Show All Solutions

Rosen, Discrete Mathematics and Its Applications, 7th edition
Extra Examples
Section 1.5—Nested Quantifiers

Extra Examples — *Page references correspond to locations of Extra Examples icons in the textbook.*

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p.57, icon at Example 1
#1. Write the following statements in English, using the predicate $S(x, y)$: "x shops in y", where x represents people and y represents stores:

(a) $\forall y\, S(Margaret, y)$.

(b) $\exists x \forall y\, S(x, y)$.

See Solution

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p.57, icon at Example 1
#2. Write in symbols using predicates and quantifiers: "Every Junior in this class scored above 90 on the first exam."

See Solution

p.57, icon at Example 1
#3. Write the following statement in English, using the predicates

$S(x, y)$: "x shops in y"
$T(x)$: "x is a student"

where x represents people and y represents stores:

$$\exists y \, \forall x \, (T(x) \to \neg S(x, y)).$$

See Solution

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p.57, icon at Example 1
#4. Write the following statement in English, using the predicates

$S(x, y)$: "x shops in y"
$T(x)$: "x is a student"

where x represents people and y represents stores:

$$\forall y \, \exists x \, (T(x) \wedge S(x, y)).$$

See Solution

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p.57, icon at Example 1
#5. Write the following statement in English, using the predicate $S(x, y)$ for "x shops in y", where x represents people and y represents stores:

$$\exists x_1 \, \exists y \, \forall x_2 \, [S(x_1, y) \wedge (x_1 \neq x_2 \to \neg S(x_2, y))].$$

See Solution

p.57, icon at Example 1

#6. Write the following statement in English, using the predicates

$C(x)$: "x is a Computer Science major"
$M(y)$: "y is a math course"
$T(x, y)$: "x is taking y"

where x represents students and y represents courses:

$$\forall x\, \exists y\, (C(x) \rightarrow M(y) \wedge T(x, y)).$$

See Solution

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p.57, icon at Example 1

#7. Write the following statement in English, using the predicates

$C(x)$: "x is a Computer Science major"
$T(x, y)$: "x is taking y"

where x represents students and y represents courses:

$$\forall y\, \exists x\, (\neg C(x) \wedge T(x, y)).$$

See Solution

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p.57, icon at Example 1

#8. Write the following statement in English, using the predicates

$F(x)$: "x is a Freshman"
$M(y)$: "y is a math course"
$T(x, y)$: "x is taking y"

where x represents students and y represents courses:

$$\neg \exists x\, [F(x) \wedge \forall y\, (M(y) \rightarrow T(x, y))].$$

See Solution

p.57, icon at Example 1
#9. Write the following statement using quantifiers and the predicate $S(x, y)$ for "x shops in y", where the universe for x consists of people and the universe for y consists of stores:

"Will shops in Al's Record Shoppe."

See Solution

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p.57, icon at Example 1
#10. Write the following statement using quantifiers and the predicates

$S(x, y)$: "x shops in y"
$T(x)$: "x is a student"

where the universe for x consists of people and the universe for y consists of stores:

"There is no store that has no students who shop there."

See Solution

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p.57, icon at Example 1
#11. Write the following statement using quantifiers and the predicates

$S(x, y)$: "x shops in y"
$T(x)$: "x is a student"

where the universe for x consists of people and the universe for y consists of stores:

"The only shoppers in some stores are students."

See Solution

p.57, icon at Example 1

#12. Suppose that the universe for x and y is $\{1, 2, 3\}$. Also, assume that $P(x, y)$ is a predicate that is true in the following cases, and false otherwise: $P(1, 3), P(2, 1), P(2, 2), P(3, 1), P(3, 2), P(3, 3)$. Determine whether each of the following is true or false:

(a) $\forall y \exists x\, (x \neq y \wedge P(x, y))$.

(b) $\forall x \exists y\, (x \neq y \wedge \neg P(x, y))$.

