



# Decisions Under Risk and Uncertainty

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## Learning Objectives

After reading Chapter 15 and working the problems for Chapter 15 in the textbook and in this Workbook, you should be able to:

- Distinguish between decision making under uncertainty and under risk.
- Compute the expected value, variance, standard deviation, and coefficient of variation of a probability distribution.
- Apply the expected value rule, the mean-variance rules, and the coefficient of variation rule to make decisions under risk.
- Define three risk preference categories: risk averse, risk neutral, and risk loving, and relate these attitudes toward risk to the shape of the utility of profit curve.
- Find the optimal level of a risky activity (when the variances of marginal benefit and marginal cost are constant) by setting  $E(MB) = E(MC)$ .
- Apply (1) the maximax rule, (2) the maximin rule, (3) the minimax regret rule, and (4) the equal probability rule to make decisions under uncertainty.

## Essential Concepts

1. Conditions of *risk* occur when a manager must make a decision for which the outcome is not known with certainty. Under conditions of risk, the manager can make a list of all possible outcomes and assign probabilities to the various outcomes. *Uncertainty* exists when a decision maker cannot list all possible outcomes and/or cannot assign probabilities to the various outcomes.
2. In order to measure the risk associated with a decision, the manager can examine several characteristics of the probability distribution of outcomes for the decision. A *probability distribution* is a table or graph showing all possible outcomes or payoffs for a decision and the probability that each outcome will occur.
3. In order to measure the risk associated with a decision, several statistical characteristics of the probability distribution can be employed:
  - a. The *expected value (or mean)* of a probability distribution is

$$E(X) = \text{Expected value of } X = \sum_{i=1}^n p_i X_i$$

where  $X_i$  is the  $i^{\text{th}}$  outcome of a decision,  $p_i$  is the probability of the  $i^{\text{th}}$  outcome, and  $n$  is the total number of possible outcomes in the probability distribution. The expected value of a distribution does *not* give the actual

value of the random outcome, but rather indicates the “average” value of the outcomes if the risky decision were to be repeated a large number of times.

- b. The *variance* (a measure of *absolute* risk) of a probability distribution measures the dispersion of the outcomes about the mean or expected outcome. The variance is calculated as

$$\text{Variance}(X) = \sigma_x^2 = \sum_{i=1}^n p_i (X_i - E(X))^2$$

The higher (lower) the variance, the greater (lower) the risk associated with a probability distribution.

- c. The *standard deviation* is the square root of the variance:

$$\sigma_x = \sqrt{\text{Variance}(X)}$$

The higher (lower) the standard deviation, the greater (lower) the risk.

- d. When the expected values of outcomes differ substantially, managers should measure the riskiness of a decision relative to its expected value using the coefficient of variation (a measure of *relative* risk):

$$v = \frac{\text{Standard deviation } \sigma}{\text{Expected value } E(X)}$$

4. While no single decision rule guarantees that profits will actually be maximized, there are a number of decision rules that managers can use to help them make decisions under risk. Decision rules do not eliminate the risk surrounding a decision, they just provide a method of systematically including risk in the process of decision making. The three rules presented in this chapter are (1) the expected value rule, (2) the mean-variance rules, and (3) the coefficient of variation rule. These three rules are summarized below:

#### Summary of Decision Rules Under Conditions of Risk

|                               |  |
|-------------------------------|--|
| Expected value rule           | Choose the decision with the highest expected value.   |
| Mean-variance rules           | <p>Given two risky decisions A and B:</p> <p>If decision A has a higher expected outcome and a lower variance than decision B, decision A should be made.</p> <p>If both decisions A and B have identical variances (or standard deviations), the decision with the higher expected value should be made.</p> <p>If both decisions A and B have identical expected values, the decision with the lower variance (standard deviation) should be made.</p> |
| Coefficient of variation rule | Choose the decision with the smallest coefficient of variation.  |

5. Which rule is best?

When a decision is to be made repeatedly, with identical probabilities each time, the expected value rule provides managers with the most reliable rule for maximizing (expected) profit. The average return of a given risky course of action repeated many times will approach the expected value of that action.

