Estimating and Forecasting Industry Demand for Price-Taking Firms

As we explained in the textbook discussion of statistical estimation of demand and statistical forecasting, estimating the parameters of the empirical demand function can be accomplished by using regression analysis, and the method of estimating the parameters depends on whether the price of the product is determined by the intersection of demand and supply curves (a market-determined price) or is set by the manager of a firm (a manager-determined price). This important distinction also carries over to forecasting prices and quantities with the estimated demand (and supply) equation. In this Special Topic Module, we will show you the correct method of estimating the industry demand (and supply) curve for price-taking firms.

As we mentioned in the textbook, estimating the demand for a price-taking industry is a bit more challenging than estimating demand for a single, price-setting firm for two reasons. First, since price is determined by the interplay of both demand and supply, demand cannot be correctly estimated without also accounting for variation in price and quantity data attributable to both demand-side and supply-side variables simultaneously. As you will see shortly, this complexity calls for the researcher to collect data on all the relevant demand-determining variables and all the relevant supply-determining variables. Thus much more data are required to estimate industry demand than to estimate a single (price-setting) firm's demand. Second, ordinary least-squares (OLS) regression, which works fine for estimating parameters of a single firm's demand curve, fails to produce unbiased estimates when used to estimate parameters for an industry demand curve. To fix this problem, a different estimation technique, called two-stage least-squares (2SLS), must be utilized. We will show you how to do this and how to interpret the estimation results.

In this Special Topic Module, we also show you how to use the estimated industry demand and estimated industry supply curves to make forecasts of future prices and output levels in price-taking industries. This forecasting technique is more sophisticated than the simpler time-series forecasting method presented in the textbook. This is not meant to disparage time-series methods, however, since they often forecast more accurately in the short term than more complex multiple-equation econometric techniques, which often tend to perform better than time-series methods in longer-term forecasting situations.

We begin this module with discussion of the important statistical differences between estimating demand for a single price-setting firm and demand for a competitive industry. In section II of this module, we set forth the estimation methodology and present an application by estimating the world demand for copper using real-world data. In section III, we show you how to forecast price and quantities for price-taking industries. We again use the copper market data to illustrate how to forecast with simultaneous demand and supply equations. Technical Problems are provided in this module, just as they are in the textbook. Answers to these Technical Problems can be found at the end of this module. We also provide a Statistical Appendix to show you in more detail how two-stage least-squares estimation solves the problem of simultaneous equations bias.

I. MARKET-DETERMINED VERSUS MANAGER-DETERMINED PRICES

As noted earlier, estimating the parameters of the empirical demand function can be accomplished using regression analysis, and the method of estimating the parameters depends on whether the price of the product is determined by the intersection of demand and supply curves (a *market-determined* price) or is set by the manager of a firm (a *manager-determined* price). As we stressed throughout the text, our goal is to show you how to *use* estimated values of parameters in decision making, rather than to show you the statistical details involved in computing the estimates. In this section, we will briefly discuss *why* the parameters of a demand function in which price is market-determined cannot be correctly estimated using the same estimation method that is appropriate for estimating parameters of a demand function in which price is manager-determined. While it is not important for the purposes of managerial decision making to understand the computational procedure required to estimate correctly demand curves with market-determined prices, it is quite important that you know which statistical estimation procedure is the appropriate one for your firm. As you will see, the statistical problems that sometimes arise in demand estimation can be routinely handled by "asking" the computer to use the correct estimation procedure.

For some firms, a manager does not set the price of the firm's product; rather, as you saw in Chapter 2, price is determined by the point where the industry's supply curve crosses its demand curve. Recall from Chapter 1 that firms in this situation are called *price-taking* firms. For example, firms producing agricultural commodities must generally accept market-determined prices for their products. When price is determined by the simultaneous interaction of demand and supply, price is being "set" within a system of demand and supply equations. When a variable is determined by a system of equations, it is said to be an **endogenous variable** in that system. For price-taking firms, managers must accept the price of the product as it is determined by market forces in a system of demand and supply equations.

endogenous variable
A variable whose value is determined by a system of equations.

When a firm produces a differentiated product or competes with a relatively small number of rivals, the firm can choose the price of its product and the associated quantity along the firm's downward-sloping demand curve. Recall from our discussion in Chapter 1 that firms such as these are called *price-setting* firms and possess market power. For price-setting firms, price is not determined by a system of demand and supply equations. Price, in this situation, is an **exogenous variable** because it is *not* determined within a system of demand and supply equations. A force outside the system, the manager of the firm, determines price. We can summarize our discussion with a relation:

exogenous variable
A variable in a system of equations that is determined outside the system.

Relation Managers of price-taking firms do not set the price of the product they sell; rather, prices are endogenous or market-determined by the intersection of demand and supply. Managers of price-setting firms set the price of the product they sell by producing the quantity associated with the chosen price on the downward-sloping demand curve facing the firm. Because price is manager-determined rather than market-determined, price is exogenous for price-setting firms.

The distinction between price-taking firms and price-setting firms plays an important role in determining how the parameters of an empirical demand function must be estimated in order to obtain estimates that are not biased. In order for the least-squares method of estimating the parameters of a regression equation to yield unbiased estimates of the regression parameters, the explanatory variables cannot be correlated with the random error term of the equation. We did not mention this fact in our discussion of regression analysis in Chapter 4 because virtually all the applications covered in the book involve explanatory variables that are not likely to be correlated with the random error term in the equation. There is one important exception, however, and it involves the estimation of demand when the price of the product or service—an explanatory variable in all demand functions—is endogenously determined by demand and supply. In the next section, we discuss estimation of demand when price is endogenous and show how to use a method of estimation called *two-stage least-squares* that is appropriate for estimating the industry demand for price-taking firms.

II. ESTIMATING AND FORECASTING INDUSTRY DEMAND FOR PRICE-TAKING FIRMS

A fundamental difficulty arises in estimating and forecasting industry demand for price-taking firms because the observed quantity and price data used in a regression analysis of demand are determined simultaneously by the intersection of demand and supply. Consequently, the observed variation in equilibrium quantity and price is caused by all the factors that can shift either demand or supply, and estimation of industry demand for price-taking firms involves a bit more of a challenge than estimation of demand curves for price-setting firms. The problem of estimating demand when price is market-determined is frequently referred to as the **simultaneity problem.**

simultaneity problem
The problem in estimating industry demand that arises because variation in observed values of market quantity and price is simultaneously determined by changes in both demand and supply.

