

18

Power Transmission Case Study

Chapter Outline

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Transmission of power from a source, such as an engine or motor, through a machine to an output actuation is one of the most common machine tasks. An efficient means of transmitting power is through rotary motion of a shaft that is supported by bearings. Gears, belt pulleys, or chain sprockets may be incorporated to provide for torque and speed changes between shafts. Most shafts are cylindrical (solid or hollow), and include stepped diameters with shoulders to accommodate the positioning and support of bearings, gears, etc.

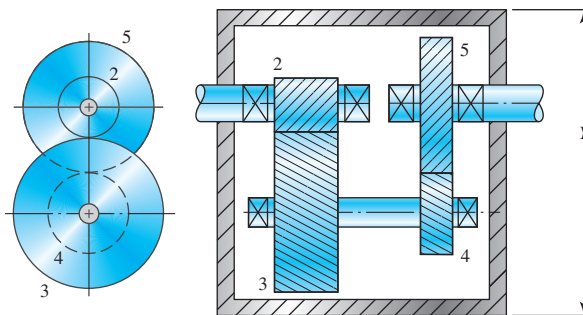
The design of a system to transmit power requires attention to the design and selection of individual components (gears, bearings, shaft, etc.). However, as is often the case in design, these components are not independent. For example, in order to design the shaft for stress and deflection, it is necessary to know the applied forces. If the forces are transmitted through gears, it is necessary to know the gear specifications in order to determine the forces that will be transmitted to the shaft. But stock gears come with certain bore sizes, requiring knowledge of the necessary shaft diameter. It is no surprise that the design process is interdependent and iterative, but where should a designer start?

The nature of machine design textbooks is to focus on each component separately. This chapter will focus on an overview of a power transmission system design, demonstrating how to incorporate the details of each component into an overall design process. A typical two-stage gear reduction such as shown in Fig. 18–1 will be assumed for this discussion. The design sequence is similar for variations of this particular transmission system.

The following outline will help clarify a logical design sequence. Discussion of how each part of the outline affects the overall design process will be given in sequence in this chapter. Details on the specifics for designing and selecting major components are covered in separate chapters, particularly Chap. 7 on shaft design, Chap. 11 on bearing selection, and Chaps. 13 and 14 on gear specification. A complete case study is presented as a specific vehicle to demonstrate the process.

Figure 18–1

A compound reverted gear train.



CASE STUDY PART 1 PROBLEM SPECIFICATION

Section 1-18, p. 34, presents the background for this case study involving a speed reducer. A two-stage, compound reverted gear train such as shown in Fig. 18-1 will be designed. In this chapter, the design of the intermediate shaft and its components is presented, taking into account the other shafts as necessary.

A subset of the pertinent design specifications that will be needed for this part of the design are given here.

Power to be delivered: 20 hp

Input speed: 1750 rpm

Output speed: 82–88 rev/min

Usually low shock levels, occasional moderate shock

Input and output shafts extend 4 in outside gearbox

Maximum gearbox size: 14-in × 14-in base, 22-in height

Output shaft and input shaft in-line

Gear and bearing life > 12 000 hours; infinite shaft life

18–1 Design Sequence for Power Transmission

There is not a precise sequence of steps for any design process. By nature, design is an iterative process in which it is necessary to make some tentative choices, and to build a skeleton of a design, and to determine which parts of the design are critical. However, much time can be saved by understanding the dependencies between the parts of the problem, allowing the designer to know what parts will be affected by any given change. In this section, only an outline is presented, with a short explanation of each step. Further details will be discussed in the following sections.

- *Power and torque requirements.* Power considerations should be addressed first, as this will determine the overall sizing needs for the entire system. Any necessary speed or torque ratio from input to output must be determined before addressing gear/pulley sizing.
- *Gear specification.* Necessary gear ratios and torque transmission issues can now be addressed with selection of appropriate gears. Note that a full force analysis of the shafts is not yet needed, as only the transmitted loads are required to specify the gears.
- *Shaft layout.* The general layout of the shaft, including axial location of gears and bearings must now be specified. Decisions on how to transmit the torque from the gears to the shaft need to be made (keys, splines, etc.), as well as how to hold gears and bearings in place (retaining rings, press fits, nuts, etc.). However, it is not necessary at this point to size these elements, since their standard sizes allow estimation of stress-concentration factors.
- *Force analysis.* Once the gear/pulley diameters are known, and the axial locations of the gears and bearings are known, the free-body, shear force, and bending moment diagrams for the shafts can be produced. Forces at the bearings can be determined.
- *Shaft material selection.* Since fatigue design depends so heavily on the material choice, it is usually easier to make a reasonable material selection first, then check for satisfactory results.
- *Shaft design for stress (fatigue and static).* At this point, a stress design of the shaft should look very similar to a typical design problem from the shaft chapter (Chap. 7). Shear force and bending moment diagrams are known, critical locations can be predicted, approximate stress concentrations can be used, and estimates for shaft diameters can be determined.

- *Shaft design for deflection.* Since deflection analysis is dependent on the entire shaft geometry, it is saved until this point. With all shaft geometry now estimated, the critical deflections at the bearing and gear locations can be checked by analysis.
- *Bearing selection.* Specific bearings from a catalog may now be chosen to match the estimated shaft diameters. The diameters can be adjusted slightly as necessary to match the catalog specifications.
- *Key and retaining ring selection.* With shaft diameters settling in to stable values, appropriate keys and retaining rings can be specified in standard sizes. This should make little change in the overall design if reasonable stress-concentration factors were assumed in previous steps.
- *Final analysis.* Once everything has been specified, iterated, and adjusted as necessary for any specific part of the task, a complete analysis from start to finish will provide a final check and specific safety factors for the actual system.

