Linear Programming

Linear programming is a mathematical technique used to determine the optimal solutions to certain specific problems. This tool is frequently used to find the least-cost combinations of inputs necessary to produce some desired level of output; that is, cost minimization problems. However, the same technique can be used to solve other types of optimization problems, such as the optimal level of inventory, the least-cost method of transporting commodities, and so on.

Basic Concepts

Let's begin with a practical problem: A firm produces two products, X_1 and X_2, which it can sell at fixed prices, P_1 and P_2. The production of X_1 and X_2 requires the use of three different types of machines, which can be used for eight hours a day. The firm currently owns three type-1 machines, two type-2 machines, and five type-3 machines. Therefore, given the daily capacity of each machine, the firm has available 24 type-1 machine-hours, 16 type-2 machine-hours, and 40 type-3 machine-hours per period. In the short run, the firm cannot buy or sell any machines; but it can employ various amounts of labor or other inputs at prevailing market prices.[1]

Since labor and other inputs are obtainable in unlimited supplies, the firm first calculates the gross profit on each product net of labor and other input costs from the market prices for X_1 and X_2. These net prices,

$p_1 = P_1 -$ Labor cost per unit of $X_1 -$ Other costs per unit of X_1
$p_2 = P_2 -$ Labor cost per unit of $X_2 -$ Other costs per unit of X_2

are the accountant's measure of gross profit. The problem for the firm is to choose the output combination that maximizes total (gross) profit.

To solve this problem, we must first know something about the actual productive capacity of each machine. Suppose that the number of type-1 machine-hours required per unit of X_1 is six, while only three type-1 machine-hours are required to produce a unit of X_2. Likewise, suppose each unit of X_1 requires two type-2 machine-hours and X_2 requires four hours per unit. Finally, suppose that eight type-3 machine-hours are required to produce a unit of either X_1 or X_2. Given the respective fixed quantities of machine-hours per period, these production relations may be written in the form of constraints:

$6X_1 + 3X_2 = 24$
$2X_1 + 4X_2 = 16$
$8X_1 + 8X_2 = 40$

which show the possible combinations of X_1 and X_2, given machine availability. For example, the first constraint indicates that if no X_2 is produced, the maximum daily production of X_1 is 4 units because each unit of X_1 requires 6 hours of a type-1 machine and only 24 hours of type-1 are available. Similarly from the first constraint, if 4 units of X_2 are produced, only 12 hours of type-1 machine time are left to produce X_1; thus, only 2 units of X_1 can be produced. The other two constraints are interpreted similarly. Thus, all three constraints put a limit on the combinations of X_1 and X_2 that the firm can produce daily.

Suppose that the net prices of X_1 and X_2 are \$12 and \$8 per unit, respectively. The problem facing the firm is to choose the combination of X_1 and X_2 (X_1 and X_2 are the *choice variables*) that maximizes total gross income:

$$p = 12X_1 + 8X_2$$

subject to the physical constraints imposed by the production processes and the limited availability of machines.

In general, we write this type of problem, a *linear program*, as

$$\begin{aligned} \max\ p &= p_1X_1 + p_2X_2 \\ \text{subject to } & a_{11}X_1 + a_{12}X_2 = r_1 \\ & a_{21}X_1 + a_{22}X_2 = r_2 \\ & a_{31}X_1 + a_{32}X_2 = r_3 \\ & X_1, X_2 = 0 \end{aligned}$$

where a_{ij} (i = 1, 2, 3; j = 1, 2) is the required number of type-i machine-hours per unit of output j and r_i represents the restrictions on the program—in our example, the fixed quantities of machine-hours available. Of course, it should be noted that there could be any number of choice variables and constraints in any given linear program.

The first equation in the program, the total (gross) profit function, constitutes the objective function of the linear program; that is, it is the firm's *objective* to maximize total gross profits per production period. The three inequalities that follow are the *constraints* imposed on the linear program by the technological relation and the restrictions. Finally, by the last two inequalities (X_1, X_2 = 0), referred to as the *nonnegativity restrictions*, we impose the restriction that negative outputs are impossible. Therefore, there are three essential ingredients to every linear program: an objective function, a set of constraints, and a set of nonnegativity restrictions.

