

Rosen, Discrete Mathematics and Its Applications, 7th edition
Extra Examples
Section 1.2—Applications of Propositional Logic



— Page references correspond to locations of Extra Examples icons in the textbook.

p.17, icon at Example 1

#1. Suppose u represents “you understand the material”, s represents “you study the theory”, and w represents “you work on exercises”. Write the following compound proposition using u , s , w , and appropriate connectives.

“You study the theory and work on exercises, but you don’t understand the material.”

Solution:

This compound statement makes three assertions: you study the theory, you work on exercises, you don’t understand the material. The word “but” acts like “and”. Thus, the proposition is $s \wedge w \wedge \neg u$.

p.17, icon at Example 1

#2. Suppose u represents “you understand the material” and s represents “you study the theory”. Write the following compound proposition using u , s , and appropriate connectives.

“Studying the theory is sufficient for understanding the material.”

Solution:

We can rewrite the statement as “If you study the theory, then you understand the material,” or in symbols $s \rightarrow u$.

p.17, icon at Example 1

#3. Suppose s represents “you study the theory” and w represents “you work on exercises”. Write the following compound proposition using s , w , and appropriate connectives.

“In order to work on exercises, you need to study the theory.”

Solution:

The statement is “If you do not study the theory, then you cannot work on the exercises,” or $\neg s \rightarrow \neg w$. We can also rewrite this as $w \rightarrow s$.

p.17, icon at Example 1

#4. Suppose u represents “you understand the material”, s represents “you study the theory”, and w represents “you work on exercises”. Write the following compound proposition using u , s , w , and appropriate connectives.

“When you study the theory and work on exercises, you understand the material.”

Solution:

Using “if” for “when”, we have $s \wedge w \rightarrow u$.

p.17, icon at Example 1

#5. Suppose u represents “you understand the material”, s represents “you study the theory”, and w represents “you work on exercises”. Write the following compound proposition using u , s , w , and appropriate connectives.

“You don’t understand the material unless you study the theory and work on exercises.”

Solution:

The word “unless” conveys the meaning “if not”. We can rewrite the proposition as “If you do not study the theory and work on exercises, then you don’t understand the material,” which is $\neg(s \wedge w) \rightarrow \neg u$, or, equivalently, $u \rightarrow (s \wedge w)$.

p.17, icon at Example 3

#1. Translate this system specification into symbols:

“The online user is sent a notification of a link error if the network link is down.”

Solution:

The statement is equivalent to “If the network link is down, then the online user is sent a notification of a link error.” Using d for “the network link is down” and s for “the online user is sent a notification of a link error,” the statement becomes $d \rightarrow s$.

p.17 icon at Example 3

#2. Translate this system specification into symbols:

“Whenever the file is locked or the system is in executive clearance mode, the user cannot make changes in the data.”

Solution:

The statement is equivalent to “If the file is locked or the system is in executive clearance mode, the user cannot make changes in the data.” Using l for “the file is locked,” e for “the system is in executive clearance mode,” and u for “the user can make changes in the data,” the statement is $(l \vee e) \rightarrow \neg u$. Note that the parentheses are not necessary because the order of precedence of operations requires that the disjunction be

performed before the implication; thus we can also write $l \vee e \rightarrow \neg u$.

p.17, icon at Example 3

#3. Write these system specifications in symbols using the propositions

v : “The user enters a valid password,”

a : “Access is granted to the user,”

c : “The user has contacted the network administrator,”

and logical connectives. Then determine if the system specifications are consistent.

(i) “The user has contacted the network administrator, but does not enter a valid password.”

(ii) “Access is granted whenever the user has contacted the network administrator or enters a valid password.”

(iii) “Access is denied if the user has not entered a valid password or has not contacted the network administrator.”

Solution:

(i) The word “but” means “and”, so we have $c \wedge \neg v$.

(ii) The statement says that if either of two conditions is satisfied, then access is granted. Therefore we have $(c \vee v) \rightarrow a$.