When a manager makes a one-time decision under risk, there will not be any follow-up repetitions of the decision to “average out” a bad outcome (or a good outcome). Unfortunately, there is no best rule to follow when decisions are not repetitive.

The rules for risky decision making should be used by managers to help analyze and guide the decision-making process. Ultimately, making decisions under risk (or uncertainty) is as much an art as it is a science.

6. The actual decisions made by a manager depend upon the manager’s willingness to accept risk. To allow for different attitudes toward risk-taking in decision making, modern decision theory treats managers as deriving utility or satisfaction from the profits earned by their firms. Just as consumers derived utility from consumption of goods in Chapter 6, in *expected utility theory*, managers are assumed to derive utility from earning profits.
7. Expected utility theory postulates that managers make risky decisions in a way that maximizes the expected utility of the profit outcomes, where the expected utility of a risky decision is the sum of the probability-weighted utilities of each possible profit outcome:

$$E[U(\pi)] = p_1U(\pi_1) + p_2U(\pi_2) + \dots + p_nU(\pi_n)$$

$U(B)$  is the manager’s utility function for profit that measures the utility associated with a particular level of profit. The utility function for profit gives an index value to measure the level of utility experienced when a given amount of profit is earned. The relation between an index of utility and the level of profit earned is assumed to be an upward-sloping curve.

8. A manager’s attitude toward risk is determined by the manager’s *marginal utility of profit*:

$$MU_{\text{profit}} = \Delta U(\pi) / \Delta \pi$$

Marginal utility, then, measures the slope of the upward-sloping  $U(B)$  curve. It is the slope of the utility curve, or marginal utility, that determines a manager’s attitude toward risk:

- People are said to be *risk averse* if, facing two risky decisions with equal expected profits, they choose the less risky decision.
  - Someone who chooses the more risky of two decisions when the expected profits are the same is said to be *risk loving*.
  - A *risk neutral* person is indifferent between risky decisions that all have the same expected profit.
9. A manager’s attitude toward risky decisions can be related to his or her marginal utility of profit. Someone who experiences diminishing (increasing) marginal utility for profit will be a risk averse (risk loving) decision maker. Someone whose marginal utility of profit is constant is risk neutral.

10. If a manager maximizes expected utility for profit, the decisions can differ from decisions reached using the three decision rules discussed for making risky decisions. In the case of a risk-neutral manager, however, the decisions are the same under maximization of expected profit and maximization of expected utility of profit.
11. In the case of uncertainty, decision science can provide very little guidance to managers beyond offering them some simple decision rules to aid them in their analysis of uncertain situations. Four basic rules for decision making under uncertainty are summarized in the following table.

**Summary of Decision Rules Under Conditions of Uncertainty**

|                        |  |
|------------------------|--|
| Maximax rule           | Identify the best outcome for each possible decision and choose the decision with the maximum payoff.  |
| Maximin rule           | Identify the worst outcome for each decision and choose the decision associated with the maximum worst payoff.   |
| Minimax regret rule    | Determine the worst potential regret associated with each decision, where the potential regret associated with any particular decision and state of nature is the improvement in payoff the manager could have experienced had the decision been the best one when that state of nature actually occurred. The manager chooses the decision with the minimum worst potential regret. |
| Equal probability rule | Assume each state of nature is equally likely to occur and compute the average payoff for each equally likely possible state of nature. Choose the decision with the highest average payoff.   |

## Matching Definitions

certainty equivalent  
coefficient of variation  
coefficient of variation rule  
equal probability rule  
expected utility  
expected utility theory  
expected value  
expected value rule  
marginal utility of profit  
maximax rule  
maximin rule  
mean of the distribution

mean-variance analysis  
minimax regret rule  
payoff matrix  
potential regret  
probability distribution  
risk  
risk averse  
risk loving  
risk neutral  
standard deviation  
uncertainty  
variance