18-2 Power and Torque Requirements

Power transmission systems will typically be specified by a power capacity, for example, a 40-horsepower gearbox. This rating specifies the combination of torque and speed that the unit can endure. Remember that, in the ideal case, *power in* equals *power out*, so that we can refer to the power being the same throughout the system. In reality, there are small losses due to factors like friction in the bearings and gears. In many transmission systems, the losses in the rolling bearings will be negligible. Gears have a reasonably high efficiency, with about 1 to 2 percent power loss in a pair of meshed gears. Thus, in the double-reduction gearbox in Fig. 18-1, with two pairs of meshed gears the output power is likely to be about 2 to 4 percent less than the input power. Since this is a small loss, it is common to speak of simply the power of the system, rather than input power and output power. Flat belts and timing belts have efficiencies typically in the mid to upper 90 percent range. V belts and worm gears have efficiencies that may dip much lower, requiring a distinction between the necessary input power to obtain a desired output power.

Torque, on the other hand, is typically not constant throughout a transmission system. Remember that power equals the product of torque and speed. Since *power in* = *power out*, we know that for a gear train

$$H = T_i \omega_i = T_o \omega_o \quad (18-1)$$

With a constant power, a gear ratio to decrease the angular velocity will simultaneously increase torque. The gear ratio, or train value, for the gear train is

$$e = \omega_o / \omega_i = T_i / T_o \quad (18-2)$$

A typical power transmission design problem will specify the desired power capacity, along with either the input and output angular velocities, or the input and output torques. There will usually be a tolerance specified for the output values. After the specific gears are specified, the actual output values can be determined.

18-3 Gear Specification

With the gear train value known, the next step is to determine appropriate gears. As a rough guideline, a train value of up to 10 to 1 can be obtained with one pair of gears. Greater ratios can be obtained by compounding additional pairs of gears (See Sec. 13-13, p. 690). The compound reverted gear train in Fig. 18-1 can obtain a train value of up to 100 to 1.

Since numbers of teeth on gears must be integers, it is best to design with teeth numbers rather than diameters. See Ex. 13–3, 13–4, and 13–5, pp. 692–694, for details on designing appropriate numbers of teeth to satisfy the gear train value and any necessary geometry condition, such as in-line condition of input and output shaft. Care should be taken at this point to find the best combination of teeth numbers to minimize the overall package size. If the train value only needs to be approximate, use this flexibility to try different options of tooth numbers to minimize the package size. A difference of one tooth on the smallest gear can result in a significant increase in size of the overall package.

If designing for large production quantities, gears can be purchased in large enough quantities that it is not necessary to worry about preferred sizes. For small lot production, consideration should be given to the tradeoffs between smaller gearbox size and extra cost for odd gear sizes that are difficult to purchase off the shelf. If stock gears are to be used, their availability in prescribed numbers of teeth with anticipated diametral pitch should be checked at this time. If necessary, iterate the design for numbers of teeth that are available.

CASE STUDY PART 2 SPEED, TORQUE, AND GEAR RATIOS

Continue the case study by determining appropriate tooth counts to reduce the input speed of $\omega_i = 1750$ rev/min to an output speed within the range

$$82 \text{ rev/min} < \omega_o < 88 \text{ rev/min}$$

Once final tooth counts are specified, determine values of

- Speeds for the intermediate and output shafts
- Torques for the input, intermediate and output shafts, to transmit 20 hp.

Solution

Use the notation for gear numbers from Fig. 18-1. Choose mean value for initial design, $\omega_5 = 85$ rev/min.

$$e = \frac{\omega_5}{\omega_2} = \frac{85}{1750} = \frac{1}{20.59} \quad \text{Eq. (18-2)}$$

For a compound reverted geartrain,

$$e = \frac{1}{20.59} = \frac{N_2 N_4}{N_3 N_5} \quad \text{Eq. (13-30), p. 691}$$

For smallest package size, let both stages be the same reduction. Also, by making the two stages identical, the in-line condition on the input and output shaft will automatically be satisfied.

$$\frac{N_2}{N_3} = \frac{N_4}{N_5} = \sqrt{\frac{1}{20.59}} = \frac{1}{4.54}$$

For this ratio, the minimum number of teeth from Eq. (13-11), p. 678, is 16.

$$N_2 = N_4 = 16 \text{ teeth}$$

$$N_3 = 4.54(N_2) = 72.64$$

Try rounding down and check if ω_5 is within limits.

$$\omega_5 = \left(\frac{16}{72}\right)\left(\frac{16}{72}\right)(1750) = 86.42 \text{ rev/min} \quad \text{Acceptable}$$

Proceed with

$$N_2 = N_4 = 16 \text{ teeth}$$

$$N_3 = N_5 = 72 \text{ teeth}$$

$$e = \left(\frac{16}{72}\right)\left(\frac{16}{72}\right) = \frac{1}{20.25}$$

$$\omega_5 = 86.42 \text{ rev/min}$$

$$\omega_3 = \omega_4 = \left(\frac{16}{72}\right)(1750) = 388.9 \text{ rev/min}$$

To determine the torques, return to the power relationship,

$$H = T_2\omega_2 = T_5\omega_5 \quad \text{Eq. (18-1)}$$

$$T_2 = H/\omega_2 = \left(\frac{20 \text{ hp}}{1750 \text{ rev/min}}\right)\left(550 \frac{\text{ft}\cdot\text{lbf/s}}{\text{hp}}\right)\left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right)\left(60 \frac{\text{s}}{\text{min}}\right)$$

$$T_2 = 60.0 \text{ lbf}\cdot\text{ft}$$

$$T_3 = T_2 \frac{\omega_2}{\omega_3} = 60.0 \frac{1750}{388.9} = 270 \text{ lbf}\cdot\text{ft}$$

$$T_5 = T_2 \frac{\omega_2}{\omega_5} = 60.0 \frac{1750}{86.42} = 1215 \text{ lbf}\cdot\text{ft}$$

If a maximum size for the gearbox has been specified in the problem specification, a minimum diametral pitch (maximum tooth size) can be estimated at this point by writing an expression for the gearbox size in terms of gear diameters, and converting to numbers of teeth through the diametral pitch. For example, from Fig. 18-1, the overall height of the gearbox is

$$Y = d_3 + d_2/2 + d_5/2 + 2/P + \text{clearances} + \text{wall thicknesses}$$

where the $2/P$ term accounts for the addendum height of the teeth on gears 3 and 5 that extend beyond the pitch diameters. Substituting $d_i = N_i/P$ gives

$$Y = N_3/P + N_2/(2P) + N_5/(2P) + 2/P + \text{clearances} + \text{wall thicknesses}$$

Solving this for P , we find

$$P = (N_3 + N_2/2 + N_5/2 + 2)/(Y - \text{clearances} - \text{wall thicknesses}) \quad (18-3)$$

This is the minimum value that can be used for diametral pitch, and therefore the maximum tooth size, to stay within the overall gearbox constraint. It should be rounded *up* to the next standard diametral pitch, which reduces the maximum tooth size.