To understand the nature of the simultaneity problem, consider the following simple model of industry demand and supply curves for gasoline (say, unleaded 89 octane grade):

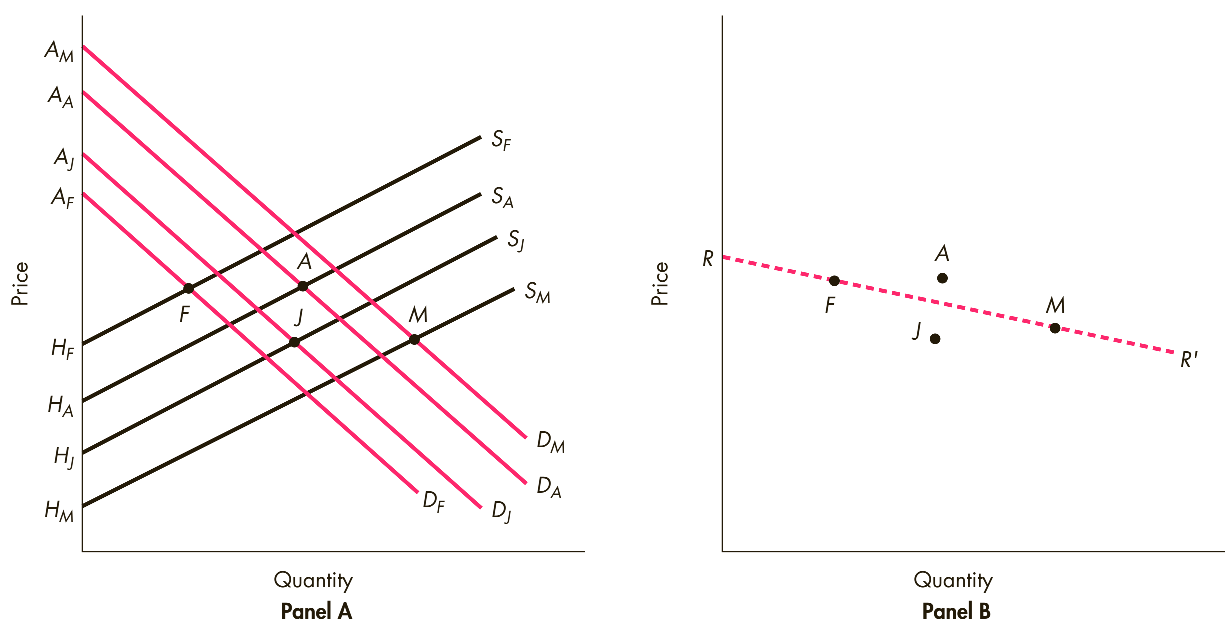
$$\text{Demand:} \quad Q = a + bP + cM + \varepsilon_d$$
$$\text{Supply:} \quad Q = h + kP + lP_I + \varepsilon_s$$

where Q is the number of gallons of gasoline sold in a month (i.e., equilibrium Q), P is the average price of gasoline (before taxes are added), M is consumer income, P_I is the average price of crude oil (a key ingredient input for gasoline production), and ε_d and ε_s are the random error terms representing random influences on demand and supply, respectively. These two equations make up a system of two simultaneous equations with two *endogenous* variables: Q and P. The values of the other economic variables in the system, M and P_I, are determined outside this system of equations and are exogenous variables.

Panel A of Figure 1 shows how four monthly observations, from January through April, on P and Q are generated by the demand and supply curves for gasoline. The demand and supply equations in Panel A can be represented as

$$\text{Demand:} \quad Q = A + bP, \quad \text{where } A = a + cM + \varepsilon_d$$
$$\text{Supply:} \quad Q = H + kP, \quad \text{where } H = h + lP_I + \varepsilon_s$$

FIGURE 1
The Nature of Simultaneity



The location of the demand curve in any one of the four months is determined by the value of the demand intercept, A, for that month. The demand intercept is itself determined by the value of the exogenous variable M and the random error term ε_d, which accounts for random variation in monthly gasoline demand. Similarly, the location of monthly supply is determined by the values of P_I and ε_s. The four monthly values of the demand and supply intercepts are shown as A_J, A_F, A_M, A_A, and H_J, H_F, H_M, H_A, respectively. The observed values of price and quantity at points J, F, M, and A in Panel A of Figure 1 are determined solely by the values of the exogenous variables of the system and the random errors in both demand and supply. Consequently, the equilibrium values of price and quantity, P_E and Q_E, can be expressed as functions of M, P_I, ε_d, and ε_s:

$$P_E = f(M, P_I, \varepsilon_d, \varepsilon_s) \quad \text{and} \quad Q_E = g(M, P_I, \varepsilon_d, \varepsilon_s)$$

reduced-form equations
Equations expressing each endogenous variable as functions of all exogenous variables and random errors in the system.

These equations, which express the endogenous variables as functions of the exogenous variables and the random error terms, are called the **reduced-form equations** of the system. The reduced-form equations show two things clearly: (1) The observed values of P and Q are each determined by *all* the exogenous variables and random errors in both the demand *and* the supply equations, and (2) the observed values of price are correlated with the random errors in both demand and supply. The first point shows why, in estimating industry demand, information about variation in supply-shifting variables is required to properly explain the observed variation in quantity demanded (which is Q_E). The second point explains why price is correlated with the random errors. As we mentioned, when explanatory variables are correlated with the random error term of the equation to be estimated, the ordinary method of least-squares estimation will produce biased estimates of the parameters of a demand equation. Because price must always be one of the explanatory variables in demand estimation, the **ordinary least-squares** method (OLS) presented in Chapter 4 is not the best way to estimate an industry demand equation when price is market-determined.

ordinary least-squares (OLS)
Another name for standard regression analysis

Panel B of Figure 1 illustrates the challenge of estimating the true demand equation that is generating the observed price-quantity combinations J, F, M, and A. Fitting a regression line through the scatter of data points at J, F, M, and A produces a regression line RR' that does not accurately reflect the true demand function. The slope of RR' is too flat, and the intercept of RR' is smaller than it should be for any of the four monthly values of M.

identification of demand
The process of making sure the sample data will trace out the true demand curve.

two-stage least-squares (2SLS)
A method of estimating parameters of demand when price is endogenous or market-determined.

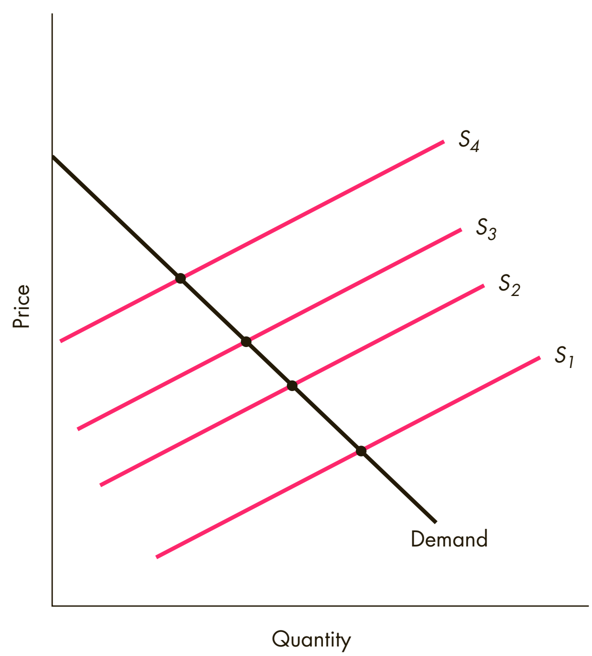
To properly estimate industry demand when price is endogenously determined by the intersection of demand and supply, two steps must be followed. The first step, called **identification of demand,** involves determining whether it is possible to trace out the true demand curve from the sample data generated by the underlying system of equations. If the demand curve can be identified from sample data—as it can be in most cases—then the second step involves using the method of **two-stage least-squares (2SLS)** to estimate the parameters of the industry demand equation. A complete discussion of the identification of demand and the use of two-stage least-squares is quite complex and unnecessary for our purpose, which is to show you how to use and interpret the parameters estimated using regression analysis. We turn now to a brief intuitive discussion of identification and then to the use of two-stage least-squares regression.