1. \_\_\_\_\_ A decision-making situation in which a manager can list all possible outcomes and assign probabilities to the occurrence of each one.
2. \_\_\_\_\_ A situation in which the manager cannot list all possible outcomes and/or cannot assign probabilities to the outcomes.
3. \_\_\_\_\_ A table or graph showing all possible outcomes for a decision and the probabilities that each outcome will occur.
4. \_\_\_\_\_ The weighted average of the outcomes, with the probabilities of each outcome serving as the respective weights.
5. \_\_\_\_\_ Expected value of a distribution.
6. \_\_\_\_\_ A measure of the dispersion of a distribution around its mean.
7. \_\_\_\_\_ Square root of the variance.
8. \_\_\_\_\_ Standard deviation divided by the expected value of the probability distribution.
9. \_\_\_\_\_ Choosing the decision with the highest expected value.
10. \_\_\_\_\_ Method of decision making that uses both the mean and the variance to make decisions.
11. \_\_\_\_\_ A decision-making method that chooses the decision with the lowest coefficient of variation.
12. \_\_\_\_\_ A decision-making theory that accounts for the decision-maker's attitude toward risk.
13. \_\_\_\_\_ The sum of the probability-weighted utilities of each possible profit outcome.
14. \_\_\_\_\_ The amount by which total utility increases for each additional dollar of profit that the firm makes.

15. \_\_\_\_\_ A decision maker who chooses the less risky project when two projects have an equal expected profit.
16. \_\_\_\_\_ A decision maker who chooses the project with the higher risk when two projects have an equal expected profit.
17. \_\_\_\_\_ The decision maker who ignores risk and focuses only on the expected value of decisions.
18. \_\_\_\_\_ The dollar amount to be received with certainty that a manager would be just willing to trade for the opportunity to engage in a risky decision.
19. \_\_\_\_\_ A decision-making guide in which the manager identifies for each possible decision the best outcome that could occur and then chooses the decision that would give the maximum payoff of all the best outcomes.
20. \_\_\_\_\_ A table with rows corresponding to the various decisions and columns corresponding to the various states of nature.
21. \_\_\_\_\_ A decision-making guide in which the manager identifies the worst outcome for each decision and makes the decision associated with the maximum worst payoff.
22. \_\_\_\_\_ For a given decision and state of nature, the improvement in payoff the manager could have experienced had the decision been the best one when that state of nature actually occurred.
23. \_\_\_\_\_ A decision-making guide that requires managers make to the decision with the minimum worst potential regret.
24. \_\_\_\_\_ A decision-making guide that assumes each state of nature has an equal probability of occurring; the manager then calculates the average payoff for each decision and chooses the decision with the highest average payoff.

## Study Problems

1. Consider the following two probability distributions for sales:

| Sales | Probability                 |                             |
|-------|-----------------------------|-----------------------------|
|       | Distribution A<br>(percent) | Distribution B<br>(percent) |
| 100   | 20                          | 5                           |
| 200   | 40                          | 20                          |
| 300   | 20                          | 50                          |
| 400   | 15                          | 20                          |
| 500   | 5                           | 5                           |

- a. Calculate the expected sales for both of these probability distributions.

$$E(\text{Sales}_A) = \underline{\hspace{2cm}}$$

$$E(\text{Sales}_B) = \underline{\hspace{2cm}}$$

- b. Calculate the variance and standard deviation for both of the probability distributions.

$$\sigma_A^2 = \underline{\hspace{2cm}} \text{ and } \sigma_A = \underline{\hspace{2cm}}$$

$$\sigma_B^2 = \underline{\hspace{2cm}} \text{ and } \sigma_B = \underline{\hspace{2cm}}$$

Distribution \_\_\_\_\_ is more risky than distribution \_\_\_\_\_.

- c. Calculate the coefficient of variation for both distributions

$$v_A = \underline{\hspace{2cm}}$$

$$v_B = \underline{\hspace{2cm}}$$

Distribution \_\_\_\_\_ has greater risk relative to its mean than distribution \_\_\_\_\_.

