

Rosen, Discrete Mathematics and Its Applications, 7th edition
Extra Examples
Section 9.6—Partial Orderings



— Page references correspond to locations of Extra Examples icons in the textbook.

p.619, icon at Example 1

#1. Let $A = \{(x, y) \mid x, y \text{ integers}\}$. Define a relation R on A by the rule

$$(a, b)R(c, d) \leftrightarrow a \leq c \text{ or } b \leq d.$$

Determine whether R is a partial order relation on A .

Solution:

R is reflexive: $(a, b)R(a, b)$ for all elements (a, b) because $a \leq a$ or $b \leq b$ is always true.

R is not antisymmetric: For example, $(1, 4)R(3, 2)$ because $1 \leq 3$, and $(3, 2)R(1, 4)$ because $2 \leq 4$. But $(1, 4) \neq (3, 2)$.

R is not transitive: For example, $(1, 4)R(3, 2)$ because $1 \leq 3$, and $(3, 2)R(0, 3)$ because $2 \leq 3$. But $(1, 4) \not R(0, 3)$ because $1 \not\leq 0$ and $4 \not\leq 3$.

Therefore, R is not a partial order relation because R is neither antisymmetric nor transitive.

p.619, icon at Example 1

#2. Let $A = \{(x, y) \mid x, y \text{ integers}\}$. Define a relation R on A by the rule

$$(a, b)R(c, d) \leftrightarrow a = c \text{ or } b = d.$$

Determine whether R is a partial order relation on A .

Solution:

R is reflexive: $(a, b)R(a, b)$ for all elements (a, b) because $a = a$ and $b = b$ are always true.

R is not antisymmetric: For example, $(1, 2)R(1, 3)$ and $(1, 3)R(1, 2)$ because $1 = 1$, but $(1, 2) \neq (1, 3)$.

R is not transitive: For example, $(1, 2)R(1, 3)$ because $1 = 1$, and $(1, 3)R(4, 3)$ because $3 = 3$. But $(1, 2) \not R(4, 3)$ because $1 \neq 4$ and $2 \neq 3$.

Therefore, R is not a partial order relation because R is neither antisymmetric nor transitive.

p.619, icon at Example 4

#1. Let R be the relation on the set of words in the English language where xRy if x precedes (that is, comes before) y in the dictionary. Show that R is not a partial ordering.

Solution:

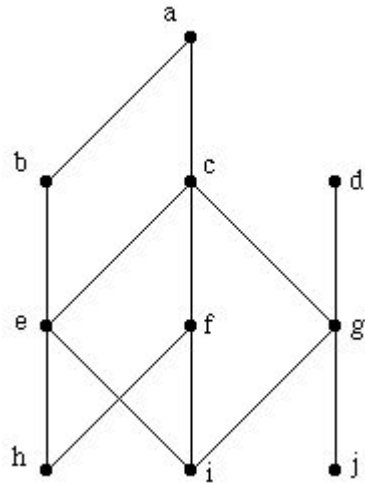
Note that R is antisymmetric because if x precedes y in the dictionary, where x and y are English words, then y does not precede x . Also note that R is transitive, for if x precedes y in the dictionary and y precedes

z in the dictionary, where x , y , and z are English words, then x precedes z in the dictionary. However, R is not reflexive because no word precedes itself in the dictionary. This means that R is not a partial ordering.

p.624, icon at Example 20

#1. Referring to this Hasse diagram of a partially ordered set, find the following:

- (a) all upper bounds of $\{d, e\}$.
- (b) the least upper bound of $\{d, e\}$.
- (c) all lower bounds of $\{a, e, g\}$.
- (d) the greatest lower bound of $\{a, e, g\}$.
- (e) greatest lower bound of $\{b, c, f\}$.
- (f) least upper bound of $\{h, i, j\}$.
- (g) greatest lower bound of $\{g, h\}$.
- (h) least upper bound of $\{f, i\}$.



Solution:

- (a) There are no upper bounds of $\{d, e\}$.
 - (b) Because there are no upper bounds of $\{d, e\}$, there is no least upper bound of $\{d, e\}$.
 - (c) The only lower bound of $\{a, e, g\}$ is i .
 - (d) The glb of $\{a, e, g\}$ is the only lower bound of $\{a, e, g\}$, namely i .
 - (e) Both h and i are lower bounds of $\{b, c, f\}$. But there is no greatest lower bound.
 - (f) Both a and c are upper bounds of $\{h, i, j\}$. The element c is the least upper bound.
 - (g) There is no lower bound of $\{g, h\}$. Hence there is no greatest lower bound.
 - (h) The elements a , c , and f are upper bounds of $\{f, i\}$. The element f is the least upper bound.
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