Identification of Industry Demand

The observed quantities sold and the observed prices are not simply points on a specific demand curve but, rather, points of market equilibrium that occur at the intersection of the demand and supply curves. As you saw in Panel B of Figure 1, the observed price-quantity combinations (*J*, *F*, *M*, *A*) may not trace out a picture of the underlying industry demand curve. Before a researcher runs a regression analysis to estimate an industry demand equation, the researcher must be sure that the data generated by the underlying system of demand and supply equations will trace out the true demand equation.

There are several ways to identify an industry demand equation, but we will show you only the most widely used method here. Figure 2 illustrates this method of identifying demand. When the supply equation contains an exogenous supply-shifting variable that does not also cause the demand curve to shift, then changes in this exogenous variable shift the supply curve along a stationary demand curve. The resulting points of intersection along the demand curve generate observable points of equilibrium that trace out the true underlying demand curve; demand is identified.

FIGURE 2
Identification of Industry Demand



Typically, the identification problem is solved when, in addition to the price of the product, quantity supplied is a function of at least one of the supply-shifting variables discussed in Chapter 2 (technology, input prices, prices of goods related in production, price expectations, or the number of sellers). Since a supply-shifting variable will not generally also be a demand-shifting variable, the industry demand function is identified in most commonly occurring situations. We can summarize this method of identifying demand in a relation:

Relation An industry demand equation is identified when it is possible to estimate the true demand function from a sample of observations of equilibrium output and price. Industry demand is identified when supply includes at least one exogenous variable that is not also in the demand equation.

Estimation of Industry Demand Using Two-Stage Least-Squares (2SLS)

In order for the ordinary least-squares is method of estimating the parameters of a regression equation to yield unbiased estimates of the regression parameters, the right-hand-side explanatory variables cannot be correlated with the random error term of the equation. Since all demand functions will have price as one of the explanatory variables, OLS estimation is not a suitable method of estimating industry demand when price is an endogenous variable. As can be seen by examining the reduced-form equations, random variations in either the demand or the supply equations will cause variation in price, and, consequently, price will be correlated with the random error term in the demand (and supply) equation(s). Thus when price is market-determined—as it will be for price-taking firms—price will be correlated with the random error term in the demand equation, and the least-squares estimates of the parameter of the demand equation will be biased. Recall that a parameter estimate is biased if the average (or expected) value of the estimate does not equal the true value of the parameter. The bias that occurs when the OLS estimation method is employed to estimate parameters of an equation for which one, or more, of the right-hand-side variables (price in this case) is an endogenous variable is called a **simultaneous equations bias.**

simultaneous equations bias
Bias in estimation that occurs when the ordinary least-squares estimation method is used to estimate the parameters of an equation for which one, or more, of the explanatory variables is an endogenous variable.

Econometricians employ the two-stage least-squares (2SLS) estimation technique to address the problem of simultaneous equations bias. As its name suggests, the estimation proceeds in two steps. In the first stage, a proxy variable for the endogenous variable (price in this case) is created in such a way that the proxy variable is correlated with market price but uncorrelated with the random error term in the demand equation. In the second stage, price is replaced with the proxy variable created in the first stage, and the usual least-squares procedure is then employed to estimate the parameters of the demand equation.[1] Two-stage least-squares can be applied only to demand equations that are identified. If industry demand is not identified, there is no estimation technique that will correctly

[1] A more complete presentation of 2SLS estimation is given in the appendix to this module.

estimate the parameters of the demand equation. We summarize this discussion in a principle:

Principle When market price is an endogenous variable, price will be correlated with the random error term in the demand equation, causing a simultaneous equations bias if the ordinary least-squares (OLS) method of estimation is applied. To avoid simultaneous equations bias, the two-stage least-squares method of estimation (2SLS) can be employed if the industry demand equation is identified.

Before we illustrate in the next section how to estimate an industry demand function using 2SLS, we will summarize the previous theoretical discussion with a step-by-step guide to estimating an industry demand function:[2]

Step 1: Specify the industry demand and supply equations

Since price is determined by the intersection of industry demand and supply curves, *both* a demand and a supply equation must be specified in order to estimate the demand function. For example, a rather typical specification of demand and supply functions can be written as

$$\text{Demand:} \quad Q = a + bP + cM + dP_R$$
$$\text{Supply:} \quad Q = h + kP + iP_I$$

where Q is market quantity, P is price, M is income, P_R is price of a good related in consumption, and P_I is price of a production input. Other exogenous demand-shifting or supply-shifting variables, could, of course, be utilized when needed, and nonlinear functional forms also can be estimated.

Step 2: Check for identification of industry demand

As explained earlier, estimation of demand cannot proceed unless the industry demand is identified. You cannot successfully estimate parameters of an industry demand function, even using the two-stage least-squares procedure, if demand is not identified.[3] As you can verify, the demand equation specified in Step 1 is identified because the specification of *supply* includes at least one exogenous variable—P_I in this instance—that is not also in the demand equation.

Step 3: Collect data for the variables in demand and supply

Data must be collected for the endogenous and exogenous variables in *both* the demand and the supply equations, even if only one of the equations is to be estimated. The 2SLS procedure requires data for the exogenous variables in both functions in order to correct for simultaneous equations bias in estimating either *one* of the equations.

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[2]Industry supply can be estimated by following the same steps as for estimating industry demand. As we will show you later in this chapter, forecasting future industry prices and quantities requires estimation of *both* the demand and the supply equations in the system.

[3]Should you accidentally attempt to use 2SLS to estimate a function that is not identified, the 2SLS procedure will not be able to calculate parameter estimates, and the computer software will generate an error message.

Step 4: Estimate industry demand using 2SLS

Many regression programs are available, even for personal computers, that have a two-stage least-squares routine, and these 2SLS packages perform the two stages of estimation automatically. It is normally necessary for the user to specify which variables are endogenous and which are exogenous in the system equations. Once the estimates for the parameters of a demand (or supply) equation have been obtained from the second stage of the regression, their significance can be evaluated using either a *t*-test or the *p*-values in precisely the same manner as for any other regression equation.[4] Demand elasticities can then be computed as explained at the beginning of this module.

To illustrate how to implement these steps to estimate the industry demand when price is market-determined and to illustrate how to calculate and interpret estimates of the associated demand elasticities, we will now estimate the world-
T 2 wide demand for copper using data from the world copper market.