2. Texas Petroleum Company is a producer of crude oil that is considering two drilling projects with the following profit outcomes and associated probabilities:

| Drilling Project A |                          | Drilling Project B |                          |
|--------------------|--------------------------|--------------------|--------------------------|
| Profit             | Probability<br>(percent) | Profit             | Probability<br>(percent) |
| -\$300,000         | 10                       | -\$600,000         | 15                       |
| 100,000            | 60                       | 100,000            | 25                       |
| 500,000            | 20                       | 300,000            | 40                       |
| 600,000            | 10                       | 1,000,000          | 20                       |

- a. Compute the expected profit for both drilling projects.  
 $E(\text{Profit}_A) = \underline{\hspace{2cm}}$  and  $E(\text{Profit}_B) = \underline{\hspace{2cm}}$
- b. Based on the expected value rule, Texas Petroleum should choose drilling project           .
- c. Compute the standard deviations of both projects:  
 $\sigma_A = \underline{\hspace{2cm}}$  and  $\sigma_B = \underline{\hspace{2cm}}$
- d. Which drilling project has the greater (absolute) risk?
- e. Use mean-variance rules, if possible, to decide which drilling project to undertake. Explain.
- f. Compute the coefficient of variation for both projects:  
 $v_A = \underline{\hspace{2cm}}$  and  $v_B = \underline{\hspace{2cm}}$   
 Using the coefficient of variation rule, Texas Petroleum should choose project       .

3. A manager's utility function for profit is  $U(\pi) = 35\pi$ , where  $\pi$  is the dollar amount of profit. The manager is considering a risky decision with the four possible profit outcomes shown below. The manager makes the following subjective assessments about the probability of each profit outcome:

| <i>Probability</i> | <i>Profit Outcome</i> |
|--------------------|-----------------------|
| 0.05               | −\$15,000             |
| 0.40               | −\$1,000              |
| 0.50               | \$5,000               |
| 0.05               | \$10,000              |

- a. The expected profit is                                 .
- b. The expected utility of profit is                                 .
- c. The marginal utility of an extra dollar of profit is                                 .
- d. The manager is risk                                  because the marginal utility of profit is                                 .



4. Suppose the manager of a firm has a utility function for profit  $U(\pi) = 12\ln(\pi)$ , where  $\pi$  is the dollar amount of profit. The manager is considering a risky project with the following profit payoffs and probabilities:

| <i>Probability</i> | <i>Profit Outcome</i> | <i>Marginal Utility of Profit</i> |
|--------------------|-----------------------|-----------------------------------|
| 0.10               | \$1,000               | xx                                |
| 0.20               | \$2,000               | _____                             |
| 0.30               | \$3,000               | _____                             |
| 0.40               | \$4,000               | _____                             |

- a. The expected profit is \_\_\_\_\_.
- b. The expected utility of profit is \_\_\_\_\_.
- c. Fill in the blanks in the following table showing the marginal utility of an additional \$1,000 of profit.
- d. The manager is risk \_\_\_\_\_ because the marginal utility of profit is \_\_\_\_\_.
5. A firm is making production plans for next quarter, but the manager does not know what the price of the product will be next month. She believes there is a 30 percent chance that price will be \$500 and a 70 percent chance that price will be \$750. The four possible profit outcomes are

|                                      | <i>Profit (loss) when price is</i> |           |
|--------------------------------------|------------------------------------|-----------|
|                                      | \$500                              | \$750     |
| <i>Option A: produce 1,000 units</i> | –\$12,000                          | \$80,000  |
| <i>Option B: produce 2,000 units</i> | –\$20,000                          | \$150,000 |