The AGMA approach, as described in Chap. 14, for both bending and contact stress should be applied next to determine suitable gear parameters. The primary design parameters to be specified by the designer include material, diametral pitch, and face width. A recommended procedure is to start with an estimated diametral pitch. This allows determination of gear diameters ($d = N/P$), pitch-line velocities [Eq. (13–34), p. 699], and transmitted loads [Eq. (13–35) or (13–36), p. 699]. Typical spur gears are available with face widths from 3 to 5 times the circular pitch p . Using an average of 4, a first estimate can be made for face width $F = 4p = 4\pi/P$. Alternatively, the designer can simply perform a quick search of on-line gear catalogs to find available face widths for the diametral pitch and number of teeth.

Next, the AGMA equations in Chap. 14 can be used to determine appropriate material choices to provide desired safety factors. It is generally most efficient to attempt to analyze the most critical gear first, as it will determine the limiting values of diametral pitch and material strength. Usually, the critical gear will be the smaller gear, on the high-torque (low-speed) end of the gearbox.

If the required material strengths are too high, such that they are either too expensive or not available, iteration with a smaller diametral pitch (larger tooth) will help. Of course, this will increase the overall gearbox size. Often the excessive stress will be in one of the small gears. Rather than increase the tooth size for all gears, it is sometimes better to reconsider the design of tooth counts, shifting more of the gear ratio to the pair of gears with less stress, and less ratio to the pair of gears with the excessive stress. This will allow the offending gear to have more teeth and therefore larger diameter, decreasing its stress.

If contact stress turns out to be more limiting than bending stress, consider gear materials that have been heat treated or case hardened to increase the surface strength. Adjustments can be made to the diametral pitch if necessary to achieve a good balance of size, material, and cost. If the stresses are all much lower than the material strengths, a larger diametral pitch is in order, which will reduce the size of the gears and the gearbox.

Everything up to this point should be iterated until acceptable results are obtained, as this portion of the design process can usually be accomplished independently from the next stages of the process. The designer should be satisfied with the gear selection before proceeding to the shaft. Selection of specific gears from catalogs at this point will be helpful in later stages, particularly in knowing overall width, bore size, recommended shoulder support, and maximum fillet radius.

CASE STUDY PART 3 GEAR SPECIFICATION

Continue the case study by specifying appropriate gears, including pitch diameter, diametral pitch, face width, and material. Achieve safety factors of at least 1.2 for wear and bending.

Solution

Estimate the minimum diametral pitch for overall gearbox height = 22 in.

From Eq. (18-3) and Fig. 18-1,

$$P_{\min} = \frac{\left(N_3 + \frac{N_2}{2} + \frac{N_5}{2} + 2\right)}{(Y - \text{clearances} - \text{wall thickness})}$$

Allow 1.5 in for clearances and wall thickness:

$$P_{\min} = \frac{\left(72 + \frac{16}{2} + \frac{72}{2} + 2\right)}{(22 - 1.5)} = 5.76 \text{ teeth/in}$$

Start with $P = 6$ teeth/in

$$d_2 = d_4 = N_2/P = 16/6 = 2.67 \text{ in}$$

$$d_3 = d_5 = 72/6 = 12.0 \text{ in}$$

Shaft speeds were previously determined to be

$$\omega_2 = 1750 \text{ rev/min} \quad \omega_3 = \omega_4 = 388.9 \text{ rev/min} \quad \omega_5 = 86.4 \text{ rev/min}$$

Get pitch-line velocities and transmitted loads for later use.

$$V_{23} = \frac{\pi d_2 \omega_2}{12} = \frac{\pi(2.67)(1750)}{12} = \underline{1223 \text{ ft/min}} \quad \text{Eq. (13-34), p. 699}$$

$$V_{45} = \frac{\pi d_5 \omega_5}{12} = \underline{271.5 \text{ ft/min}}$$

$$W_{23}^t = 33\,000 \frac{H}{V_{23}} = 33\,000 \left(\frac{20}{1223}\right) = \underline{540.0 \text{ lbf}} \quad \text{Eq. (13-35), p. 699}$$

$$W_{45}^t = 33\,000 \frac{H}{V_{45}} = 33\,000 \left(\frac{20}{271.5}\right) = \underline{2431 \text{ lbf}}$$

Start with gear 4, since it is the smallest gear, transmitting the largest load. It will likely be critical. Start with wear by contact stress, since it is often the limiting factor.

Gear 4 Wear

$$I = \frac{\cos 20^\circ \sin 20^\circ}{2(1)} \left(\frac{4.5}{4.5 + 1}\right) = 0.1315 \quad \text{Eq. (14-23), p. 747}$$

$$\text{For } K_v, \text{ assume } Q_v = 7. B = 0.731, A = 65.1 \quad \text{Eq. (14-29), p. 748}$$

$$K_v = \left(\frac{65.1 + \sqrt{271.5}}{65.1}\right)^{0.731} = 1.18 \quad \text{Eq. (14-27), p. 748}$$

Face width F is typically from 3 to 5 times circular pitch. Try

$$F = 4 \left(\frac{\pi}{P}\right) = 4 \left(\frac{\pi}{6}\right) = 2.09 \text{ in.}$$

Since gear specifications are readily available on the Internet, we might as well check for commonly available face widths. On www.globalspec.com, entering $P = 6$ teeth/in and $d = 2.67$ in, stock spur gears from several sources have face widths of 1.5 in or 2.0 in. These are also available for the meshing gear 5 with $d = 12$ in.