(iii) In this case, if either of two negations happens, then access is not granted to the user. Therefore we have $(\neg v \vee \neg c) \rightarrow \neg a$.

In symbols, the three propositions are:

$$c \wedge \neg v$$

$$(c \vee v) \rightarrow a$$

$$(\neg v \vee \neg c) \rightarrow \neg a.$$

In order for the first proposition to be true, c must be true and v must be false. Because c is true, $c \vee v$ is true. Therefore, from the second proposition, a must be true.

But because v is false, $\neg v$ is true. Therefore $\neg v \vee \neg c$ is true. Therefore, the implication in the third proposition forces $\neg a$ to be true. Thus, a is false. But this contradicts that fact that a must be true. Therefore the three propositions are not consistent.

p.18, icon at Example 4

#1. How would you do a Boolean search for the appropriate Web pages for each of these:

(a) hotels in New England.

(b) hotels in England.

(c) hotels in England or New England.

Solution:

(a) We need to examine “hotels” and both “New” and “England”; that is, **HOTELS AND (NEW AND ENGLAND)**.

(b) To avoid getting hotels in New England, we use ENGLAND NOT NEW. Therefore we have HOTELS AND (ENGLAND NOT NEW).

(c) The two key words here are “hotels” and “England” (which will include both the country and the part of the United States). Therefore we can search for HOTELS AND ENGLAND.

p.19, icon at Example 7

#1. Suppose you have three cards: one red on both sides (red/red), one green on both sides (green/green), and one red on one side and green on the other side (red/green). The three cards are placed in a row on a table. Explain how to determine the identity of all three cards by selecting one card and turning it over.

Solution:

When the three cards are put in a row, exactly two of the three must have the same color showing — say red. Pick one of these two red cards and turn it over.

If the other side is also red, then you have found the red/red card. The other card with red showing must be the red/green card, and the card with green showing must be the green/green card.

If the card you turn over has green on the other side, you have located the red/green card. The other card with red showing must be the red/red card, and the card with green showing must be the green/green card.

(A similar procedure will determine the identity of the three cards, if two cards have green showing.)

p.19 icon at Example 7

#2. Another of Smullyan’s puzzles poses this problem. You meet two people, *A* and *B*. Each person either always tells the truth (i.e., the person is a knight) or always lies (i.e., the person is a knave). Person *A* tells you, “We are not both truth-tellers.”

Determine, if possible, which type of person each one is.

Solution:

One way to solve the problem is by considering each of the four possible cases: both lie, both tell the truth, *A* lies and *B* tells the truth, *A* tells the truth and *B* lies. We examine each of the four cases separately.

Both lie: That would mean that neither one is a truth-teller. The statement “We are not both truth-tellers” is true because they are not both truth-tellers. Thus *A* is telling the truth, which contradicts the assumption that both are liars. Therefore, this case cannot happen.

Both tell the truth: Therefore the statement “We are not both truth-tellers” is a lie. Thus *A* is a liar, contradicting the assumption that both are truth-tellers. Therefore, this case cannot happen.

A lies and B tells the truth: If *A* is a liar, *A*’s statement “We are not both truth-tellers” must be a lie. Therefore *A* and *B* must both be truth-tellers. But this contradicts the assumption that *A* is a liar. Therefore, this case cannot happen.

A tells the truth and B lies: In this case *A*’s statement “We are not both truth-tellers” is true and no contradiction is obtained. Therefore, this case is not ruled out.

Therefore, the only possibility is that *A* is a truth-teller and *B* is a liar.

Another way to solve the problem is to see where the assumption “ A is a liar” leads us. If we assume that A always lies, then A ’s statement “We are not both truth-tellers” must be false. Therefore both A and B must be truth-tellers. Therefore A ’s statement “We are not both truth-tellers” must be true, a contradiction of the fact that the statement is false.

This says that A is not a liar. Therefore A is a truth-teller. Therefore A ’s statement “We are not both truth-tellers” must be true. Because A is a truth-teller, A ’s statement forces B to be a liar.