The World Demand for Copper: Estimating Industry Demand Using 2SLS

To illustrate how an industry demand function is estimated using two-stage least-squares, we estimate the world demand for copper (i.e., the market demand for all countries buying copper). In its simplest form, the world demand for copper is a function of the price of copper, income, and the price of any related commodities. Using aluminum as the related commodity, because it is the primary substitute for copper in manufacturing, the demand function can be written in linear form as

$$Q_{\text{copper}} = a + bP_{\text{copper}} + cM + dP_{\text{aluminum}}$$

It is tempting to simply regress copper consumption on the price of copper, income, and the price of aluminum. Because the price of copper and the quantity of copper consumed are determined simultaneously by the intersection of industry demand and supply, it is necessary first to determine if copper demand is identified; then, if it is, the empirical demand function for copper can be estimated using 2SLS. The copper demand function is identified if it is reasonable to believe that the copper supply equation includes at least one exogenous variable not found in the copper demand function. We turn now to the specification of copper supply.

Begin by letting the quantity supplied of copper depend on the price of copper and the level of available technology. Next consider inventories, which play a particularly important role in the market for copper. When inventories rise, current production usually falls. To measure *changes* in copper inventory, define a variable denoted by X to be the ratio of consumption to production in the preceding period. As consumption declines relative to production, X will fall, and current

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[4]Due to the manner in which 2SLS estimates are calculated, the R^2 and F-statistics are not particularly meaningful and are not reported in many instances.

production is expected to decline. Thus the supply function can be reasonably specified in linear form as

$$Q_{\text{copper}} = e + fP_{\text{copper}} + gT + hX$$

Since the supply function includes two exogenous variables that are excluded from the demand equation (T and X), the demand function is identified and may be estimated using 2SLS.

The data needed to estimate demand are (1) the world consumption (sales) of copper in 1,000 metric tons; (2) the price of copper and aluminum in cents per pound, deflated by a price index to obtain the real (i.e., constant-dollar) prices; (3) an index of real per capita income; and (4) the world production of copper (to calculate the inventory variable, X). Time serves as a proxy for available technology (this assumes that the level of technology increased steadily over time). The resulting data set is presented in Table A of the appendix at the end of this module.

Using these data, the demand function is estimated using 2SLS. The results of these estimations are presented here:

Two-Stage Least-Squares Estimation

DEPENDENT VARIABLE:

OBSERVATIONS:

VARIABLE	PARAMETER ESTIMATE	STANDARD ERROR	T-RATIO	P-VALUE
INTERCEPT	−6837.800	1264.500	−5.408	0.0001
PC	−66.495	31.534	−2.109	0.0472
M	13997.7	1306.300	10.715	0.0001
PA	107.662	44.510	2.419	0.0247

Before estimating the industry demand equation, we determined whether the estimated coefficients $\hat{b}$, $\hat{c}$, and $\hat{d}$ should be positive or negative based on theoretical considerations. We expected that (1) due to a downward-sloping demand curve for copper, $b < 0$; (2) because copper is a normal good, $c > 0$; and (3) because copper and aluminum are substitutes, $d > 0$. The estimated coefficients do conform to this sign pattern. Examining the p-values for the parameter estimates shows that all parameter estimates are statistically significant at the 5 percent level, or better.

Now, we calculate estimates of the demand elasticities. While the elasticity can be evaluated at any point on the demand curve, we choose to estimate the elasticities for the values of P_c, M, and P_A in the last year of the sample. From Table A in the appendix, we obtain, for the 25th observation in the sample, the values $P_c = 36.33$, $M = 1.07$, and $P_A = 22.75$. At the point associated with the 25th observation on the estimated demand curve, the estimated quantity of copper demanded is calculated to be 8,172.49 (= −6,837.8 − 66.495 × 36.33 + 13,997 × 1.07 + 107.66 × 22.75). The price elasticity of demand is estimated to be

$$\hat{E} = \hat{b}\,\frac{P_c}{Q_c} = -66.495 \times \frac{36.33}{8{,}172.49} = -0.296$$

Similarly, the estimated income elasticity of demand is

$$\hat{E}_M = \hat{c}\,\frac{M_c}{Q_c} = 13{,}997 \times \frac{1.07}{8{,}172.49} = 1.833$$

and the estimated cross-price elasticity of demand is

$$\hat{E}_{CA} = \hat{d}\,\frac{P_A}{Q_c} = 107.66 \times \frac{22.75}{8{,}172.49} = 0.300$$

Thus the demand for copper—when evaluated at the point associated with the 25th observation in the sample—is inelastic ($|E| < 1$), copper is a normal good ($E_M > 0$), and copper is a substitute for aluminum ($E_{CA} > 0$). Note that copper is a rather poor substitute for aluminum since a 10 percent increase in the price of aluminum increases the quantity demanded of copper by only 3 percent.

We have stressed that estimation of demand for a price-taking industry must be carried out using the technique of two-stage least-squares (2SLS) rather than with the somewhat easier method of ordinary least-squares (OLS), which can be used to estimate the demand facing a price-setting firm. Unfortunately, business statisticians and forecasters sometimes ignore this important principle, perhaps because they don't know better or they think it really doesn't matter all that much. As stated previously, a simultaneous equations bias results when OLS is used when 2SLS should be used. In Technical Problem 4 at the end of this module, you will see that estimating the copper demand equation using OLS instead of 2SLS does indeed cause problems. Having established this most important distinction, we now turn to forecasting price and quantities by using demand and supply equations estimated with two-stage least-squares.

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III. ECONOMETRIC FORECASTING

econometric model
A statistical model that employs an explicit structural model to explain the underlying economic relations.

An alternative to time-series methods of statistical forecasting and decision making is econometric modeling. The primary characteristic of **econometric models,** which differentiates this approach from time-series approaches, is the use of an explicit structural model that attempts to *explain* the underlying economic relations. More specifically, if we wish to employ an econometric model to forecast future sales, we must develop a model that incorporates the variables that actually determine the level of sales (e.g., income, the price of substitutes, and so on).

The use of econometric models has several advantages. First, econometric models require analysts to define explicit causal relations. This specification of an explicit model helps eliminate problems such as spurious (false) correlation between normally unrelated variables and may make the model more logically consistent and reliable.

Second, this approach allows analysts to consider the sensitivity of the variable to be forecasted to changes in the exogenous explanatory variables. Using estimated elasticities, forecasters can determine which of the variables are most important in determining changes in the variable to be forecasted. Therefore, the analyst can examine the behavior of these variables more closely.

Econometric forecasting can be utilized to forecast either future industry price and quantity for price-taking firms or future demand for a price-setting firm. We begin our discussion of econometric models by showing you, in a step-by-step fashion, how to forecast future industry price and sales for price-taking firms. We then apply this procedure using data from the world copper market to forecast future copper price and sales.

Forecasting Future Industry Price and Sales

Using econometric models to forecast future price and sales in an industry is slightly more complicated and requires more information than forecasting future demand for a price-setting firm. To forecast industry price and sales, an analyst must estimate not only demand but also supply. You will see, in the following step-by-step discussion, that the process of making forecasts using simultaneous demand and supply equations is not particularly difficult.