Returning to our specific example, we may write our problem in this general form:

max p = $12X_1 + 8X_2$
subject to $6X_1 + 3X_2 = 24$
$2X_1 + 4X_2 = 16$
$8X_1 + 8X_2 = 40$
$X_1, X_2 = 0$

Since our problem involves only two choice variables, X_1 and X_2, the linear program may be solved graphically. In Figure A.1 (23.0K) we plot X_1 along the horizontal axis and X_2 along the vertical axis. Because of the nonnegativity restrictions, we need concern ourselves only with the positive (nonnegative) quadrant.

To see what the constraints look like graphically, first treat them as equalities and plot them as straight lines as in Panel A. Since each constraint is of the "less-than-or-equal-to" type, only the points lying on the line or below it will satisfy the constraint. To satisfy all three constraints simultaneously, we can accept only those points that lie interior to all three constraint lines. The collection of all points that satisfy all three constraints simultaneously is called the *feasible region*, shown as the shaded region in Panel B. Each individual point in that region is known as *feasible solution.* It should be noted that the feasible region includes the points on the *boundary*, or the heavy line in Panel B. Note that in the present (two-dimensional) case, the corner points on the boundary are called *extreme points.* They occur either at the intersection of two constraints [(2, 3) and (3, 2)] or at the intersection of one constraint and one of the axes [(0, 4) and (4, 0)].

The feasible region contains all output combinations satisfying all three constraints and the nonnegativity restrictions. However, some of these points may entail a lower level of total profits than others. To maximize profits, we must consider the objective function. To plot the profit function in (X_1, X_2) space we rewrite it as

X_2 = p/8 − 3/2 X_1

This equation represents a family of parallel straight lines corresponding to different levels of profits or values of p. Since each of these lines is associated with a specific value of p, they are sometimes called *isoprofit curves.* Three isoprofit curves are shown in Figure A.2 (24.0K) as dashed lines, labeled I, II, and III.

The firm's objective is, of course, to attain the highest possible isoprofit curve while still remaining in the feasible region. In Figure A.2 (24.0K) , isoprofit curve II satisfies this objective. While isoprofit curve III represents the highest level of profits, the combinations of X_1 and X_2 on this line are not in the feasible region, so this level of profit cannot be attained. Combinations on isoprofit line I clearly lie in the feasible region; however, a higher level of profit can be reached. Isoprofit line II represents the highest possible profit level that still incorporates a point in the feasible region. It coincides with the output combination of 3 units of X_1 and 2 units of X_2. Thus, the point (3, 2) is the *optimal solution* to our linear program. Total profits for this optimal output combination can easily be obtained by using the values $X_1 = 3$ and $X_2 = 2$ in the objective function to yield the maximum profit, p = $52 per production period.

Note that the optimal solution is an extreme point. In fact, the optimal solution to *any* linear program is always an extreme point. This fact will prove useful in developing a general solution methodology for linear programs.

General Solution Method

With two choice variables, the graphical method provided an optimal solution with little difficulty. This situation holds regardless of the number of constraints; additional constraints simply increase the number of extreme points, not the dimension of the diagram. When there are more than three choice variables, however, the graphical method becomes intractable, since we cannot draw a four-dimensional graph. Therefore, we need an analytical method to find the optimal solution to linear programs involving any number of choice variables.

As suggested above, the optimal solution of a linear program is one of the extreme points. Given a two-dimensional feasible region, it is relatively easy to find its extreme points, but finding them for the nongraphable *n*-variable case is more complex. Before considering the *n*-variable case, it will be instructive to return to our example. In Figure A.2 (24.0K) , note that there are five extreme points—(0, 4), (2, 3), (3, 2), (4, 0), and (0, 0)—all of which can be placed in one of three general categories.

The first category consists of those extreme points occurring at the intersection of two constraints. In our example, these points are (3, 2) and (2, 3). While such points exactly fulfill two of the three constraints, the remaining constraint is inexactly fulfilled. Consider the output combination (3, 2). While the type-1 and type-3 machine constraints are exactly fulfilled, (3, 2) lies inside the type-2 machine constraint and hence there is *slack* (or underutilization) in the use of type-2 machines.

Extreme points in the second category, illustrated by (0, 4) and (4, 0), occur at the intersection of a constraint and one of the axes. Because they are located on only one of the constraints, these points exactly fulfill only one constraint; therefore, at such points there will be slack in the two remaining constraints. Last, the third category of extreme points consists of a single output combination, the origin (0, 0), where there exists slack in all the constraints.

