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Some Notes on Free-Body Diagrams

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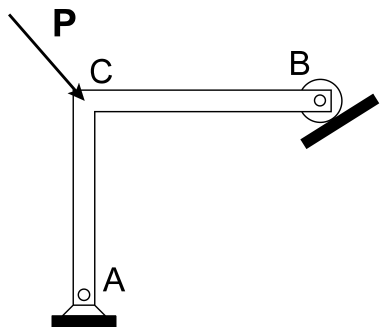
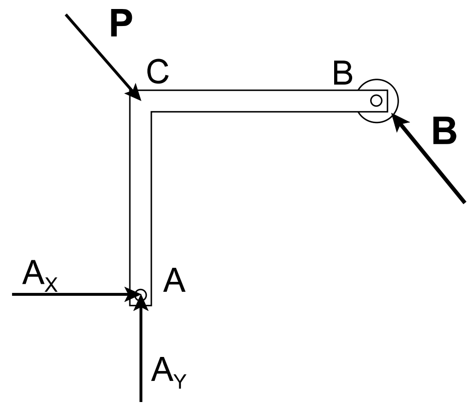
Successful problem solving in mechanics depends on proper free-body diagrams (FBD's). This note summarizes the essentials of good free-body diagram practice.

1. Direction/Sign of Unknown Forces

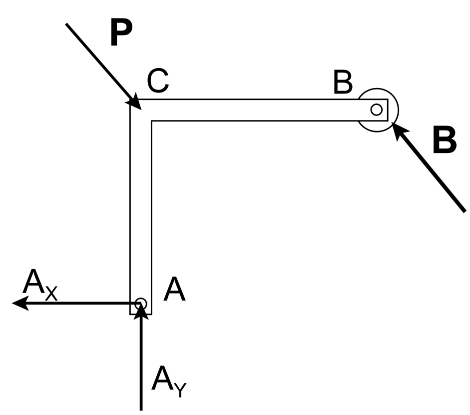
In drawing unknown forces on an FBD, the angle of the force may be known (*e.g.* a cable or thin link, for which the line of action is along the cable, or a frictionless slide or support, for which the force is normal to the contact surface). If it is not known (*e.g.* pin joint, fixed support), two unknown components should be specified. In both cases, one has to decide which direction or sign to give the force arrows on the diagram. Most people draw FBD's with unknown forces acting in the positive co-ordinate directions. If this turns out to be wrong for a particular force, the algebra will tell you by giving you a negative sign for the force.





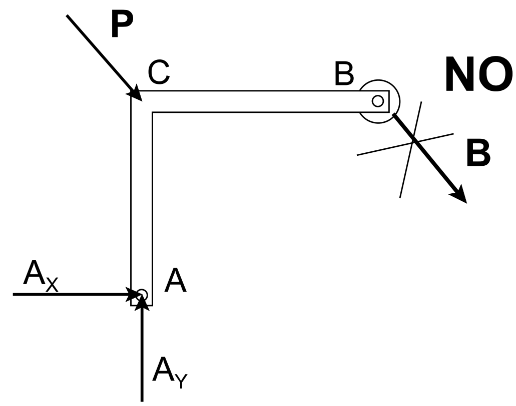
In the example above, force **A** has been entered as components in the positive x and y directions. Calculations will give a negative sign for A_X, showing that A_X in reality points in the negative x direction. If this happens, **DO NOT** go back and change the free-body diagram. Simply note in your solution that "positive force directions are as shown in the FBD", or show the force direction unambiguously in giving your answer.

As an alternative to drawing all forces in positive co-ordinate directions, you can try to guess the force directions by mentally taking ΣM and ΣF as you draw the diagram. In this example, we know the directions of **P** (given) and **B** (frictionless support); mentally taking ΣM_C shows that A_X points in the negative x-direction. You will not always be able to get all force directions this way - for example, you cannot get the correctdirection for A_Y by ΣM or ΣF_Y without doing actual calculations - but the exercise will



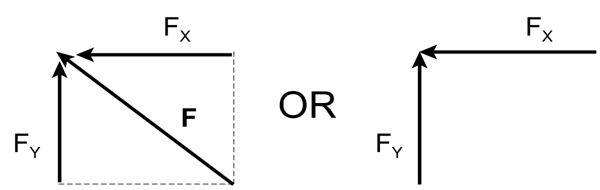
help you understand how forces work, and will also help you plan your solution strategy. Both this FBD and the previous one are correct.

