credit of

$$416{,}000 \times \frac{0.1}{100} \times 3 = 1248\ \text{kWh}$$

Total energy billed $= 416{,}000 - 1{,}248 = 414{,}752$ kWh
Energy cost $= \$0.06 \times 414{,}752 =$ **$24,885.12**