Stress-Life Fatigue Design

This section considers the design for fatigue of components that are assumed to be able to withstand an infinite number of stress cycles if the maximum stress is kept below the fatigue (endurance) limit. For materials that do not have a defined fatigue limit, the design is based on *fatigue strength*, defined as the stress amplitude that the material can support for at least 10^8 fatigue cycles.

Example of Use of Stress-Life Approach to Fatigue Design

A steel shaft heat-treated to a Brinell hardness of 200 has a major diameter of 1.5 in. and a small diameter of 1.0 in. There is a 0.10-in. radius at the shoulder between the diameters. The shaft is subjected to completely reversed cycles of stress of pure bending. The fatigue limit determined on polished specimens of 0.2-in. diameter is 42,000 psi. The shaft is produced by machining from bar stock. What is the best estimate of the fatigue limit of the shaft?

Since an experimental value for fatigue limit is known, we start with it, recognizing that tests on small, unnotched polished specimens represent an unrealistically high value of the fatigue limit of the actual part.[1] The procedure, then, is to factor down the idealized value. We start with the stress concentration (notch) produced at the shoulder between two diameters of the shaft. A shaft with a fillet in bending is a standard situation covered in all machine design books. If D = 1.5, d = 1.0, and r = 0.10, the important ratios are D/d = 1.5 and r/d = 0.1 . Then, from standard curves, the *theoretical stress concentration factor* is K_t = 1.68. However, K_t is determined for a brittle elastic solid and most ductile materials exhibit a lesser value of stress concentration when subjected to fatigue. The extent to which the plasticity of the material reduces K_t is given by the *fatigue notch sensitivity* q .

$$q = \frac{K_f - 1}{K_t - 1} \qquad (1)$$

Where K_t = theoretical stress concentration factor

$$K_f = \text{fatigue notch factor} = \frac{\text{fatigue limit unnotched}}{\text{fatigue limit notched}}$$

From design charts we find that a steel with a Brinell hardness of 200 has a q of 0.8. From Eq. (1), K_f is 1.54. This information will be used later in the design.

Returning to the fatigue limit for a small polished specimen, S_e = 42,000 psi, we need to reduce this value because of size effect, surface finish, and type of loading and for statistical scatter

$$S_e' = S_e C_S C_F C_L C_Z \qquad (2)$$

Where C_S = factor for size effect

C_F = factor for surface finish

C_L = factor for type of loading

C_Z = factor for statistical scatter

Increasing the specimen size increases the probability of surface defects, and hence the fatigue limit decreases with increasing size. Typical values of C_S are given in Table 1. In this example we use C_S = 0.9.

1. If fatigue property data are not given, they must be determined from the published literature or estimated from other mechanical properties of the material; see H. O. Fuchs and R. I. Stephens , *Metal Fatigue in Engineering,* pp. 156–160, John Wiley & Sons, New York , 1980 ; *ASM Handbook,* Vol. 19, 1996 , pp. 589–955.

TABLE 1
Fatigue Reduction Factor
Due to Size Effect

Diameter, in.	C_S
$D \leq 0.4$	1.0
$0.4 \leq D \leq 2.0$	0.9
$2.0 \leq D \leq 9.0$	$1 - \frac{D - 0.03}{15}$

Curves for the reduction in fatigue limit due to various surface finishes are available in standard sources.[1] For a standard machined finish in a steel of BHN 200, $C_F = 0.8$.

Laboratory fatigue data (as opposed to simulated service fatigue tests) commonly are determined in a reversed bending loading mode. Other types of loading, such as axial and torsional, generate different stress gradients and stress distributions and do not produce the same fatigue limit for the same material. Thus, fatigue data generated in reversed bending must be corrected by a load factor C_L, if the data are to be used in a different loading mode. Table 2 gives typical values. Since the bending fatigue data are used for an application involving bending, $C_L = 1.0$.

Fatigue tests show considerable scatter in results. Fatigue limit values are normally distributed with a standard deviation that can be up to 8 percent of the mean value. If the test or literature value is taken as the mean value of fatigue limit (which in itself is a big assumption), then this value is reduced by a statistical factor [2] according to the reliability level that is desired (Table 3).

TABLE 2
Loading Factor for
Fatigue Tests

Loading Type	C_L
Bending	1.0
Torsion	0.58
Axial	0.9

TABLE 3
Statistical Factor for
Fatigue Limit

Reliability, percent	C_Z
50	1.0
99	0.814
99.9	0.752

If we assume that a 99 percent reliability level is acceptable, then $C_Z = 0.814$. Therefore, the unnotched fatigue limit corrected for these factors is

$$S'_e = S_e C_S C_F C_Z = 42{,}000(0.9)(0.8)(1.0)(0.81) = 24{,}494 \text{ psi}$$

Since K_f is 1.54, the fatigue limit of the shaft, with a notch created by the shoulder, is estimated to be

$$S'_{e,notch} = \frac{24{,}494}{1.54} = 15{,}900\, psi$$

This example is fairly realistic, but it has not included the important situation in which the mean stress is other than zero.[3] Including mean stress in the calculation also permits consideration of fatigue strengthening from compressive residual stresses.

1. R.C. Juvinall, op. cit., p.234.
2. G. Castleberry, *Machine Design*, pp. 108-110, Feb. 23, 1978.
3. See, for example, R.C. Juvinall, op. cit., chap. 14.