Step 1: Estimate the industry demand and supply equations

The process begins with the specification of industry demand and supply equations. As we explained earlier, the parameters of a demand or a supply equation in a simultaneous system cannot be estimated unless the equation is identified. Because both equations in a system must be estimated in order to forecast future price and sales, demand and supply must be specified in such a way that they are *both* identified. Recall that demand is identified when supply includes at least one exogenous explanatory variable that is not also in the demand equation. In the same fashion, supply is identified when demand includes at least one exogenous explanatory variable that is not also in the supply equation. The two-stage least-squares (2SLS) estimation procedure can now be employed to estimate the parameters of the identified demand and supply equations. For example, the forecasting department of a firm can use 2SLS to estimate the industry demand function:

$$Q = a + bP + cM + dP_R$$

and the industry supply function

$$Q = e + fP + gP_I$$

where Q is industry sales, P is the market-determined price, M is income, P_R is the price of a good related in consumption, and P_I is the price of an input used in production. Notice that both demand and supply are identified: Each equation contains an exogenous variable not contained in the other equation.

Step 2: Locate industry demand and supply in the forecast period

To forecast price and sales in a future period, a forecaster must know where demand and supply will be located in the future period of the forecast. The process of locating demand and supply in a future period is straightforward. The forecaster obtains future values of *all* of the exogenous explanatory variables in the system of demand and supply equations and then substitutes these values into the *estimated* demand and supply equations. As already mentioned, forecasted values of exogenous variables can be acquired either by using time-series techniques to generate predicted values of the exogenous explanatory variables or by purchasing forecasts of the exogenous explanatory variables from forecasting firms.

To locate the industry demand and supply equations in the future period 2009, for example, a forecaster must obtain forecasts of all exogenous variables for that year: M_{2009}, $P_{R,2009}$, and $P_{I,2009}$. Then the forecasted future demand equation for 2009 is

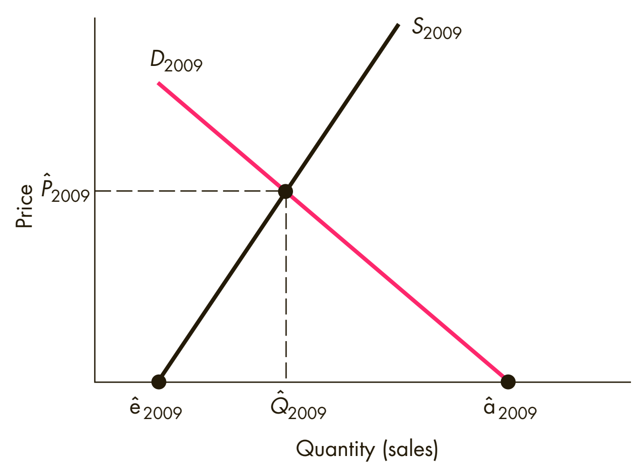
$$\begin{aligned} Q_{2009} &= \hat{a} + \hat{b}P_{2009} + \hat{c}M_{2009} + \hat{d}P_{R,2009} \\ &= (\hat{a} + \hat{c}M_{2009} + \hat{d}P_{R,2009}) + \hat{b}P_{2009} \\ &= \hat{a}_{2009} + \hat{b}P_{2009} \end{aligned}$$

and the forecasted future supply equation for 2009 is

$$\begin{aligned} Q_{2009} &= \hat{e} + \hat{f}P_{2009} + \hat{g}P_{I,2009} \\ &= (\hat{e} + \hat{g}P_{I,2009}) + \hat{f}P_{2009} \\ &= \hat{e}_{2009} + \hat{f}P_{2009} \end{aligned}$$

These future demand and supply equations are illustrated in Figure 3.

FIGURE 3
Locating Future Industry Demand and Supply



Step 3: Calculate the intersection of future demand and supply

The intersection of the forecasted industry demand and supply equations provides the forecasted industry price and sales in the future period. In Figure 3, the forecasted price $\hat{P}_{2009}$, and the forecasted level of sales, $\hat{Q}_{2009}$, are found by solving for the intersection of demand and supply equations in precisely the same way that you found equilibrium price and quantity in Chapter 2.

To illustrate the implementation of these steps, we turn now to the world copper market to forecast the industry price and quantity of copper. We continue here to use the data from the copper market that were presented and discussed previously in this module.

The World Market for Copper: A Simultaneous Equations Forecast

Recall that the copper data consist of 25 annual observations on world consumption of copper, copper price, and the exogenous variables required to estimate industry demand and supply equations. Using these data, we now follow the steps set forth above to forecast industry price and sales of copper in year 26.

Step 1: Estimate the copper industry demand and supply equations

Recall from our earlier discussion that world demand for copper was specified as

$$Q_{\text{copper}} = a + bP_{\text{copper}} + cM + dP_{\text{aluminum}}$$

and world supply as

$$Q_{\text{copper}} = e + fP_{\text{copper}} + gT + hX$$

where time (T) is a proxy for the level of available technology and X is the ratio of consumption of copper to production of copper in the previous period to reflect inventory changes. Both of these equations are identified and can be estimated using two-stage least-squares (2SLS). Recall that the estimated demand for copper is

$$\hat{Q}_{\text{copper}} = -6{,}837.8 - 66.495P_{\text{copper}} + 13{,}997.9M + 107.662P_{\text{aluminum}}$$

The estimated supply function, using the 2SLS procedure, is

$$\hat{Q}_{\text{copper}} = 149.104 + 18.154P_{\text{copper}} + 213.88T + 1{,}819.7X$$

Step 2: Locate copper demand and supply in year 26

To locate demand and supply in year 26, we must obtain forecast values for the exogenous variables in year 26. Because time is a proxy for technology, the period-26 value of T is simply $T_{26} = 26$. As previously mentioned, the value of X in any period is the ratio of consumption to production in the preceding period. Since both of these values are known (consumption was 7,157.2 and production was 8,058.0), $\hat{X}_{26} = 0.88821$ ($= 7{,}157.2/8{,}058.0$). For the other two exogenous explanatory variables, M and P_R, values must be obtained using time-series forecasting.

To obtain values for $\hat{M}_{26}$ and $\hat{P}_{R,26}$, a linear trend method of forecasting was used to obtain

$$\hat{M}_{26} = 1.13 \quad \text{and} \quad \hat{P}_{R,26} = 23.79$$

Using $\hat{M}_{26}$ and $\hat{P}_{R,\,26}$, the demand function in time period 26 is

$$\begin{aligned}\hat{Q}_{\text{copper, 26}} &= -6{,}837.8 - 66.495\,P_{\text{copper, 26}} + 13{,}997(1.13) + 107.662(23.79)\\ &= 11{,}540.09 - 66.495\,P_{\text{copper, 26}}\end{aligned}$$

Likewise, using $\hat{T}_{26} = 26$ and $\hat{X}_{26} = 0.88821$, the supply function in time period 26 is

$$\begin{aligned}\hat{Q}_{\text{copper, 26}} &= 149.104 + 18.154\,P_{\text{copper, 26}} + 213.88(26) + 1{,}819.7(0.88821)\\ &= 7{,}326.26 + 18.154\,P_{\text{copper, 26}}\end{aligned}$$

Step 3: Calculate the intersection of the demand and supply functions.