- a. Option \_\_\_\_\_ maximizes expected profit.
- b. Option \_\_\_\_\_ is the riskier of the two options.
- c. The manager \_\_\_\_\_ (can, cannot) apply mean-variance rules in this decision. If the manager can use mean-variance rules, the manager would choose Option \_\_\_\_\_.
- d. Using the coefficient of variation rule, the manager chooses Option \_\_\_\_\_.
6. Suppose the manager in Problem 5 has absolutely no idea about the probabilities of the two prices occurring. Which option would the manager choose under each of the following rules?
- a. Maximax rule \_\_\_\_\_
- b. Maximin rule \_\_\_\_\_
- c. Minimax regret rule \_\_\_\_\_
- d. Equal probability rule \_\_\_\_\_

## Multiple Choice / True-False

Answer questions 1–3 using the following probability distribution for profit:

| <i>Profit</i> | <i>Probability</i> |
|---------------|--------------------|
| \$30          | 0.10               |
| \$40          | 0.30               |
| \$50          | 0.50               |
| \$60          | 0.10               |

- The expected profit for the profit distribution above is \_\_\_\_\_.
  - $0.10(30) + 0.3(40) + 0.5(50) + 0.10(60)$
  - $0.10^2(30) + 0.3^2(40) + 0.5^2(50) + 0.10^2(60)$
  - $0.10(30 - 46)^2 + 0.30(40 - 46)^2 + 0.50(50 - 46)^2 + 0.10(60 - 46)^2$
  - $(30 + 40 + 50 + 60)/2$
  - both *b* and *d*.
- The variance for the profit distribution above is \_\_\_\_\_.
  - 64
  - 46
  - 54
  - 8
- The coefficient of variation for the distribution above is \_\_\_\_\_.
  - 0.17
  - 5.75
  - 0.50
  - 1.39

Answer questions 4–10 using the following table that shows the various profit outcomes for different projects when the price of the product is \$10 or \$20.

| <i>Project</i> | <i>Profit</i>   |                 |
|----------------|-----------------|-----------------|
|                | <i>P = \$10</i> | <i>P = \$20</i> |
| A              | \$40            | \$120           |
| B              | \$55            | \$70            |
| C              | –\$10           | \$200           |

4. Using the maximax rule, a manager would choose \_\_\_\_\_.
  - a. Project A
  - b. Project B
  - c. Project C
  - d. Either Project A or Project C
5. Following the maximin rule, a manager would choose \_\_\_\_\_.
  - a. Project A
  - b. Project B
  - c. Project C
6. Which project has the minimum worst potential regret?
  - a. Project A
  - b. Project B
  - c. Project C
7. Under the equal probability rule, which project should be chosen?
  - a. Project A
  - b. Project B
  - c. Project C
  - d. Either Project A or C

Now suppose the manager in questions 4–7 above is able to determine that product price is likely to follow the probability distribution below:

| <i>Price</i> | <i>Profitability</i> |
|--------------|----------------------|
| \$10         | 60%                  |
| \$20         | 40%                  |

8. Which project should be chosen under the expected value rule?
  - a. Project A
  - b. Project B
  - c. Project C
9. Using mean-variance rules, which project would the manager choose?
  - a. Project A
  - b. Project B
  - c. Project C
  - d. Cannot apply mean-variance rules for these projects
10. Which project should be chosen under the coefficient of variation rule?
  - a. Project A
  - b. Project B
  - c. Project C

11. Risk exists when
  - a. all possible outcomes are known but probabilities can't be assigned to the outcomes.
  - b. all possible outcomes are known and probabilities can be assigned to each.
  - c. all possible outcomes are known but only objective probabilities can be assigned to each.
  - d. future events can influence the payoffs but the decision maker has some control over their probabilities.
  - e. both *c* and *d*.
12. Using the minimax regret rule the manager makes the decision
  - a. with the smallest worst-potential regret.
  - b. with the largest worst-potential regret.
  - c. knowing he will not regret it.
  - d. that has the highest expected value relative to the other decisions.
13. In the maximin strategy, a manager choosing between two options will choose the option that
  - a. has the highest expected profit.
  - b. provides the best of the worst possible outcomes.
  - c. minimizes the maximum loss.
  - d. both *a* and *b*.
  - e. both *b* and *c*.
14. In making decisions under risk
  - a. maximizing expected value is always the best rule.
  - b. mean variance analysis is always the best rule.
  - c. the coefficient of variation rule is always best.
  - d. maximizing expected value is most reliable for making repeated decisions with identical probabilities.
  - e. none of the above.
15. A probability distribution
  - a. is a way of dealing with uncertainty.
  - b. lists all possible outcomes and the corresponding probabilities of occurrence.
  - c. shows only the most likely outcome in an uncertain situation.
  - d. both *a* and *b*.
  - e. both *a* and *c*.
16. The variance of a probability distribution is used to measure risk because a higher variance is associated with
  - a. a wider spread of values around the mean.
  - b. a more compact distribution.
  - c. a lower expected value.
  - d. both *a* and *b*.
  - e. all of the above.