Choose $F = 2.0$ in.

$$\text{For } K_m, \quad C_{pf} = 0.0624 \quad \text{Eq. (14-32), p. 752}$$

$$C_{mc} = 1 \text{ uncrowned teeth} \quad \text{Eq. (14-31), p. 752}$$

$$C_{pm} = 1 \text{ straddle-mounted} \quad \text{Eq. (14-33), p. 752}$$

$$C_{ma} = 0.15 \text{ commercial enclosed unit} \quad \text{Eq. (14-34), p. 752}$$

$$C_e = 1 \quad \text{Eq. (14-35), p. 752}$$

$$K_m = 1.21 \quad \text{Eq. (14-30), p. 751}$$

$$C_p = 2300 \quad \text{Table 14-8, p. 749}$$

$$K_o = K_s = C_f = 1$$

$$\sigma_c = 2300 \sqrt{\frac{2431(1.18)(1.21)}{2.67(2)(0.1315)}} = \underline{161\,700 \text{ psi}} \quad \text{Eq. (14-16), p. 738}$$

Get factors for $\sigma_{c,\text{all}}$. For life factor Z_N , get number of cycles for specified life of 12 000 h.

$$L_4 = (12\,000 \text{ h}) \left(60 \frac{\text{min}}{\text{h}}\right) \left(389 \frac{\text{rev}}{\text{min}}\right) = 2.8 \times 10^8 \text{ rev}$$

$$Z_N = 0.9 \quad \text{Fig. 14-15, p. 755}$$

$$K_R = K_T = C_H = 1$$

For a design factor of 1.2,

$$\sigma_{c,\text{all}} = S_c Z_N / S_H = \sigma_c \quad \text{Eq. (14-18), p. 742}$$

$$S_c = \frac{S_H \sigma_c}{Z_N} = \frac{1.2(161\,700)}{0.9} = \underline{215\,600 \text{ psi}}$$

From Table 14-6, p. 743, this strength is achievable with Grade 2 carburized and hardened with $S_c = 225\,000 \text{ psi}$. To find the achieved factor of safety, $n_c = \sigma_{c,\text{all}} / \sigma_c$ with $S_H = 1$. The factor of safety for wear of gear 4 is

$$n_c = \frac{\sigma_{c,\text{all}}}{\sigma_c} = \frac{S_c Z_N}{\sigma_c} = \frac{225\,000(0.9)}{161\,700} = \underline{1.25}$$

Gear 4 Bending

$$J = 0.27 \quad \text{Fig. 14-6, p. 745}$$

$$K_B = 1$$

Everything else is the same as before.

$$\sigma = W_t K_v \frac{P_d K_m}{F J} = (2431)(1.18) \left(\frac{6}{2}\right) \left(\frac{1.21}{0.27}\right) \quad \text{Eq. (14-15), p. 738}$$

$$\underline{\sigma = 38\,570 \text{ psi}}$$

$$Y_N = 0.9 \quad \text{Fig. 14-14, p. 755}$$

Using Grade 2 carburized and hardened, same as chosen for wear, find $S_t = 65\,000$ psi (Table 14-3, p. 740).

$$\sigma_{\text{all}} = S_t Y_N = 58\,500 \text{ psi}$$

The factor of safety for bending of gear 4 is

$$n = \frac{\sigma_{\text{all}}}{\sigma} = \frac{58\,500}{38\,570} = 1.52$$

Gear 5 Bending and Wear

Everything is the same as for gear 4, except J , Y_N , and Z_N .

$$J = 0.41$$

Fig. 14-6, p. 745

$$L_5 = (12\,000 \text{ h})(60 \text{ min/h})(86.4 \text{ rev/min}) = 6.2 \times 10^7 \text{ rev}$$

$$Y_N = 0.97$$

Fig. 14-14, p. 755

$$Z_N = 1.0$$

Fig. 14-15, p. 755

$$\sigma_c = 2300 \sqrt{\frac{2431(1.18)(1.21)}{2.67(2)(0.1315)}} = 161\,700 \text{ psi}$$

$$\sigma = (2431)(1.18) \left(\frac{6}{2}\right) \left(\frac{1.21}{0.41}\right) = 25\,400 \text{ psi}$$

Choose Grade 2 carburized and hardened, the same as gear 4

$$n_c = \frac{\sigma_{c,\text{all}}}{\sigma_c} = \frac{225\,000}{161\,700} = 1.39$$

$$n = \frac{\sigma_{\text{all}}}{\sigma} = \frac{65\,000(0.97)}{25\,400} = 2.48$$

Gear 2 Wear

Gears 2 and 3 are evaluated similarly. Only selected results are shown.

$$K_v = 1.37$$

Try $F = 1.5$ in, since the loading is less on gears 2 and 3.

$$K_m = 1.19$$

All other factors are the same as those for gear 4.

$$\sigma_c = 2300 \sqrt{\frac{(539.7)(1.37)(1.19)}{2.67(1.5)(0.1315)}} = 94\,000 \text{ psi}$$

$$L_2 = (12\,000 \text{ h})(60 \text{ min/h})(1750 \text{ rev/min}) = 1.26 \times 10^9 \text{ rev} \quad Z_N = 0.8$$

Try grade 1 flame-hardened, $S_c = 170\,000$ psi

$$n_c = \frac{\sigma_{c,\text{all}}}{\sigma_c} = \frac{170\,000(0.8)}{94\,000} = 1.40$$