The point is that whenever the number of constraints exceeds the number of choice variables, every extreme point will involve slack in at least one of the constraints. Furthermore, as is evident from Panel A of Figure A.1 (23.0K) , the magnitude of the slack in any particular constraint can be calculated. Therefore, when we choose a particular extreme point as the optimal solution, we are choosing not only the optimal output combination (X_1, X_2) but also the optimal amount of slack in at least one constraint. Let us consider these slacks explicitly and denote the slack in the ith constraint (i = 1, 2, 3 in our example) by S_i. We call these S_i's *slack variables.* Since we now explicitly consider the possible slack in each constraint, we can transform each inequality constraint into a strict equality by adding these S_i's to the left-hand side of the ith constraint.

Returning to our example and adding a slack variable to each constraint, we may rewrite our linear program as

max $p = 12X_1 + 8X_2$
subject to $6X_1 + 3X_2 + S_1 = 24$
$2X_1 + 4X_2 + S_2 = 16$
$8X_1 + 8X_2 + S_3 = 40$
$X_1, X_2, S_1, S_2, S_3 = 0$

There are now five choice variables: X_1, X_2, S_1, S_2, and S_3. When $S_i > 0$, there is slack in the ith constraint (a *nonbinding* constraint); if $S_i = 0$, there is no slack and the ith constraint is exactly fulfilled (a *binding* constraint). Slack in a constraint for this particular problem could best be thought of as excess capacity or overcapitalization of a certain type of machine.

It is easy to determine the values of the slacks implied by each extreme point. If we start with the origin (0, 0), we substitute $X_1 = 0$ and $X_2 = 0$ into the transformed constraints and find that $S_1 = 24$, $S_2 = 16$, and $S_3 = 40$. Thus, the extreme point (0, 0) in output space can be mapped into *solution space* as the point

$(X_1, X_2, S_1, S_2, S_3) = (0, 0, 24, 16, 40)$

Similarly, we may map each extreme point in outer space into solution space. The results are presented in Table A.1 (44.0K) .

From Table A.1 (44.0K) the profit contribution at each extreme point can be calculated by inserting the values for X_1 and X_2 into the objective function. The point that yields the maximum profit is the constrained profit-maximizing output point—the solution to our linear programming problem. The profit contributions of each point in solution space are shown in Table A.2 (27.0K) . Again, we confirm that output combination (3, 2) is the profit-maximizing point. Note that $S_2 > 0$ at the optimum indicates that the constraint on the type-2 machine is nonbinding.

The procedure described above is used in solving more complex linear programming problems. Computer programs are available which find solution values for the variables at all extreme points, evaluate total profits at each potential extreme point, and then determine the extreme point at which the objective function is maximized.

The Dual in Linear Programming

For every maximization problem in linear programming there exists a symmetrical minimization problem and vice versa. The original programming problem is referred to as the *primal* program, and its symmetrical counterpart is referred to as the *dual* program. The concept of this duality is quite significant because the optimal values of the objective functions in the primal and in the dual are always identical. Therefore, the analyst can pick the program, the primal or the dual, that is easiest to solve.

The linear program we have been using as an illustration—our *primal*—is a maximization problem: we wish to maximize total (gross) profit subject to the constraints imposed by the technology and machine time availability:

Primal

max p = $12X_1 + 8X_2$
subject to $6X_1 + 3X_2 = 24$

$$2X_1 + 4X_2 = 16$$
$$8X_1 + 8X_2 = 40$$
$$X_1, X_2 = 0$$

Corresponding to this maximization problem there exists a *dual* minimization problem: minimize the (opportunity) cost of using available machine-hours for the three machines subject to the constraints imposed by the production process and (gross) profitability of the two outputs:

Dual

min p° = $24y_1 + 16y_2 + 40y_3$
subject to $6y_1 + 2y_2 + 8y_3 = 12$

$$3y_1 + 4y_2 + 8y_3 = 8$$
$$y_1, y_2, y_3 = 0$$

In the primal, the choice variables X_1 and X_2 are the output levels of the two products. In the dual, the choice variables y_1, y_2, and y_3 represent the shadow prices (or premiums) for the inputs. For example, the variable y_1 is the shadow price of using one hour of machine type-1, and since we have 24 type-1 hours, the total cost of using machine type-1 is $24y_1$. A shadow price can be viewed as the implicit value to the firm of having 1 more unit of the input; that is, the marginal profit contribution of the input. We then attempt to determine minimum values, or shadow prices, for each of the inputs, such that these shadow prices will be just sufficient to absorb the firm's total profit. In other words, we seek to assign values to each input so as to minimize the total inputted value of the firm's resources.