The next three questions refer to the following probability distribution for profit:

| <i>Profit</i> | <i>Probability</i> |
|---------------|--------------------|
| \$30          | 0.05               |
| 40            | 0.25               |
| 50            | 0.60               |
| 60            | 0.10               |

17. What is the expected profit for this distribution?
  - a. \$11,875
  - b. \$46
  - c. \$47.50
  - d. \$48.75
  - e. none of the above.
18. What is the variance of this distribution?
  - a. 48.75
  - b. 2,376
  - c. 525
  - d. 70
  - e. 11.875
19. What is the coefficient of variation for this distribution?
  - a. 1.67
  - b. 0.675
  - c. 18.6
  - d. 0.147
  - e. 1.03
20. T F Given two projects, A and B, if  $E(\pi_A) > E(\pi_B)$  and  $\sigma_A^2 > \sigma_B^2$ , then the manager should select project A using mean-variance analysis.
21. T F Given two projects, A and B, if  $E(\pi_A) = E(\pi_B)$  and  $\sigma_A^2 < \sigma_B^2$ , then the manager should select project A using mean-variance analysis.
22. T F Employing the expected value rule guarantees that a manager will always earn the greatest return possible, a return equal to the expected value.
23. T F Suppose a person has two alternatives: (A) receive \$1,000 with certainty, or (B) flip a coin and receive \$2,000 if a head comes up or nothing (\$0) if a tail comes up. A risk-averse person will take the \$1,000 with certainty (alternative A).
24. T F When net benefit has the same variance at all relevant levels of activity, a risk-loving manager will undertake more of the risky activity than will a risk-averse manager.

# Answers

## MATCHING DEFINITIONS

1. risk
2. uncertainty
3. probability distribution
4. expected value
5. mean of the distribution
6. variance
7. standard deviation
8. coefficient of variation
9. expected value rule
10. mean-variance analysis
11. coefficient of variation rule
12. expected utility theory
13. expected utility
14. marginal utility of profit
15. risk averse
16. risk loving
17. risk neutral
18. certainty equivalent
19. maximax rule
20. payoff matrix
21. maximin rule
22. potential regret
23. minimax regret rule
24. equal probability rule

## STUDY PROBLEMS

1. a.  $E(\text{Sales}_A) = (0.20 \times 100) + (0.40 \times 200) + (0.20 \times 300) + (0.15 \times 400) + (0.05 \times 500) = 245$   
 $E(\text{Sales}_B) = (0.05 \times 100) + (0.20 \times 200) + (0.50 \times 300) + (0.20 \times 400) + (0.05 \times 500) = 300$
- b.  $\sigma_A^2 = (100 - 245)^2(0.2) + (200 - 245)^2(0.4) + (300 - 245)^2(0.2) + (400 - 245)^2(0.15) + (500 - 245)^2(0.05) = 12,475$   
 $\sigma_A = (12,475)^{.5} = 111.69$   
 $\sigma_B^2 = (100 - 300)^2(0.05) + (200 - 300)^2(0.2) + (300 - 300)^2(0.5) + (400 - 300)^2(0.20) + (500 - 300)^2(0.05) = 8,000$   
 $\sigma_B = (8,000)^{.5} = 89.44$   
 Distribution A is more risky than distribution B because  $\sigma_A^2 < \sigma_B^2$ .
- c.  $\Lambda_A = \text{standard deviation} / \text{expected value} = 111.69/245 = 0.456$   
 $\Lambda_B = \text{standard deviation} / \text{expected value} = 89.44/300 = 0.298$   
 Distribution A has greater relative risk than distribution B because  $\Lambda_A > \Lambda_B$ .
2. a.  $E(\text{Profit}_A) = (-300,000 \times 0.10) + (100,000 \times 0.60) + (500,000 \times 0.20) + (600,000 \times 0.10) = \$190,000$