Gear 2 Bending

$$J = 0.27 \quad Y_N = 0.88$$

$$\sigma = 539.7(1.37) \frac{(6)(1.19)}{(1.5)(0.27)} = 13\,040 \text{ psi}$$

$$n = \frac{\sigma_{\text{all}}}{\sigma} = \frac{45\,000(0.88)}{13\,040} = 3.04$$

Gear 3 Wear and Bending

$$J = 0.41 \quad Y_N = 0.9 \quad Z_N = 0.9$$

$$\sigma_c = 2300 \sqrt{\frac{(539.7)(1.37)(1.19)}{2.67(1.5)(0.1315)}} = 94\,000 \text{ psi}$$

$$\sigma = 539.7(1.37) \frac{(6)(1.19)}{1.5(0.41)} = 8584 \text{ psi}$$

Try Grade 1 steel, through-hardened to 300 H_B . From Fig. 14-2, p. 739, $S_t = 36\,000$ psi and from Fig. 14-5, p. 742, $S_c = 126\,000$ psi.

$$n_c = \frac{126\,000(0.9)}{94\,000} = 1.21$$

$$n = \frac{\sigma_{\text{all}}}{\sigma} = \frac{36\,000(0.9)}{8584} = 3.77$$

In summary, the resulting gear specifications are:

All gears, $P = 6$ teeth/in

Gear 2, Grade 1 flame-hardened, $S_c = 170\,000$ psi and $S_t = 45\,000$ psi
 $d_2 = 2.67$ in, face width = 1.5 in

Gear 3, Grade 1 through-hardened to 300 H_B , $S_c = 126\,000$ psi and $S_t = 36\,000$ psi
 $d_3 = 12.0$ in, face width = 1.5 in

Gear 4, Grade 2 carburized and hardened, $S_c = 225\,000$ psi and $S_t = 65\,000$ psi
 $d_4 = 2.67$ in, face width = 2.0 in

Gear 5, Grade 2 carburized and hardened, $S_c = 225\,000$ psi and $S_t = 65\,000$ psi
 $d_5 = 12.0$ in, face width = 2.0 in

18-4 Shaft Layout

The general layout of the shafts, including axial location of gears and bearings, must now be specified in order to perform a free-body force analysis and to obtain shear force and bending moment diagrams. If there is no existing design to use as a starter, then the determination of the shaft layout may have many solutions. Section 7-3, p. 353, discusses the issues involved in shaft layout. In this section the focus will be on how the decisions relate to the overall process.

A free-body force analysis can be performed without knowing shaft diameters, but can not be performed without knowing axial distances between gears and bearings. It is extremely important to keep axial distances small. Even small forces can create large bending moments if the moment arms are large. Also, recall that beam deflection equations typically include length terms raised to the third power.

It is worth examining the entirety of the gearbox at this time, to determine what factors drive the length of the shaft and the placement of the components. A rough sketch, such as shown in Fig. 18-2, is sufficient for this purpose.

CASE STUDY PART 4 SHAFT LAYOUT

Continue the case study by preparing a sketch of the gearbox sufficient to determine the axial dimensions. In particular, estimate the overall length, and the distance between the gears of the intermediate shaft, in order to fit with the mounting requirements of the other shafts.

Solution

Fig. 18-2 shows the rough sketch. It includes all three shafts, with consideration of how the bearings are to mount in the case. The gear widths are known at this point. Bearing widths are guessed, allowing a little more space for larger bearings on the intermediate shaft where bending moments will be greater. Small changes in bearing widths will have minimal effect on the force analysis, since the location of the ground reaction force will change very little. The 4-in distance between the two gears on the countershaft is dictated by the requirements of the input and output shafts, including the space for the case to mount the bearings. Small allotments are given for the retaining rings, and for space behind the bearings. Adding it all up gives the intermediate shaft length as 11.5 in.

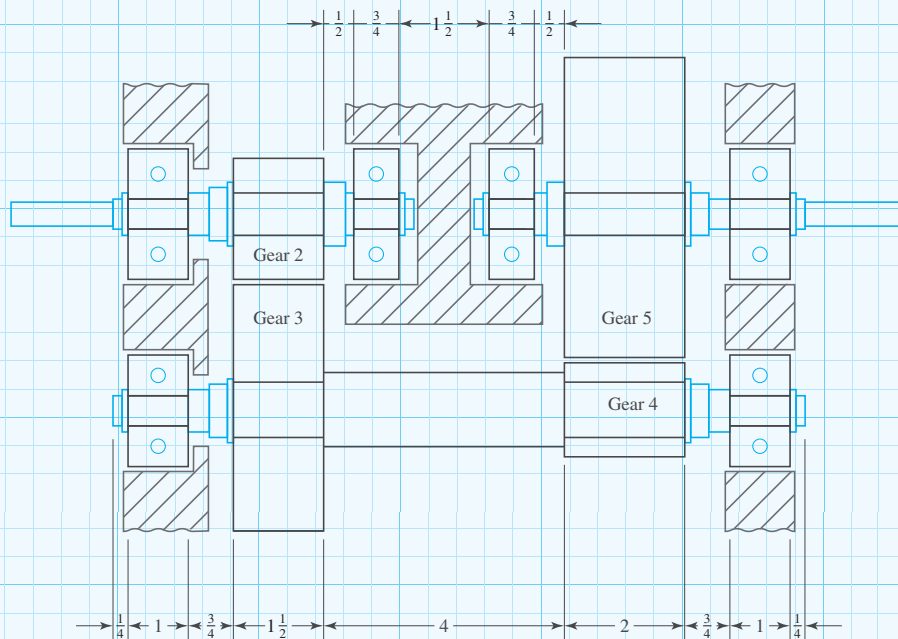


Figure 18-2

Sketch for shaft layout. Dimensions are in inches.

Wider face widths on gears require more shaft length. Originally, gears with hubs were considered for this design to allow the use of set screws instead of high-stress-concentration retaining rings. However, the extra hub lengths added several inches to the shaft lengths and the gearbox housing.