(c) $\forall y \exists x\, (x \neq y \wedge \neg P(x, y))$.

See Solution

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p.57, icon at Example 1

#13. Suppose that the universe for x and y is $\{1, 2, 3, 4\}$. Assume that $P(x, y)$ is a predicate that is true in the following cases and false otherwise: $P(1, 4), P(2, 1), P(2, 2), P(3, 4), P(4, 1), P(4, 4)$. Determine whether each of the following is true or false:

(a) $\exists y \forall x P(x, y)$.

(b) $\forall x P(x, x)$.

(c) $\forall x \exists y\, (x \neq y \wedge P(x, y))$.

See Solution

p.57, icon at Example 1

#14. Consider this sentence, which is Amendment 3 to the U. S. Constitution: "No soldier shall, in time of peace, be quartered in any house, without the consent of the owner, nor in time of war, but in a manner to be prescribed by law."

(a) The sentence has the form of a conjunction of two conditional sentences. Write the given sentence in this form.

(b) Using the six predicates, $S(x)$: "x is a soldier," $P(t)$ "t is a peaceful time," $Q(x, y, h)$: "x is required to allow y to be quartered in h," $O(x, h)$: "x owns h," $C(x, y, h)$: "x consents to quarter y in h,', $A(x, h)$: "the law allows x to be quartered in h," where the universe for x and y consists of all people, the universe for t consists of all points in time, and the universe for h consists of all houses, rewrite the sentence using quantifiers and predicates.

See Solution

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p.57, icon at Example 1

#15. Consider these lines of code from a C++ program:

```
if (!(x!=0 && y/x < 1) || x==0)
  cout << "True";
else
  cout << "False"
```

(a) Express the code in this statement as a compound statement using the logical connectives $\neg, \vee, \wedge, \rightarrow$, and these predicates

$$E(x)\colon x = 0$$
$$L(x, y)\colon y/x < 1$$
$$A(z)\colon \text{“}z \text{ is assigned to } \texttt{cout}\text{”}$$

where x and y are integers and z is a Boolean variable (with values True and False).

(b) Use the laws of propositional logic to simplify the statement by expressing it in a simpler form.

(c) Translate the answer in part (b) back into C++.

See Solution

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p.58, icon at Example 3

#1. What are the truth values of each of these? Assume that in each case the universe consists of all real numbers.

(a) $\exists x\, \exists y\, (xy = 2)$

(b) $\exists x\, \forall y\, (xy = 2)$

(c) $\forall x\, \exists y\, (xy = 2)$

(d) $\forall x\, \forall y\, (xy = 2)$

See Solution

p.58, icon at Example 3

#2. Write the following statements in English, using the predicate $S(x, y)$: "x shops in y", where x represents people and y represents stores:

$$(a)\ \exists y\, \forall x\, S(x, y).$$

$$(b)\ \forall x\, \exists y\, S(x, y).$$

See Solution

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p.58, icon at Example 3

#3. Suppose that the universe for x and y is $\{1, 2, 3\}$. Also, assume that $P(x, y)$ is a predicate that is true in the following cases, and false otherwise: $P(1, 3), P(2, 1), P(2, 2), P(3, 1), P(3, 2), P(3, 3)$. Determine whether each of the following is true or false:
(a) $\exists x \forall y\, (y < x \rightarrow P(x, y))$.
(b) $\forall y \exists x\, (y < x \vee P(x, y))$.
(c) $\exists x \exists y\, (P(x, y) \wedge P(y, x))$.
(d) $\forall y \exists x\, (P(x, y) \rightarrow \neg P(y, x))$.

See Solution

p.58, icon at Example 3

#4. Suppose $P(x, y, z)$ is a predicate where the universe for x, y, and z is $\{1, 2\}$. Also suppose that the predicate is true in the following cases $P(1, 1, 1)$, $P(1, 2, 1)$, $P(1, 2, 2)$, $P(2, 1, 1)$, $P(2, 2, 2)$, and false otherwise. Determine the truth value of each of the following quantified statements:

(a) $\forall x\,\exists y\,\exists z\,P(x, y, z)$.