- $E(\text{Profit}_B) = (-600,000 \times 0.15) + (100,000 \times 0.25) + (300,000 \times 0.40) + (1,000,000 \times 0.20) = \$255,000$
- b. Project B (It has the larger expected profit.)
- c. 
$$\text{Variance}_A = (-300,000 - 190,000)^2(0.10) + (100,000 - 190,000)^2(0.60) + (500,000 - 190,000)^2(0.20) + (600,000 - 190,000)^2(0.10)$$

$$= 64,900,000,000$$

$$\sigma_A = (64,900,000,000)^{0.5} = 254,755$$

$$\text{Variance}_B = (-600,000 - 255,000)^2(0.15) + (100,000 - 255,000)^2(0.25) + (300,000 - 255,000)^2(0.40) + (1,000,000 - 255,000)^2(0.20)$$

$$= 27,475,000,000$$

$$\sigma_B = (27,475,000,000)^{0.5} = 165,756$$
- d. Project A has higher (absolute) risk than Project B since  $\sigma_A > \sigma_B$ .
- e. The expected profit in project B exceeds the expected profit in project A, but project B has a higher variance than A. The manager at Texas Petroleum must make a tradeoff between risk and return in order to decide which of the two projects to choose. Mean-variance rules cannot be employed to make decision when a tradeoff between risk and return is involved.
- f.  $v_A = 254,755/190,000 = 1.34$  and  $v_B = 165,756/255,000 = 0.65$   
Project B has higher relative risk. Under the coefficient of variation rule, Project A is chosen.
3. a.  $E(\pi) = 0.05(-\$15,000) + 0.40(-\$1,000) + 0.50(\$5,000) + 0.05(\$10,000)$   
 $= \$1,850$
- b.  $E[U(\pi)] = 0.05 \times U(-\$15,000) + 0.40 \times U(-\$1,000) + 0.50 \times U(\$5,000) + 0.05 \times U(\$10,000)$   
 $= 0.05(-525,000) + 0.40(-35,000) + 0.50(175,000) + 0.05(350,000)$   
 $= 64,750$
- c. 35
- d. neutral; constant
4. a.  $E(\pi) = 0.10(\$1,000) + 0.20(\$2,000) + 0.30(\$3,000) + 0.40(\$4,000)$   
 $= \$3,000$
- c.  $E[U(\pi)] = 0.10 \times U(\$1,000) + 0.20 \times U(\$2,000) + 0.30 \times U(\$3,000) + 0.40 \times U(\$4,000)$   
 $= 0.10(82.89) + 0.20(91.21) + 0.30(96.08) + 0.40(99.53)$   
 $= 95.17$
- c. 8.32; 4.87; 3.45 (in the three blanks)
- d. averse; decreasing
5. a. B
- b. B
- c. cannot; blank
- d. B
6. a. B
- b. A