Several points are worth noting in the layout in Fig. 18–2. The gears and bearings are positioned against shoulders, with retaining rings to hold them in position. While it is desirable to place gears near the bearings, a little extra space is provided between them to accommodate any housing that extends behind the bearing, and to allow for a bearing puller to have space to access the back of the bearing. The extra change in diameter between the bearings and the gears allows the shoulder height for the bearing and the bore size for the gear to be different. This diameter can have loose tolerances and a large fillet radius.

Each bearing is restrained axially on its shaft, but only one bearing on each shaft is axially fixed in the housing, allowing for slight axial thermal expansion of the shafts.

18–5 Force Analysis

Once the gear diameters are known, and the axial locations of the components are set, the free-body diagrams and shear force and bending moment diagrams for the shafts can be produced. With the known transmitted loads, determine the radial and axial loads transmitted through the gears (see Secs. 13–14 through 13–17, pp. 697–706). From summation of forces and moments on each shaft, ground reaction forces at the bearings can be determined. For shafts with gears and pulleys, the forces and moments will usually have components in two planes along the shaft. For rotating shafts, usually only the resultant magnitude is needed, so force components at bearings are summed as vectors. Shear force and bending moment diagrams are usually obtained in two planes, then summed as vectors at any point of interest. A torque diagram should also be generated to clearly visualize the transfer of torque from an input component, through the shaft, and to an output component.

See the beginning of Ex. 7–2, p. 366, for the force analysis portion of the case study for the intermediate shaft. The bending moment is largest at gear 4. This is predictable, since gear 4 is smaller, and must transmit the same torque that entered the shaft through the much larger gear 3.

While the force analysis is not difficult to perform manually, if beam software is to be used for the deflection analysis, it will necessarily calculate reaction forces, along with shear force and bending moment diagrams in the process of calculating deflections. The designer can enter guessed values for diameters into the software at this point, just to get the force information, and later enter actual diameters to the same model to determine deflections.

18–6 Shaft Material Selection

A trial material for the shaft can be selected at any point before the stress design of the shaft, and can be modified as necessary during the stress design process. Section 7–2, p. 352, provides details for decisions regarding material selection. For the case study, an inexpensive steel, 1020 CD, is initially selected. After the stress analysis, a slightly higher strength 1050 CD is chosen to reduce the critical stresses without further increasing the shaft diameters.

18-7 Shaft Design for Stress

The critical shaft diameters are to be determined by stress analysis at critical locations. Section 7-4, p. 358, provides a detailed examination of the issues involved in shaft design for stress.

CASE STUDY PART 5 DESIGN FOR STRESS

Proceed with the next phase of the case study design, in which appropriate diameters for each section of the shaft are estimated, based on providing sufficient fatigue and static stress capacity for infinite life of the shaft, with minimum safety factor of 1.5.

Solution

The solution to this phase of the design is presented in Ex. 7-2, p. 366.

Since the bending moment is highest at gear 4, potentially critical stress points are at its shoulder, keyway, and retaining ring groove. It turns out that the keyway is the critical location. It seems that shoulders often get the most attention. This example demonstrates the danger of neglecting other stress concentration sources, such as keyways.

The material choice was changed in the course of this phase, choosing to pay for a higher strength to limit the shaft diameter to 2 in. If the shaft were to get much bigger, the small gear would not be able to provide an adequate bore size. If it becomes necessary to increase the shaft diameter any more, the gearing specification will need to be redesigned.

18-8 Shaft Design for Deflection

Section 7-5, p. 371, provides a detailed discussion of deflection considerations for shafts. Typically, a deflection problem in a shaft will not cause catastrophic failure of the shaft, but will lead to excess noise and vibration, and premature failure of the gears or bearings.

CASE STUDY PART 6 DEFLECTION CHECK

Proceed with the next phase of the case study by checking that deflections and slopes at the gears and bearings on the intermediate shaft are within acceptable ranges.

Solution

The solution to this phase of the design is presented in Ex. 7-3, p. 372.

It turns out that in this problem all the deflections are within recommended limits for bearings and gears. This is not always the case, and it would be a poor choice to neglect the deflection analysis. In a first iteration of this case study, with longer shafts due to using gears with hubs, the deflections were more critical than the stresses.

18-9 Bearing Selection

Bearing selection is straightforward now that the bearing reaction forces and the approximate bore diameters are known. See Chap. 11 for general details on bearing selection. Rolling-contact bearings are available with a wide range of load capacities and dimensions, so it is usually not a problem to find a suitable bearing that is close to the estimated bore diameter and width.

CASE STUDY PART 7 BEARING SELECTION

Continue the case study by selecting appropriate bearings for the intermediate shaft, with a reliability of 99 percent. The problem specifies a design life of 12 000 h. The intermediate shaft speed is 389 rev/min. The estimated bore size is 1 in, and the estimated bearing width is 1 in.

Solution

From the free-body diagram (see Ex. 7-2, p. 366),

$$R_{Az} = 115.0 \text{ lbf} \quad R_{Ay} = 356.7 \text{ lbf} \quad R_A = 375 \text{ lbf}$$

$$R_{Bz} = 1776.0 \text{ lbf} \quad R_{By} = 725.3 \text{ lbf} \quad R_B = 1918 \text{ lbf}$$

At the shaft speed of 389 rev/min, the design life of 12 000 h correlates to a bearing life of $L_D = (12\,000 \text{ h})(60 \text{ min/h})(389 \text{ rev/min}) = 2.8 \times 10^8 \text{ rev}$.

