Assignment 12: Integration and Riemann Sums (4.1–4) **Name____________________**
Please provide a handwritten response.

1a. The `Integrate` command is used to find both indefinite and definite integrals. Execute `f[x_] = 4x - 2Sqrt[x]` followed by

```
Integrate[f[x], x]
```

to find the indefinite integral $\int \left(4x - 2\sqrt{x}\right)dx$, and record the result below. Is the answer correct? Note *Mathematica* omits the arbitrary constant "$+c$".

1b. Next execute `Clear[f]` followed by `f[x_] = 2x^3/(x^4 + 1)` and

```
F[x_] = Integrate[f[x], x]
```

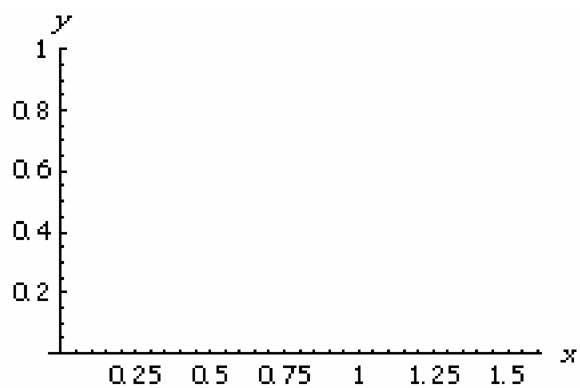
to calculate $F(x) = \int \frac{2x^3}{x^4+1}dx$; record the result below.

1c. By definition of antiderivative, what should $F'(x)$ be? Execute `F'[x]` and record the result below; is it correct?

2a. Exercise 9, Section 4.3 calls for an approximation of the area under the graph of $f(x) = \cos x$ on the interval $\left[0, \frac{\pi}{2}\right]$. Execute `Clear[f]` followed by

```
f[x_] = Cos[x]
```

and then use the `Plot` command to graph *f* over $\left[0, \frac{\pi}{2}\right]$. (Remember that π is denoted by `Pi` in *Mathematica*.) Sketch the result on the axes at right.



2b. Exercise 9 calls for the use of $n = 50$ rectangles in our approximation; moreover, in this case, our endpoints *a* and *b* are given by $a = 0$ and $b = \frac{\pi}{2}$. Execute in order the commands `a = 0.0`, `b = Pi/2`, `n = 50` and `deltax = (b - a)/n`. (The decimal point in the value for `a` is a handy way to make sure that the final answer will be reported as a decimal, not a cumbersome "exact" value.) What value for Δx did *Mathematica* give? Is this correct?

2c. It will be convenient to enter $x_i = a + i\Delta x$ as a separate *Mathematica* function. Execute

```
x[i_] = a + i*deltax
```

and record the result below.

2d. The Riemann sum $\sum_{i=1}^{n} f(x_i)\Delta x$ for right–hand evaluation can be found using the **`Sum`** command; execute

```
Sum[f[x[i]]*deltax, {i, 1, n}]
```

and record the result below. Is this a plausible approximation to the area?

2e. The Riemann sum for left–hand evaluation is $\sum_{i=1}^{n} f(x_{i-1})\Delta x$. Execute

```
Sum[f[x[i - 1]]*deltax, {i, 1, n}]
```

and record the result below. Is your answer greater or less than your result in part **d**? Why should this be so?

2f. Likewise, the Riemann sum for midpoint evaluation is $\sum_{i=1}^{n} f\left[\frac{1}{2}(x_{i-1} + x_i)\right]\Delta x$. Execute

```
Sum[f[(1/2)(x[i - 1] + x[i])]*deltax, {i, 1, n}]
```

and record the result below.

2g. Execute **`Clear[a, b, n, deltax, x]`** and re–execute all of the commands in parts **b**–**f** in order with **`n = 50`** replaced by **`n = 100`** . Do the three approximations in parts **d**–**f** become more spread out or closer together? Is this what you would expect?

3. The exact value of the area we approximated in Question **2** is given by $\int_0^{\pi/2} \cos x \, dx$. The **`Integrate`** command can also find such definite integrals: Execute **`Clear[x]`** (to make *Mathematica* forget about x_i !) followed by

```
Integrate[f[x], {x, 0, Pi/2}]
```

and record the result below. Based on the evidence we have already gathered, is this answer plausible?