We set quantity demanded equal to quantity supplied and solve for equilibrium price:

$$\begin{aligned}11{,}540.09 - 66.495\,P_{\text{copper, 26}} &= 7{,}326.26 + 18.154\,P_{\text{copper, 26}}\\ P_{\text{copper, 26}} &= 49.78\end{aligned}$$

The sales forecast is then found by substituting $P_{\text{copper, 26}}$ into either the demand or the supply equation. Using the demand function,

$$\begin{aligned}Q_{\text{copper, 26}} &= 11{,}540.09 - 66.495(49.78) = 7{,}326.26 + 18.154(49.78)\\ &= 8{,}230.0\end{aligned}$$

T 5 6

Thus we forecast that sales of copper in year 26 will be 8,230.0 (thousand) metric tons.[5] The price of copper in year 26 is forecasted to be 49.8 cents per pound.

TECHNICAL PROBLEMS

1. For each of the following sets of industry demand and supply functions, determine if the demand function is identified and explain why or why not:
   a. Demand: $Q = a + bP$
   Supply: $Q = e + fP$
   b. Demand: $Q = a + bP + cM$
   Supply: $Q = e + fP$
   c. Demand: $Q = a + bP + cW$
   Supply: $Q = e + fP + gW$
   d. Demand: $Q = a + bP + cM$
   Supply: $Q = e + fP + gT + hP_I$
2. Evaluate the following statement: "If industry demand is not identified, then two-stage least-squares (2SLS) must be used to estimate the demand equation."

[5] As we noted, the data we used for this copper market illustration are the actual data for the period 1951–1975. Hence, our forecast for year 26 can be interpreted as the forecast for 1976. The actual value for copper consumption in 1976 was 8,174.0, so the forecast error in this example was 0.54 percent—about one-half of 1 percent.

3. With the data in Table A of the appendix, the world demand for copper can be estimated by using ordinary least-squares, rather than by using 2SLS, as done in the example in the module. The estimation results using OLS are as follows:

DEPENDENT VARIABLE: QC		R-SQUARE	F-RATIO	P-VALUE ON F
OBSERVATIONS: 25		0.9648	191.71	0.0001

VARIABLE	PARAMETER ESTIMATE	STANDARD ERROR	T-RATIO	P-VALUE
INTERCEPT	−6245.43	961.291	−6.50	0.0001
PC	−13.4205	14.4504	−0.93	0.3636
M	12073.0	719.326	16.78	0.0001
PA	70.7161	31.8441	2.22	0.0375

Compare the OLS parameter estimates to the 2SLS estimates presented in this module. Do you see any problems with using OLS to estimate the parameters of world copper demand? Explain.

4. In the example dealing with the world demand for copper, we estimated the demand elasticities. Using these estimates, evaluate the impact on the world consumption of copper of
   a. The formation of a worldwide cartel in copper that increases the price of copper by 10 percent.
   b. The onset of a recession that reduces world income by 5 percent.
   c. A technical breakthrough that is expected to reduce the price of copper by 6 percent.
   d. A 10 percent reduction in the price of aluminum.
5. Supply and demand functions were specified for commodity *X:*

$$\text{Demand: } Q = a + bP + cM + dP_R$$

$$\text{Supply: } Q = e + fP + gP_I$$

Using quarterly data for the period 2000(I) through 2007(IV), these functions were estimated via 2SLS. The resulting parameter estimates are presented in the following estimated equations. (All estimated coefficients are statistically significant.)

$$\text{Demand: } Q = 500 - 300P + 1.0M - 200P_R$$

$$\text{Supply: } Q = -400 + 200P - 100P_I$$

The predicted values for the exogenous variables (M, P_R, and P_I) for the first quarter of 2009 were obtained from a macroeconomic forecasting model. These predicted values are:

Income (M) = 10,000
The price of the commodity related in consumption (P_R) = 20
The price of inputs (P_I) = 6

a. Are the signs of the estimated coefficients as would be predicted theoretically? Explain.

b. Predict the sales of commodity X in the first quarter of 2009.

c. Perform a simulation analysis to determine the sales of commodity X in 2009(I) if income were \$9,000 and \$12,000.

6. Suppose you are the market analyst for a major U.S. bank and the bank president asks you to forecast the median price of new homes and the number of new homes that will be sold in the first quarter of 2009. You specify the following demand and supply functions for the U.S. housing market:

$$\text{Demand: } Q_H = a + bP_H + cM + dP_A + eR$$

$$\text{Supply: } Q_H = f + gP_H + hP_M$$

where the endogenous variables are measured in the following way:

$$Q_H = \text{thousands of units sold quarterly}$$

$$P_H = \text{median price of a new home in thousands of dollars}$$

The exogenous variables are median income in dollars (M), average price of apartments (P_A), mortgage interest rate as a percent (R), and the price of building materials as an index (P_M).

a. Is the demand equation identified? Explain.

b. What signs do you expect each of the estimated coefficients to have? Explain.

Using quarterly data for the period 1996(I) through 2008(IV), you estimate these equations using two-stage least-squares. All the coefficients are statistically significant and the estimated equations are

$$\text{Demand: } Q_H = 504.5 - 10.0P_H + 0.01M + 0.5P_A - 11.75R$$

$$\text{Supply: } Q_H = 326.0 + 15P_H - 1.8P_M$$

The predicted values for the exogenous variables for the first quarter of 2009 are obtained from a private econometrics firm. The predicted values are:

Median income (M) = 26,000
Average price of apartments (P_A) = 400
Mortgage interest rate (R) = 14
Price of building materials (P_M) = 320 (an index)

c. Using these predicted values of the exogenous variables, forecast the median price and sales of new homes in the first quarter of 2009.

d. Suppose you feel that the predicted mortgage interest rate for the first quarter of 2009, 14 percent, is much too high. Determine how changing the forecast interest rate to 10 percent affects the forecast price and sales for the first quarter of 2009.

7. In the examination of world demand for copper, we used a linear specification. However, we could have estimated a log-linear specification. That is, we could have specified the copper demand function as

$$Q_c = aP_c^b M^c P_A^d$$

or

$$\ln Q_c = \ln a + b \ln P_c + c \ln M + d \ln P_A$$

The results of such an estimation, using the data in the appendix, are presented here:

```
Two-Stage Least-Squares Estimation

DEPENDENT VARIABLE: LNQC
     OBSERVATIONS: 25

                   PARAMETER      STANDARD
VARIABLE           ESTIMATE       ERROR          T-RATIO       P-VALUE
INTERCEPT          9.49265        1.56146        6.08          0.0001
LNPC               −0.88307       0.56457        −1.56         0.1327
LNM                2.69818        0.50542        5.34          0.0001
LNPA               0.83530        0.33400        2.50          0.0207
```

a. Using the *p*-values, discuss the statistical significance of the parameter estimates $\hat{a}$, $\hat{b}$, $\hat{c}$, and $\hat{d}$. Are the signs of $\hat{b}$, $\hat{c}$, and $\hat{d}$ consistent with the theory of demand?

b. What are the estimated values of the price ($\hat{E}$), income ($\hat{E}_M$), and cross-price ($\hat{E}_{CA}$) elasticities of demand? Compare these elasticity estimates with the estimated elasticities for the linear specification of copper demand (estimated in this chapter).

c. Which specification of copper demand, the linear or log-linear, appears to be more appropriate?