Start with bearing B since it has the higher loads and will likely raise any lurking problems. From Eq. (11-10), p. 570, assuming a ball bearing with $a = 3$ and $L = 2.8 \times 10^8 \text{ rev}$,

$$F_{RB} = (1)1918 \left[\frac{2.8 \times 10^8 / 10^6}{[0.02 + (4.459 - 0.02)(1 - 0.99)^{1/1.483}]} \right]^{1/3} = 20\,820 \text{ lbf}$$

A check on the Internet for available bearings (www.globalspec.com is one good starting place) shows that this load is relatively high for a ball bearing with bore size in the neighborhood of 1 in. Try a cylindrical roller bearing. Recalculating F_{RB} with the exponent $a = 10/3$ for roller bearings, we obtain

$$F_{RB} = 16\,400 \text{ lbf}$$

Cylindrical roller bearings are available from several sources in this range. A specific one is chosen from SKF, a common supplier of bearings, with the following specifications:

Cylindrical roller bearing at right end of shaft

$C = 18\,658 \text{ lbf}$, ID = 1.181 1 in, OD = 2.834 6 in, $W = 1.063 \text{ in}$

Shoulder diameter range = 1.45 in to 1.53 in, and maximum fillet radius = 0.043 in

For bearing *A*, again assuming a ball bearing,

$$F_{RA} = 375 \left[\frac{2.8 \times 10^8 / 10^6}{0.02 + 4.439(1 - 0.99)^{1/1.483}} \right]^{1/3} = 4070 \text{ lbf}$$

A specific ball bearing is chosen from the SKF Internet catalog.

Deep-groove ball bearing at left end of shaft

$C = 5058 \text{ lbf}$, ID = 1.000 in, OD = 2.500 in, $W = 0.75 \text{ in}$

Shoulder diameter range = 1.3 in to 1.4 in, and maximum fillet radius = 0.08 in

At this point, the actual bearing dimensions can be checked against the initial assumptions. For bearing *B* the bore diameter of 1.1811 in is slightly larger than the original 1.0 in. There is no reason for this to be a problem as long as there is room for the shoulder diameter. The original estimate for shoulder support diameters was 1.4 in. As long as this diameter is less than 1.625 in, the next step of the shaft, there should not be any problem. In the case study, the recommended shoulder support diameters are within the acceptable range. The original estimates for stress concentration at the bearing shoulder assumed a fillet radius such that $r/d = 0.02$. The actual bearings selected have ratios of 0.036 and 0.080. This allows the fillet radii to be increased from the original design, decreasing the stress-concentration factors.

The bearing widths are close to the original estimates. Slight adjustments should be made to the shaft dimensions to match the bearings. No redesign should be necessary.

18–10 Key and Retaining Ring Selection

The sizing and selection of keys is discussed in Sec. 7–7, p. 380, with an example in Ex. 7–6, p. 386. The cross-sectional size of the key will be dictated to correlate with the shaft size (see Tables 7–6 and 7–8, pp. 383, 385), and must certainly match an integral keyway in the gear bore. The design decision includes the length of the key, and if necessary an upgrade in material choice.

The key could fail by shearing across the key, or by crushing due to bearing stress. For a square key, it turns out that checking only the crushing failure is adequate, since the shearing failure will be less critical according to the distortion energy failure theory, and equal according to the maximum shear stress failure theory. Check Ex. 7–6 to investigate why.

CASE STUDY PART 8 KEY DESIGN

Continue the case study by specifying appropriate keys for the two gears on the intermediate shaft to provide a factor of safety of 2. The gears are to be custom bored and keyed to the required specifications. Previously obtained information includes the following:

Transmitted torque: $T = 3240 \text{ lbf-in}$

Bore diameters: $d_3 = d_4 = 1.625 \text{ in}$

Gear hub lengths: $l_3 = 1.5 \text{ in}$, $l_4 = 2.0 \text{ in}$

Solution

From Table 7-6, p. 383, for a shaft diameter of 1.625 in, choose a square key with side dimension $t = \frac{3}{8}$ in. Choose 1020 CD material, with $S_y = 57$ kpsi. The force on the key at the surface of the shaft is

$$F = \frac{T}{r} = \frac{3240}{1.625/2} = 3988 \text{ lbf}$$

Checking for failure by crushing, we find the area of one-half the face of the key is used.

$$n = \frac{S_y}{\sigma} = \frac{S_y}{F/(tl/2)}$$

Solving for l gives

$$l = \frac{2Fn}{tS_y} = \frac{2(3988)(2)}{(0.375)(57000)} = 0.75 \text{ in}$$

Since both gears have the same bore diameter and transmit the same torque, the same key specification can be used for both.