In the primal, the constraints reflected the fact that the total hours of each type of machine used in the production of X_1 and X_2 could not exceed the available number of hours of each type of machine. In the dual, the constraints state that the value assigned the inputs used in the production of 1 unit of X_1 or 1 unit of X_2 must not be less than the profit contribution provided by a unit of these products. Recall that $12 is the (gross) profit per unit of X_1 and $8 is the profit per unit of X_2. The constraints require that the shadow prices of the different types of machines times the hours of each type required to produce a unit of X_1 or X_2 must be greater than or equal to the gross (unit) profit of X_1 or X_2.

To solve the dual, we again introduce slack variables, which allow us to write the constraint inequalities as strict equalities. Notice that in constrained minimization problems the constraints are of the "greater than or equal to" variety. Therefore, we introduce slack variables to the left-hand side of the constraints with a negative sign. (These negative S_i's used in the solution of minimization programs are often referred to as *surplus* variables.) We can then write the dual program as:

min $p° = 24y_1 + 16y_2 + 40y_3$
subject to $6y_1 + 2y_2 + 8y_3 - S_1 = 12$

$$3y_1 + 4y_2 + 8y_3 - S_2 = 8$$
$$y_1, y_2, y_3, S_1, S_2 = 0$$

Since there are three choice variables (y_1, y_2, and y_3), a graphical solution would require a three-dimensional figure. Instead of such a complex diagram let's use the general techniques described above to find the solution space, evaluate the objective function for each feasible solution, and find that solution which minimizes the objective function.

A general rule illustrated in Table A.1 is that the maximum number of nonzero values in any solution is equal to the number of constraints. (In Table A.1 (44.0K), the number of constraints is three; so the maximum number of nonzero values in any solution is three.) Since there are two constraints in this dual problem, a maximum of two nonzero-valued variables define any solution point. Therefore, we can solve for the solutions by setting three of the variables—y_1, y_2, y_3, S_1, S_2—equal to zero and solving the constraint equations for the values of the remain-ing two.

For example, we can set y_1, y_2, and y_3 equal to zero and solve for S_1 and S_2. Using the first constraint,

$$6 * 0 + 2 * 0 + 8 * 0 - S_1 = 12$$

so $S_1 = -12$. Likewise, using the second constraint,

$$3 * 0 + 4 * 0 + 8 * 0 - S_2 = 8$$

and $S_2 = -8$. However, since S_1 and S_2 cannot be negative, this solution is outside the feasible region. Alternatively, setting y_1, y_2, and S_1 equal to zero, $y_3 = 1.5$ and $S_2 = 4$. Since all the values in this solution are positive, the solution lies in the feasible region. All the potential solutions are presented in Table A.3 (41.0K) .

It is apparent from the table that not all the solutions lie within the feasible region. Only solutions 3, 5, 7, 9, and 10 meet the nonnegativity restrictions; that is, these are the only feasible solutions.

Each of the feasible solutions is then used to calculate a corresponding value of the objective function. For example, using solution 3, the value of the objective function is

$$p° = 24 * 0 + 16 * 0 + 40 * 1.5 = 60$$

All these values are summarized in Table A.4 (34.0K) .

With solution 9, the objective function—the total value inputed to the different types of machines—is minimized. As mentioned earlier, and confirmed in this example, the optimal value of the dual objective function is equal to the optimal value of the primal objective function (see Table A.2 (27.0K)).

Note that at the optimum, the shadow price of type-2 machine-hours is zero. A zero shadow price implies that the input in question has a zero marginal value to the firm; adding another type-2 machine-hour adds nothing to the firm's maximum attainable profit. Thus, a zero shadow price for type-2 machines is consistent with our findings in the solution to the primal: the type-2 machine constraint is nonbinding. Excess capacity exists with respect to type-2 machines, so additional hours will not result in increased production of either X_1 or X_2. Analogously, we see that the shadow prices of type-1 and type-3 machines are positive. A positive shadow indicates that the fixed number of these machines' hours imposes a binding constraint on the firm and that, if an additional hour of type-1 (type-3) machine work is added, the firm can increase its total profit by \$1.33 (\$0.50).