- c. B
- d. B

**MULTIPLE CHOICE / TRUE-FALSE**

1. a This follows directly from the definition of expected value.  $E(\text{Profit}) = \$46$ .
2. a  $\text{Var}(\pi) = (30 - 46)^2(0.10) + (40 - 46)^2(0.30) + (50 - 46)^2(0.50) + (60 - 46)^2(0.10) = 64$
3. a The coefficient of variation =  $\sigma / E(\pi) = 8/46 = 0.17$
4. c Project C has the greatest best-outcome.
5. b Project B has the maximum worst possible outcome (= \$55).
6. c The worst regrets are 80, 130, and 65 for Projects A, B, and C, respectively. Project C has the smallest potential regret.
7. c The average profit for Project C is  $190/2 = \$85$ , which is higher than for Projects A or B.
8. c  $E(\pi_A) = \$72$ ,  $E(\pi_B) = \$61$ , and  $E(\pi_C) = \$74$ . Project C has the highest  $E(\pi)$ .
9. d Project C has the highest  $E(\pi)$  and the highest  $\sigma^2$ . Since a tradeoff between expected return and risk is involved, mean-variance analysis cannot be applied.
10. b Project B has the lowest coefficient of variation:  $v_A = 0.54$ ,  $v_B = 0.12$ ,  $v_C = 1.39$ .
11. b This is the definition.
12. a This is the definition.
13. e Both statements are correct.
14. d Unless a decision is made repeatedly with identical probabilities, there is no clearly best rule to follow.
15. b This is the definition of a probability distribution.
16. a The higher the variance, the greater the dispersion of outcomes and the greater is the risk.
17. c  $\$47.50 = 0.05(30) + 0.25(40) + 0.60(50) + 0.10(60)$
18. a  $48.75 = 0.05(30 - 47.50)^2 + 0.25(40 - 47.50)^2 + 0.60(50 - 47.50)^2 + 0.10(60 - 47.50)^2$
19. d  $0.147 = (48.74)^{0.5} / 47.50$
20. F Even though the expected value of Project A is higher than B's expected value, Project A has a higher variance (is riskier) than B. Projects A and B cannot be ranked using mean-variance analysis in this case.
21. T Project A should be chosen since it has a lower risk and an equal expected return.
22. F The expected value rule only guarantees the greatest return *on average*, if the decision is made a very large number of times. When the decision is made repeatedly, the expected value rule provides the most reliable rule for maximizing profit.
23. T Although the expected value of alternative B is \$1,000, a risk-averse decision maker takes the \$1,000 with certainty (alternative A).
24. F When risk is the same at all levels of activity (i.e., constant variance of net benefit), the optimal level of a risky activity is the same for both risk averse and risk loving managers. It is also the same for risk-neutral managers.



## Homework Exercises

Star Products, Inc. faces uncertain demand conditions in 2009. Management at Star Products is considering three different levels of output for 2009: 1, 1.5, or 2 million units. Management has determined that the following profit levels will occur under weak and strong demand conditions:

| <i>Output Level</i> | <i>Profit (in \$millions) if Demand is</i> |               |
|---------------------|--|---------------|
|                     | <i>Weak</i>                                | <i>Strong</i> |
| 1 million units     | 60   | 175           |
| 1.5 million units   | 50   | 200           |
| 2.0 million units   | -50  | 400           |

- Using each of the four rules for decision making under uncertainty, determine the output level of 2009.

Maximax rule \_\_\_\_\_ units of output

Maximin rule \_\_\_\_\_ units of output

Minimax regret rule \_\_\_\_\_ units of output

Equal probability rule \_\_\_\_\_ units of output

- Now suppose that management believes the probability of weak demand in 2009 is 25% and the probability of strong demand is 75%. Compute the expected profit, variance, standard deviation, and coefficient of variation for each level of output:

| <i>Output</i>     | <i><math>E(\pi)</math></i> | <i><math>\sigma^2</math></i> | <i><math>\sigma</math></i> | <i><math>v</math></i> |
|-------------------|----------------------------|------------------------------|----------------------------|-----------------------|
| 1 million units   | _____                      | _____                        | _____                      | _____                 |
| 1.5 million units | _____                      | _____                        | _____                      | _____                 |
| 2.0 million units | _____                      | _____                        | _____                      | _____                 |

- Based on the expected value rule, Star Products should produce \_\_\_\_\_ units in 2009.

- | <i>Output</i>     | $E[U(\pi)]$ |
|-------------------|-------------|
| 1 million units   | _____       |
| 1.5 million units | _____       |
| 2.0 million units | _____       |

358