STATISTICAL APPENDIX Simultaneous Equations Bias and Two-Stage Least-Squares Estimation

Simultaneous equations bias

Consider the following system of demand and supply equations

$$\text{Demand: } Q = a + bP + cM + \varepsilon_d$$

$$\text{Supply: } Q = d + eP + fP_I + \varepsilon_s$$

where P and Q are the endogenous variables, M and P_I are the exogenous variables, and ε_d and ε_s are the random error terms for demand and supply. We now solve for the *reduced-form equations*, which show how the values of the endogenous variables are determined by the exogenous variables and the random error terms. First we set $Q_d = Q_s$ and solve for P^*:

$$a + bP + cM + \varepsilon_d = d + eP + fP_I + \varepsilon_s$$

$$P(b - e) = d - a + fP_I - cM + \varepsilon_s - \varepsilon_d$$

$$P^* = \frac{d - a}{b - e} + \frac{f}{b - e}P_I + \frac{-c}{b - e}M + \frac{\varepsilon_s - \varepsilon_d}{b - e}$$

Next, we substitute P^* into either demand or supply and solve for Q^*:

$$Q^* = \frac{bd - ae}{b - e} + \frac{bf}{b - e}P_I + \frac{-ce}{b - e}M + \frac{b\varepsilon_s - e\varepsilon_d}{b - e}$$

The reduced-form equations for P^* and Q^* can be expressed in a simpler, more general form as follows:

$$P^* = f(P_I, M, \varepsilon_d, \varepsilon_s)$$

$$Q^* = g(P_I, M, \varepsilon_d, \varepsilon_s)$$

The reduced-form equations show

1. *The problem of simultaneity:* Each one of the endogenous variables, P^* and Q^* in this case, is clearly determined by all the exogenous variables in the system and by all the random error terms in the system. Thus the observed variations in both P and Q are reflecting variations in both demand- and supply-side determinants.

2. *The simultaneous equations bias:* If the ordinary least-squares estimation procedure is to produce unbiased estimates of a, b, and c in the demand equation, the explanatory variables (P and M) must *not* be correlated with the error term in the demand equation, ε_d (for a proof of this statement, see Gujarati[a]). The reduced-form equations show us clearly that all endogenous variables are functions of all random error terms in the system. P is an endogenous variable, and we have seen that P is a function of ε_d. Thus P will be correlated with the error term in the demand equation, and the estimates of a, b, and c will be biased if the ordinary least-squares procedure is employed. The bias that results because P is an endogenous explanatory variable is called simultaneous equations bias.

Two-stage least-squares estimation

If an industry demand equation is identified, it can be estimated using any number of available techniques. Perhaps the most widely used of these techniques—and the one that is most likely to be preprogrammed into the available regression packages—is two-stage least-squares (2SLS).

As shown earlier, the estimates of the parameters of the demand equation will be biased if ordinary least-squares is employed because price is an endogenous variable that is on the right-hand side of the demand equation. Because price is endogenous, it will be correlated with the error term in the demand equation, causing simultaneous equations bias.

Conceptually, the endogenous right-hand-side variable (in this case, price) must be made to behave as if it is exogenous; traditional regression techniques are used to obtain estimates of the parameters. In the linear example we have been using, we have a system of two simultaneous equations:

$$\text{Demand: } Q = a + bP + cM + \varepsilon_d$$

$$\text{Supply: } Q = d + eP + fP_I + \varepsilon_s$$

In these equations, P is an endogenous variable. To obtain unbiased estimates of a, b, and c, the estimation of the demand function proceeds in two steps or stages, which is why the technique is called two-stage least-squares:

Stage 1 The endogenous right-hand-side variable is regressed on all the exogenous variables in the system:

$$P = \alpha + \beta M + \gamma P_I$$

From this estimation, we obtain estimates of the parameters, that is, $\hat{\alpha}$, $\hat{\beta}$, and $\hat{\gamma}$. Using these estimates and the *actual values* of the exogenous variables, we generate a *new* price series—predicted price—as follows:

$$\hat{P} = \hat{\alpha} + \hat{\beta} M + \hat{\gamma} P_I$$

Note how the predicted price, $\hat{P}$, is obtained. $\hat{P}$ is simply a linear combination of the exogenous variables, so it follows that $\hat{P}$ is now also exogenous. However, given the way that the predicted price series is obtained, the values of $\hat{P}$ will correspond closely to the original values of P. In essence, this first stage forces price to behave as if it were exogenous.

Stage 2 We then use the predicted price variable (P) in the demand function we wish to estimate. That is, in the second stage, we estimate the regression equation:

$$Q = a + b\hat{P} + cM$$

Note that this estimation uses the exogenous variable constructed in the first stage. We use predicted price, P, rather than the actual price variable, $\hat{P}$, in the final regression.

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[a]Damodar N. Gujarati, *Basic Econometrics* (New York: McGraw-Hill, 2002).

ANSWERS TO TECHNICAL PROBLEMS

1. *a.* Demand is not identified because supply does not contain any exogenous variables.
   b. The supply equation contains no exogenous variables excluded from the demand equation, so the demand function is not identified.
   c. The demand function is not identified because the exogenous variable in supply is also an explanatory variable in the demand equation.
   d. Demand is identified because supply contains at least one (two in this case) exogenous variable that is not an explanatory variable in demand.
2. If a demand equation is not identified, there is no estimation technique (2SLS or otherwise) capable of estimating the parameters of the demand equation. 2SLS can be used only when the demand equation is identified.
3. Using OLS to estimate industry demand for price-taking firms when price is an endogenous variable results in a simultaneous equations bias for each of the estimated parameters of the demand equation. The most obvious problem with the OLS estimation results is the parameter estimate for copper price. The OLS estimate (−13.4205) is much smaller in absolute value than the 2SLS estimate. Further, price does not appear to have a statistically significant effect on the quantity demanded of copper (p-value = 0.3636).
4. *a.* The quantity of copper demanded will decrease 2.96 percent if the price of copper increases 10 percent.
   [$\hat{E} = -0.296 = \%\Delta QC/10\% \Rightarrow \%\Delta QC = (-0.296)(10\%) = -2.96\%$]
   b. The quantity of copper demanded will decrease 9.165 percent if income decreases 5 percent.
   [$\hat{E}_M = 1.833 = \%\Delta QC/-5\% \Rightarrow \%\Delta QC = (1.833)(-5\%) = -9.165\%$]
   c. The quantity of copper demanded will increase 1.776 percent if the price of copper decreases 6 percent.
   [$\hat{E} = -0.296 = \%\Delta QC/-6\% \Rightarrow \%\Delta QC = (-0.296)(-6\%) = +1.776\%$]
   d. The quantity of copper demanded will decrease 3.0 percent if the price of aluminum decreases 10 percent.
   [$\hat{E}_{CA} = 0.30 = \%\Delta QC/-10\% \Rightarrow \%\Delta QC = (0.30)(-10\%) = -3.0\%$]
5. *a.* Economic theory predicts that price and quantity demanded will be inversely related, income and quantity demanded will be positively related for a normal good, and the price of a complement and quantity demanded will be inversely related. The signs of the coefficients in the demand equation thus are consistent with economic theory and imply that X is a normal good and that X and R are complements. The signs of the coefficients in the supply equation are also consistent with economic theory because price and quantity supplied are positively related, while input prices and quantity supplied are inversely related.
   b. Demand $Q_{2009(\mathrm{I})} = 500 - 300P + 1(10{,}000) - 200(20) = 6{,}500 - 300P$
   Supply $Q_{2009(\mathrm{I})} = -400 + 200P - 100(6) = -1{,}000 + 200P$
   In equilibrium, $6{,}500 - 300P = -1{,}000 + 200P \Rightarrow P = \$15 \Rightarrow Q = 2{,}000$.
   c. For M = \$9,000:
   $Q_{2009(\mathrm{I})} = 500 - 300P + 1(9{,}000) - 200(20) = 5{,}500 - 300P$
   In equilibrium, $5{,}500 - 300P = -1{,}000 + 200P \Rightarrow P = \$13$ and $Q = 1{,}600$.