The dual solution has thus far not indicated the optimal output combination (X_1, X_2); however, it does provide all the information we need to determine these optimal values. Note first that the solution to the dual tells us that the type-2 machine constraint is nonbinding. Furthermore, it tells us that, at the optimal output combination, p = p°

= \$52. Now consider the three constraints in the primal, which we rewrite here for convenience:

$$6X_1 + 3X_2 + S_1 = 24 \quad \text{type-1}$$
$$2X_1 + 4X_2 + S_2 = 16 \quad \text{type-2}$$
$$8X_1 + 8X_2 + S_3 = 40 \quad \text{type-3}$$

From the solution to the dual we know that the type-1 and type-3 constraints are binding, because the dual found these inputs to have positive shadow prices. Accordingly, S_1 and S_3 equal zero in the primal program, and the binding constraints can be rewritten as

$$6X_1 + 3X_2 = 24$$
$$8X_1 + 8X_2 = 40$$

These two equations may be solved simultaneously to determine the optimal output combination. In this example, the solution is $X_1 = 3$ and $X_2 = 2$, the same output combination that was obtained in the primal problem.

Let us stress the two major points of this discussion and example. First, the choice between solving the primal or the dual of a linear programming problem is arbitrary, since both yield the same optimal value for the objective function. Second, the optimal solution obtained from the dual provides the information necessary to obtain the solution for the primal and vice versa. Thus, as we mentioned at the outset, one can elect to solve either the primal or the dual, depending on which one is easier to solve.

Activity Analysis: Linear Programming and Production Planning for a Single Output

As emphasized in Chapters 9 and 10, a decision problem faced by all firms is how to determine the least-cost combination of inputs needed to produce a particular product. If the production process satisfies certain regularity conditions, linear programming may be applied to solve the cost minimization problem.

Suppose that a firm produces a single product, Q, using two inputs, capital (K) and labor (L). As long as the production processes are subject to fixed proportions and constant returns to scale, we can characterize the relation between input usage and output as linear functions and thereby use linear programming to obtain a solution.

To illustrate how this is accomplished, consider the four production processes depicted in Figure A.3 (66.0K) . Since production is characterized by fixed proportions, the relations between input usage and output are shown by a straight line from the origin. These lines are referred to as *activity rays*—hence the title *activity analysis.*

In Figure A.3 (66.0K) , production process *A* requires 4 units of *K* and 4 units of *L* to produce 1 unit of *Q.* This requirement is illustrated by point A_1. Process *B* uses 4 units of *L* and 2 units of *K* to produce 1 unit of output. Similarly, process *C* uses 1.5 units of *K* and 5 units of *L*, while process *D* requires 8 units of *L* and 1 unit of *K* to produce 1 unit of output. These input-output relations are illustrated by B_1, C_1, and D_1, respectively. If we recall the definition of an isoquant, that is, different input combinations for which the level of output is constant, we can connect points A_1 through D_1 and derive an isoquant corresponding to a level of output equal to 1 unit of *Q.* In Figure A.3 this piecewise linear isoquant is labeled Q_1. With constant returns to scale, doubling the amount of both inputs employed results in output also doubling. In our graph, this doubling of inputs is illustrated by points A_2, B_2, C_2, and D_2. Connecting these points, we derive an isoquant corresponding to 2 units of output; it is labeled Q_2. Similarly, we may find isoquants Q_3 and Q_4 for 3 and 4 units of output, respectively.

Suppose that you are asked to determine the least-cost combination of *L* and *K* for an output level of 4 units when a unit of labor costs $4 (say the hourly wage rate is $4) and a unit of capital costs $8 (say it costs $8 to run a machine for one hour). This problem is simply a constrained minimization problem; that is, we want to minimize the total cost of producing 4 units of output. Accordingly, we may translate the problem into a linear programming problem. For illustrative purposes we will solve this program first graphically and then solve it using our general algebraic method developed above.

The isoquant for 4 units of output is reproduced in Figure A.4 (10.0K) . Since we know the price of a unit of *K* is $8 and the price of a unit of labor is $4, we can plot on this graph a family of *isocost* curves corresponding to different levels of total cost. These curves are derived by solving the total cost function $C = 8K + 4L$ to obtain

$$L = \frac{C}{4} - 2K$$

Recall from Chapter 10 that we used a tangency rule to find the least-cost combination of inputs: we find the isocost curve that is just tangent to the isoquant and, therefore, the closest to the origin. At that point of tangency, corresponding to a particular combination of inputs, total cost of production is minimized. In linear programming, the same process is used.