(b) $\forall x\,\forall y\,\exists z\,P(x, y, z)$.

(c) $\forall y\,\forall z\,\exists x\,P(x, y, z)$.

(d) $\forall x\,\exists y\,\forall z\,P(x, y, z)$.

See Solution

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p.58, icon at Example 3

#5. Suppose $P(x, y, z)$ is a predicate where the universe for x, y, and z is $\{1, 2\}$. Also suppose that the predicate is true in the following cases $P(1, 1, 1)$, $P(1, 2, 1)$, $P(1, 2, 2)$, $P(2, 1, 1)$, $P(2, 2, 2)$, and false otherwise. Determine the truth value of each of the following quantified statements:

(a) $\exists x\,\forall y\,\forall z\,P(x, y, z)$.

(b) $\forall x\,\exists z\,\forall y\,P(x, y, z)$.

(c) $\forall y\,\exists x\,\exists z\,\neg P(x, y, z)$.

(d) $\exists x\,\forall z\,\neg\forall y\,P(x, y, z)$.

See Solution

p.58, icon at Example 3

#6. Suppose that the universe for x and y is $\{1, 2, 3, 4\}$. Assume that $P(x, y)$ is a predicate that is true in the following cases and false otherwise: $P(1, 4), P(2, 1), P(2, 2), P(3, 4), P(4, 1), P(4, 4)$. Determine whether each of the following is true or false:

(a) $\forall x \exists y P(x, y)$.

(b) $\forall y \exists x P(x, y)$.

(c) $\exists x \forall y P(x, y)$.

See Solution

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p.60, icon at Example 6

#1. Write this fact about numbers using predicates and quantifiers: "Given a number, there is a number greater than it."

See Solution

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p.60, icon at Example 6

#2. Express the following statement using predicates and quantifiers: "The product of two positive numbers is positive."

See Solution

p.60, icon at Example 6

#3. Write these statements in symbols using the predicates:

$$S(x)\text{: } x \text{ is a perfect square;} \quad N(x)\text{: } x \text{ is negative.}$$

Assume that the variable x is an integer.

(a) No perfect squares are negative.

(b) No negative numbers are perfect squares.

See Solution

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p.60, icon at Example 6

#4. Write the following statement in symbols using the predicates

$$S(x)\text{: } x \text{ is a perfect square} \quad P(x)\text{: } x \text{ is positive}$$

where the universe for x is the set of all integers:

"Perfect squares are positive."

See Solution

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p.60, icon at Example 6

#5. Write the following statement in symbols using the predicate $P(x)$ to mean "x is positive", where the universe for x is the set of all integers.

"Exactly one number is positive."

See Solution

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p.60, icon at Example 6

#6. Write the following statements in symbols, using $P(x)$ to mean "x is positive" and $F(x)$ to mean "x ends in the digit 5". Assume that the universe for x is the set of all integers.

(a) Some positive integers end in the digit 5.

(b) Some positive integers end in the digit 5, while others do not.

See Solution

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p.60, icon at Example 6

#7. Write in symbols: There is no smallest positive number.

See Solution

p.60, icon at Example 6

#8. Write in symbols: If $a < b$, then $\dfrac{a+b}{2}$ lies between a and b.

See Solution

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p.60, icon at Example 6

#9. Write in symbols: For all choices of a and b, $\dfrac{a+b}{2}$ lies between a and b.

See Solution

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p.63, icon at Example 14

#1. Write the negation of the statement $\exists x \, \forall y \, (xy = 0)$ in symbols and in English. Determine the truth or falsity of the statement and its negation. Assume that the universe for x and y is the set of all real numbers.

See Solution

p.63, icon at Example 14

#2. Write the statement "There is a largest number" using predicates and quantifiers. Then give its negation in symbols.

See Solution