For $M = \$12{,}000$:

$Q_{2009(I)} = 500 - 300P + 1(12{,}000) - 200(20) = 8{,}500 - 300P$

In equilibrium, $8{,}500 - 300P = -1{,}000 + 200P \Rightarrow P = \$19$ and $Q = 2{,}800$.

Thus increasing projected income in 2009(I) from \$9,000 to \$12,000 causes forecasted price to rise by \$6 (from \$13 to \$19) and forecasted sales to rise by 1,200 units (from 1,600 to 2,800).

6. *a.* The demand equation is identified since at least one exogenous explanatory variable is in the supply equation that is not also included in the demand equation.

   b. $b < 0$: Price and quantity demanded are inversely related

   $c > 0$: Housing is a normal good.

   $d > 0$: Apartments are substitutes for new homes.

   $e < 0$: Mortgage interest rates determine how costly it is to borrow money to buy a house. Mortgage rates and sales are inversely related.

   $g > 0$: Price and quantity supplied are directly related.

   $h < 0$: Higher input prices cause supply to decrease.

   c. $Q_4 = 504.5 - 10.0P_H + 0.01(26{,}000) + 0.5(400) - 11.75(14) = 800 - 10P_H$

   $Q_4 = 326.0 + 15P_H - 1.8(320) = - 0.5(400) + 15P_H$

   $Q_4 = Q_4 \Rightarrow P_H = \$42$ and $Q_H = 380$

   Thus, the forecast for median price and sales of new homes in the first quarter of 2009 is \$42,000 and 380000 units sold quarterly, respectively.

   d. $Q_4 = 504.5 - 10.0P_H + 0.01(26{,}000) + 0.5(400) - 11.75(10) = 847 - 10P_H$

   $Q_4 = Q_4 \Rightarrow 847 - 10P_H = -250 + 15P_H \Rightarrow P_H = \$43.88$ and $Q_H = 480.2$

   Thus, the forecast when mortgage rate are 10 percent is \$43,800 and 408,200 units sold in the first quarter of 2009.

7 a. With the exception of the price of copper, all parameter estimates are highly significant. The parameter estimate for copper price (−0.913921) is significant at exactly the 15.28 percent level—possibly not a tolerable level of risk of making a Type I error. The parameter estimates are consistent with economic theory.

   b. $\hat{E} = \hat{b} = -0.913921$, $\hat{E}_M = \hat{c} = 2.734251$, and $\hat{E}_{CA} = \hat{d} = 0.793509$.

   b. The table below shows he elasticities for each specification:

	$\hat{E}$	$\hat{E}_M$	$\hat{E}_{CA}$
Linear	−0.45	2.24	0.48
Log-linear	−0.91	2.73	0.79

The estimated income elasticities are similar in the two specifications, but the price and cross-price elasticitices differ by about a factor of 2.

Data Appendix

TABLE A
The World Copper Market[a]

Year	World consumption (Q_C)	Real price, copper (P_C)	Index of real income (M)	Real price, aluminum (P_A)	World production (Q_P)	X (Q_C/Q_P)	T
	3,056.5				3,129.1		
1	3,173.0	26.56	0.70	19.76	3,052.8	0.97679	1
2	3,281.1	27.31	0.71	20.78	3,120.3	1.03937	2
3	3,135.7	32.95	0.72	22.55	3,222.3	1.05153	3
4	3,359.1	33.90	0.70	23.06	3,282.0	0.97312	4
5	3,755.1	42.70	0.74	24.93	3,606.0	1.02349	5
6	3,875.9	46.11	0.74	26.50	3,967.7	1.04135	6
7	3,905.7	31.70	0.74	27.24	3,982.6	0.97686	7
8	3,957.6	27.23	0.72	26.21	3,846.5	0.98069	8
9	4,279.1	32.89	0.75	26.09	4,138.7	1.02888	9
10	4,627.9	33.78	0.77	27.40	4,726.1	1.03392	10
11	4,910.2	31.66	0.76	26.94	4,926.0	0.97922	11
12	4,908.4	32.28	0.79	25.18	5,079.6	0.99679	12
13	5,327.9	32.38	0.83	23.94	5,177.0	0.96630	13
14	5,878.4	33.75	0.85	25.07	5,445.5	1.02915	14
15	6,075.2	36.25	0.89	25.37	5,781.9	1.07950	15
16	6,312.7	36.24	0.93	24.55	6,141.5	1.05073	16
17	6,056.8	38.23	0.95	24.98	5,891.9	1.02788	17
18	6,375.9	40.83	0.99	24.96	6,430.5	1.02799	18
19	6,974.3	44.62	1.00	25.52	6,961.0	0.99151	19
20	7,101.6	52.27	1.00	26.01	7,425.0	1.00191	20
21	7,071.7	45.16	1.02	25.46	7,294.4	0.95644	21
22	7,754.8	42.50	1.07	22.17	7,895.3	0.96947	22
23	8,480.3	43.70	1.12	18.56	8,413.6	0.98220	23
24	8,105.2	47.88	1.10	21.32	8,640.0	1.00793	24
25	7,157.2	36.33	1.07	22.75	8,054.1	0.93810	25

[a]The data presented are actual values for 1950–1975.
Q_c = world consumption (sales) of copper in 1000s of metric tons,
P_c = price of copper in cents per pound (inflation adjusted),
M = index of real per capita income (1970 = 1.00),
P_A = price of aluminum in cents per pound (inflation adjusted),
X = ratio of consumption in the previous year to production in the previous year (= Q_c/Q_P), and
T = technology (time period is a proxy).