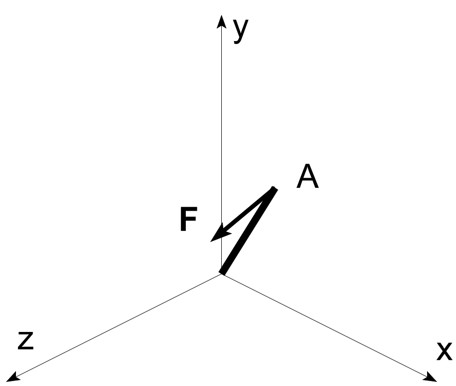
Exception: if the force direction is known (*e.g.* a cable is always in tension, the force at a support always acts to keep the surfaces in contact), the FBD **must** show the correct direction. Failure to do this will result in incorrect answers. The FBD at the right is wrong, because **B** must support the roller - as drawn, it is pulling the roller down instead. A solution based on this FBD will give **A** with the wrong direction.

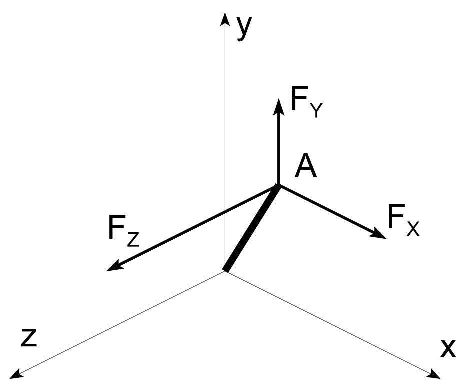


2. Components

Forces may be drawn as components if they make the problem clearer. (For a two-force member, you should *not* show components - see §4 below.) If you show both the vector and its components, use the same notation for both and connect them by dashed lines to make it clear that the components are not other forces. However, it is usually unnecessary to show both vector *and* components - one or the other will do. In three dimensions, components are often easier to draw and interpret. The two forces below are equivalent, but the diagram showing components instead of the force vector is easier to read.

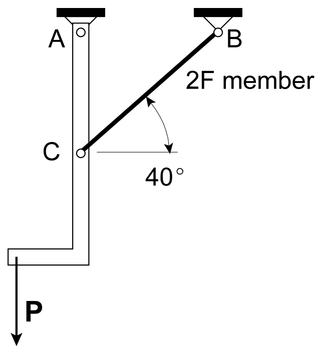


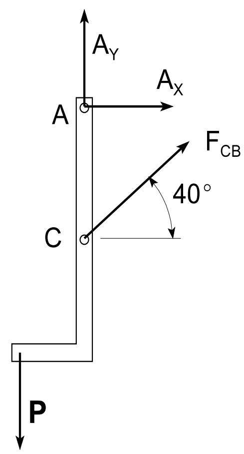


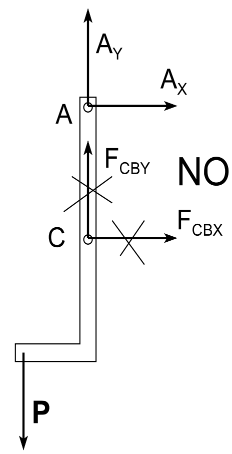


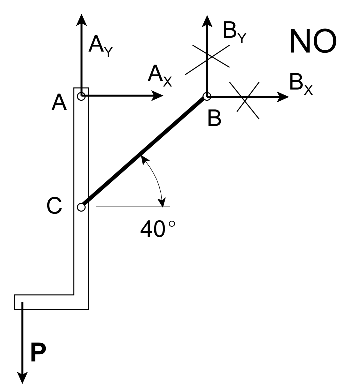
3. Forces must be labelled. Labelling is necessary to define your notation, so that the marker can understand the algebraic part of your solution. A FBD with the forces unlabelled is WRONG.

