Damage Tolerant Design

To demonstrate the damage-tolerant design methodology we use the same material and fatigue test specimen described in Example 12.3. In order to determine the crack propagation, the additional needed information is $K_{Ic} = 68\,\text{MPa}\sqrt{\text{m}}$ and $\frac{da}{dN} = 6.9 \times 10^{-12} (\Delta K)^3$ gives the cycles to propagate a crack, where da/dN is in m/cycle and ΔK is in $\text{MPa}\sqrt{\text{m}}$. The initial length of the crack is assumed to be 1 mm. It is a single side-crack in a plate 101.6 mm wide and 20 mm thick loaded in completely reversed tension to a max stress of 246 MPa. The compression part of the load cycle is ignored because in compression the crack closes up and does not grow. Equation (12.6) describes the stress intensity of the crack and is evaluated to be $K = 1.15\sigma\sqrt{\pi a}$. From Eq. (12.**6**) the final crack length at fracture, a_f, is

$$a_c = a_f = \frac{1}{\pi}\left(\frac{K_{Ic}}{1.15\sigma_{max}}\right)^2 = \frac{1}{\pi}\left(\frac{68}{1.15 \times 246}\right)^2 = 0.018\,\text{m} = 18\,\text{mm}$$

Now use the equation for the propagation of a crack in region II in a 4340 steel plate. Also,

$$\Delta K = K_{max} - K_{min} = 1.15\sigma\sqrt{\pi a}$$

$$\frac{da}{dN} = 6.9 \times 10^{-12}(\Delta K)^3 = 6.9 \times 10^{-12}(1.15 \times 246)^3 (\pi)^{3/2} a^{3/2}$$

$$\frac{da}{a^{1.5}} = 868 \times 10^{-6}\, \mathbf{d}\boldsymbol{N}$$

$$\int_{a_0=0.001}^{a_f=0.018} a^{-1.5}\,da = \frac{a^{-1.5+1}}{-1.5+1}\Big|_{0.001}^{0.018} = 868 \times 10^{-6} N \Big|_{0}^{N_P}$$

$$\frac{0.018^{-0.5}}{-0.5} - \frac{0.001^{-0.5}}{-0.5} = 868 \times 10^{-6} N_P$$

$$N_P = \frac{48.338}{868 \times 10^{-6}} = 5.56 \times 10^4 \text{ cycles}$$

The total fatigue life to failure, N_f, is the sum of the number of cycles to initiate a crack, N_i, and the cycles to grow the crack until complete fracture occurs, N_p. For the problem given in Examples 12.3 and 12.4, $N_f = N_i + N_p = 2.104 \times 10^5 + 5.56 \times 10^4 = 2.660 \times 10^5$ cycles. Note that under the given conditions the crack initiation stage is about one order of magnitude larger than the crack propagation stage.