In Figure A.4 (10.0K) isocost curves (I_1, I_2, I_3, I_4) are drawn through points B_4, C_4, D_4, and A_4. It is easy to see that isocost I_1 through point B_4 (8, 16) is closest to the origin and, therefore, represents the least cost possible of producing 4 units of output. If we use 8 units of *K* and 16 of *L* in our cost equation, we obtain a minimum total cost of production of $128.

We can use our algebraic method to solve the same problem. First note that there is only one constraint—the isoquant representing 4 units of output. Therefore, at the optimum there will be no slack or surplus in the constraint, and we can determine the solution simply by substituting the values for the extreme points into the cost function (our objective function in this program). The results are shown in Table A.5 (5.0K) . Since our objective is to minimize total cost, we pick the input combination that does just that. Again we confirm our result from the graphical solution; the combination of 8 units of capital and 16 units of labor minimizes the total cost of producing 4 units of *Q.*

APPENDIX

Statistical Tables

Student's t-Distribution (53.0K)

The table on page 725 provides critical values of the *t*-distribution at four levels of significance—0.10, 0.05, 0.02 and 0.01. It should be noted that these values are based on a two-tailed test for significance: a test to determine if an estimated coefficient is significantly different from zero. For a discussion of one-tailed hypothesis tests, a topic not covered in this text, the reader is referred to Terry Sincich, *A Course in Modern Business Statistics*, 2d ed. (New York: Dellen/Macmillan College Publishing, 1994).

To illustrate the use of this table, consider a multiple regression that uses 30 observations to estimate three coefficients, *a*, *b*, and *c.* Therefore, there are 30 − 3 = 27 degrees of freedom. If the level of significance is chosen to be 0.05 (the confidence level is 0.95 = 1 − 0.05), the critical *t*-value for the test of significance is found in the table to be 2.052. If a lower level of significance (a higher confidence level) is required, a researcher can use the 0.01 level of significance (0.99 level of confidence) to obtain a critical value of 2.771. Conversely, if a higher significance level (lower level of confidence) is acceptable, the researcher can use the 0.10 significance level (0.90 confidence level) to obtain a critical value of 1.703.

The F-Distribution (138.0K)
The table on pages 726−727 provides critical values of the *F*-distribution at 0.05 and 0.01 levels of significance (or the 0.95 and 0.99 levels of confidence, respectively). To illustrate how the table is used, consider a multiple regression that uses 30 observations to estimate three coefficients; that is, $n = 30$ and $k = 3$. The appropriate *F*-statistic has $k - 1$ degrees of freedom for the numerator and $n - k$ degrees of freedom for the denominator. Thus, in the example, there are 2 and 27 degrees of freedom. From the table the critical *F*-value corresponding to a 0.05 level of significance (or 0.95 level of confidence) is 3.35. If a 0.01 significance level is desired, the critical *F*-value is 5.49.

[1]Outputs and inputs are assumed to be infinitely divisible, and the outputs are produced according to fixed proportions, constant-returns-to-scale processes.

TECHNICAL PROBLEMS

1. Solve the following linear programming problem graphically:

   maximize $p = 2X_1 + 3X_2$
   subject to $X_1 = 8$

   $$X_2 = 6$$
   $$X_1 + 4X_2 = 16$$
   $$X_1, X_2 = 0$$

2. In problem 1, how would the optimal solution change if the restrictions imposed (i.e., the r_i's) were all cut in half?
3. Solve the following linear programming problem using the general solution method:

   minimize $C = 3X_1 + 4X_2$
   subject to $X_1 + X_2 = 2$

$$2X_1 + 4X_2 = 5$$
$$X_1, X_2 = 0$$

4. Form the dual to the linear programming problem presented in problem 3; then solve it to obtain the optimal values of X_1 and X_2.
5. Provide an explanation of the nonnegativity constraints for the problem of minimizing cost subject to a desired level of output.
6. Give some examples of managerial decisions for which linear programming can provide useful information. For each of these, suggest the type of analysis that would be employed.
7. Suppose you were hired by a firm that produces several products. This firm needs to know the amounts of the different products it should produce to maximize total profit. What information would you require? How would you analyze this problem?