4. Two-force members. Identify **all** two-force members before drawing the FBD. The essential fact of a two-force member is that the forces on it are aligned with the two points of application. Forces exerted by two-force members must therefore **always** be shown as a vector lined up with the member- *do not show two-force member forces as components!* In the example below, the first FBD allows one to recognize immediately that the direction of $\mathbf{F_{CB}}$ is known, so that the only unknown is the magnitude F_{CB}. If components are shown instead (right hand sketch), it gives the impression that there are *two* unknowns - F_{CBX} *and* F_{CBY} - instead of only one.







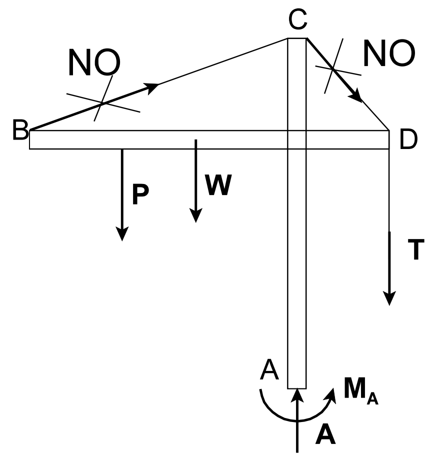


This is a common mistake, and can result in one not being able to solve a problem because it appears to have too many unknowns. If one draws the right hand FBD above, one has to recognise that $F_{CBX} = F_{CB} \cos 40°$, $F_{CBY} = F_{CB} \sin 40°$ to reduce the number of unknowns from 4 to 3. The mistake usually arises because most two-force members have pin joints, and pin joints are often drawn with two unknown force components. Make a habit, therefore, of checking each pin joint to see if it is attached to a two-force member. Another variation of this mistake is shown at right. It is not wrong to include member CB in the FBD, but the reaction at C must recognize that CB is two-force.

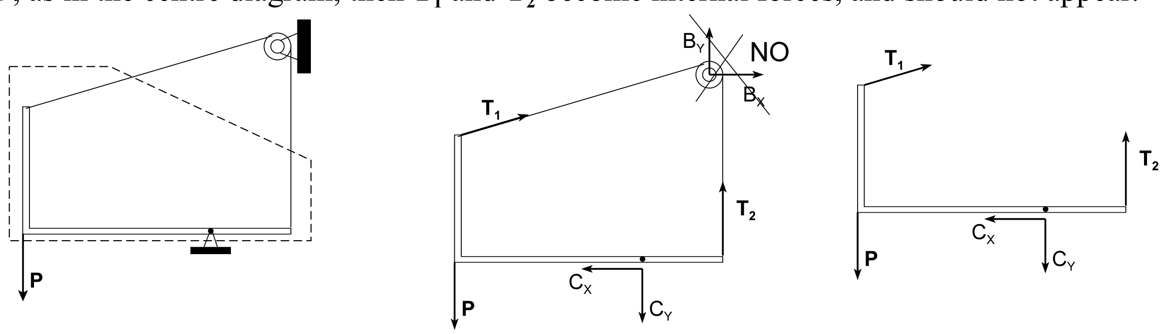
To identify two-force members, look for members that have the following characteristics:
- forces applied at only two points (usually the ends);
- no moments applied anywhere;
- negligible weight.

Most two-force members are cables, thin bars or links with pin joints. However, they can also have more complex shapes - *e.g.* curved bars or plates - and forces may be applied through point contacts or slides as well as through pin joints. A two-force member will *never* have a fixed (built-in) connection, because that would include a moment.

5. External Forces Only. A free-body diagram shows only forces acting external to the chosen free-body; internal forces must *never* be shown. The diagram at right is incorrect for this reason. All forces acting within an FB are in equilibrium, and do not affect the forces acting at the supports and external connections of the FB. At right, for example, the tensions in members BC and CD will not affect the reactions at all. Including these tensions in the solution would add non-existent forces to the object and make the solution incorrect.