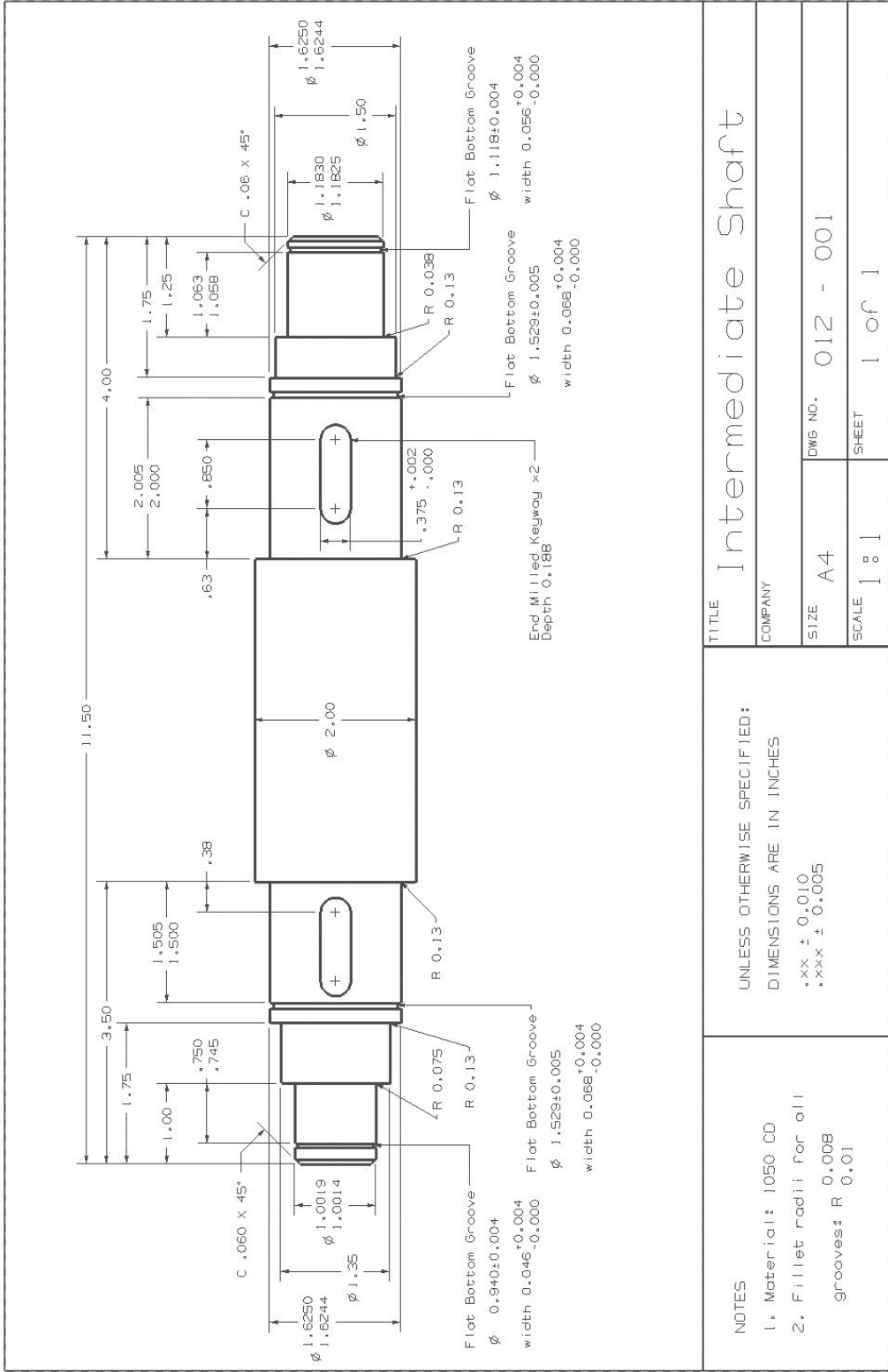
Retaining ring selection is simply a matter of checking catalog specifications. The retaining rings are listed for nominal shaft diameter, and are available with different axial load capacities. Once selected, the designer should make note of the depth of the groove, the width of the groove, and the fillet radius in the bottom of the groove. The catalog specification for the retaining ring also includes an edge margin, which is the minimum distance to the next smaller diameter change. This is to ensure support for the axial load carried by the ring. It is important to check stress-concentration factors with actual dimensions, as these factors can be rather large. In the case study, a specific retaining ring was already chosen during the stress analysis (see Ex. 7-2, p. 366) at the potentially critical location of gear 4. The other locations for retaining rings were not at points of high stress, so it is not necessary to worry about the stress concentration due to the retaining rings in these locations. Specific retaining rings should be selected at this time to complete the dimensional specifications of the shaft.

For the case study, retaining rings specifications are entered into globalspec, and specific rings are selected from Truarc Co., with the following specifications:

	Both Gears	Left Bearing	Right Bearing
Nominal Shaft diameter	1.625 in	1.000 in	1.181 in
Groove diameter	1.529 ± 0.005 in	0.940 ± 0.004 in	1.118 ± 0.004 in
Groove width	0.068 $\begin{smallmatrix} +0.004 \\ -0.000 \end{smallmatrix}$ in	0.046 $\begin{smallmatrix} +0.004 \\ -0.000 \end{smallmatrix}$ in	0.046 $\begin{smallmatrix} +0.004 \\ -0.000 \end{smallmatrix}$ in
Nominal groove depth	0.048 in	0.030 in	0.035 in
Max groove fillet radius	0.010 in	0.010 in	0.010 in
Minimum edge margin	0.144 in	0.105 in	0.105 in
Allowable axial thrust	11 850 lbf	6000 lbf	7000 lbf

These are within the estimates used for the initial shaft layout, and should not require any redesign. The final shaft should be updated with these dimensions.

Figure 18-3



18-11 Final Analysis

At this point in the design, everything seems to check out. Final details include determining dimensions and tolerances for appropriate fits with the gears and bearings. See Sec. 7-8, p. 387, for details on obtaining specific fits. Any small changes from the nominal diameters already specified will have negligible effect on the stress and deflection analysis. However, for manufacturing and assembly purposes, the designer should not overlook the tolerance specification. Improper fits can lead to failure of the design. Lack of attention to tolerance specification can make the part unfunctional or overly expensive to manufacture. Further insight on tolerance specification is given in Sec. 1-14, p. 27. The final drawing for the intermediate shaft is shown in Fig. 18-3. This drawing shows the important dimensions and dimensional tolerances in a form that is generally considered satisfactory for small production quantities where direct attention is given to manufacturing methods. A more robust method of part specification that also addresses the allowed variations from perfect form (e.g., straightness or concentricity) is known as Geometric Dimensioning and Tolerancing and is introduced in Chap. 20.

For documentation purposes, and for a check on the design work, the design process should conclude with a complete analysis of the final design. Remember that analysis is much more straightforward than design, so the investment of time for the final analysis will be relatively small.

PROBLEMS

- 18-1** For the case study problem, design the input shaft, including complete specification of the gear, bearings, key, retaining rings, and shaft.
- 18-2** For the case study problem, design the output shaft, including complete specification of the gear, bearings, key, retaining rings, and shaft.
- 18-3** For the case study problem, use helical gears and design the intermediate shaft. Compare your results with the spur gear design presented in this chapter.
- 18-4** Perform a final analysis for the resulting design of the intermediate shaft of the case study problem presented in this chapter. Produce a final drawing with dimensions and tolerances for the shaft. Does the final design satisfy all the requirements? Identify the critical aspects of the design with the lowest factor of safety.
- 18-5** For the case study problem, change the power requirement to 40 horsepower. Design the intermediate shaft, including complete specification of the gears, bearings, keys, retaining rings, and shaft.