

$$\begin{aligned}
&2^03^0 + 2^03^1 + 2^03^2 + 2^03^3 + 2^03^4 + \\
&\quad 2^13^0 + 2^13^1 + 2^13^2 + 2^13^3 + 2^13^4 + \\
&\quad\quad 2^23^0 + 2^23^1 + 2^23^2 + 2^23^3 + 2^23^4 + \\
&\quad\quad\quad 2^33^0 + 2^33^1 + 2^33^2 + 2^33^3 + 2^33^4 = \\
&2^0(3^0 + 3^1 + 3^2 + 3^3 + 3^4) + \\
&\quad 2^1(3^0 + 3^1 + 3^2 + 3^3 + 3^4) + \\
&\quad\quad 2^2(3^0 + 3^1 + 3^2 + 3^3 + 3^4) + \\
&\quad\quad\quad 2^3(3^0 + 3^1 + 3^2 + 3^3 + 3^4) = \\
&2^0 \cdot 121 + 2^1 \cdot 121 + 2^2 \cdot 121 + 2^3 \cdot 121 = 15 \cdot 121 = 1815.
\end{aligned}$$

If you are familiar with sigma notation (covered in Section 2.4), this summation process can be more compactly written as

$$\begin{aligned}
\sum_{i=0}^3 \sum_{j=0}^4 2^i 3^j &= 2^0 \sum_{j=0}^4 3^j + 2^1 \sum_{j=0}^4 3^j + 2^2 \sum_{j=0}^4 3^j + 2^3 \sum_{j=0}^4 3^j \\
&= (2^0 + 2^1 + 2^2 + 2^3)121 \\
&= 1815.
\end{aligned}$$

p.203, icon at Example 4

#1. For each pair of numbers, when the division algorithm is used to divide a by d , what are the quotient q and remainder r ?

- (a) $a = 88, d = 11.$
- (b) $a = -29, d = 9$
- (c) $a = 58^{237}, d = 58^{168}$

Solution:

- (a) Because $88 = 11 \cdot 8 + 0$, we have $q = 8, r = 0$. (The fact that $r = 0$ says that $11|88$.)
- (b) Because $-29 = 9 \cdot (-4) + 7$, we have $q = -4$ and $r = 7$. (Note that although we can write $-29 = 9 \cdot (-3) + (-2)$, we cannot use -2 as r because r is not allowed to be negative.)
- (c) We do not need to perform the exponentiations to find a and d . We need only observe that a is a multiple of d : $58^{237} = 58^{168} \cdot 58^{69}$ (recall the rule for exponents: $a^b a^c = a^{b+c}$). Therefore $58^{237} = 58^{168} \cdot 58^{69} + 0$ and we have $q = 58^{69}$ and $r = 0$.