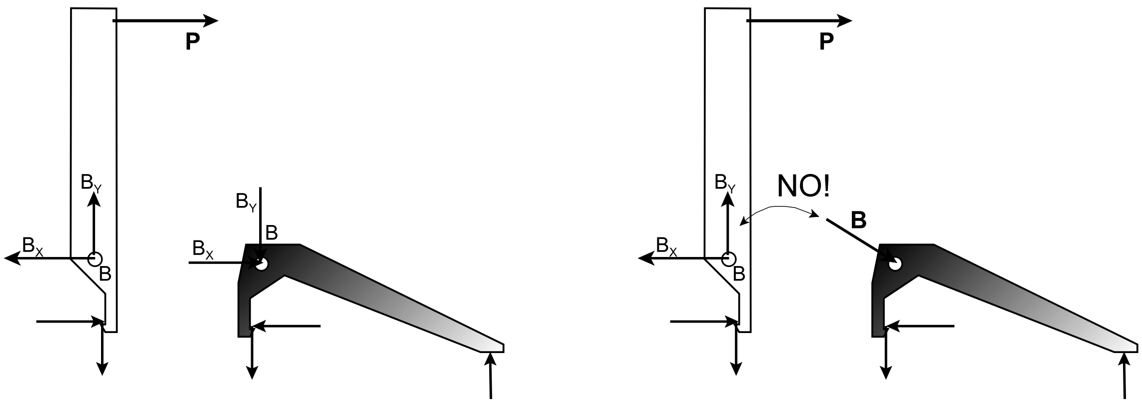
A good way to distinguish between internal and external forces is to draw a boundary around the FB: any forces or connections **intersected** by the boundary will appear on the FBD, anything **within** the boundary will not. In the centre diagram below, the lack of a clear definition for the FB has resulted in an improper FBD. One should draw a boundary as shown, excluding the pulley and resulting in the correct FBD at the right. Alternatively, if one wishes to include the pulley in the FBD, as in the centre diagram, then $\mathbf{T_1}$ and $\mathbf{T_2}$ become internal forces, and should not appear.



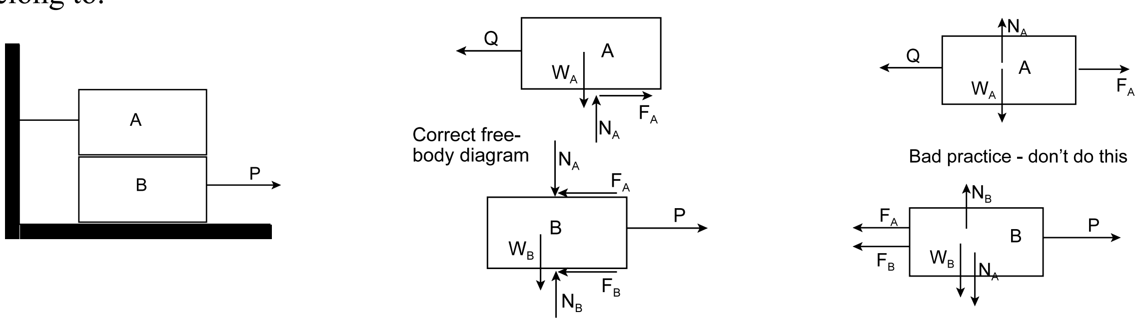
6. Newton's Third Law. In problems involving several components (*e.g.* frames and machines), in which FBD's for individual parts must be drawn, make sure that Newton's third law (action-reaction) is applied at each joint which is taken apart: force components acting on one member must be shown as equal and opposite on the other member that they connect to. The same is true of couples, for example at a fixed support. Examples are shown in the diagram at left below and in the diagram for item 7 further below. Two points to note:

(a) Draw the force in the same way on both parts. Don't draw the force as a single vector on one part and the same force as x and y components on the other - this is confusing (see sketches below).

(b) Do NOT put a negative sign on the force when you reverse the direction of the arrow. The Beer and Johnston textbook does this in a few places (*e.g.* Figures 6.20 - 6.23), but this practice is WRONG and will cause great confusion in setting up the equations. Note that the textbook is inconsistent here, because these incorrect figures are followed by examples which do *not* use this bad practice.



7. Locations of Forces. Forces should always be shown applied to the **points at which they act**. The centre sketch below shows the correct placement of forces for the friction problem at the left, with each contact surface having a frictional force and a normal force applied at the appropriate surface. The right hand sketch shows the same free-body diagram as drawn by some school physics textbooks: this is bad practice, because it is almost impossible to see which surfaces the forces belong to.



8. Weights. It is usual to neglect the weights of parts unless they are expressly given, and they will therefore not usually appear in the FBD. The weights of machine parts and structures are generally small compared with the loads they have to carry. Exceptions are large civil engineering structures, whose weights comprise a major part of the total load on the structure.

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