Assignment 18: Separable Differential Equations (7.2) **Name____________________**
Please provide a handwritten response.

1a. The separable differential equation $y' = \dfrac{x^2+\sqrt{x}}{e^{2y}+y-\sin y}$ is written $\int\left(e^{2y}+y-\sin y\right)dy = \int\left(x^2+\sqrt{x}\right)dx$ with variables separated. To solve the equation in *Mathematica* we first treat each side separately; execute

```
G[y_] = Integrate[Exp[2y] + y - Sin[y], y]
```

to calculate $G(y) = \int\left(e^{2y}+y-\sin y\right)dy$ and record the result below.

Then execute

```
H[x_] = Integrate[x^2 + Sqrt[x], x]
```

to calculate $H(x) = \int\left(x^2+\sqrt{x}\right)dx$ and record the result below.

1b. Execute `gensoln = G[y] == H[x] + c` to enter the general solution of the differential equation. (Review the comments in Assignment 9, Question **1a** regarding the single and double equal signs. Unlike that assignment, however, it is not necessary in this case to write `y[x]` instead of `y` , since we are not using *Mathematica* to find any derivatives.) Record the result below.

1c. We can form an IVP by adding the initial condition $y(1.5)=1$ to our differential equation. To extract the value of c corresponding to this initial condition, first execute

```
gensoln/.{x->1.5, y->1}
```

to watch *Mathematica* substitute $x=1.5$ and $y=1$ into the general solution using the replacement operator `/.` ; record the result below.

Now execute

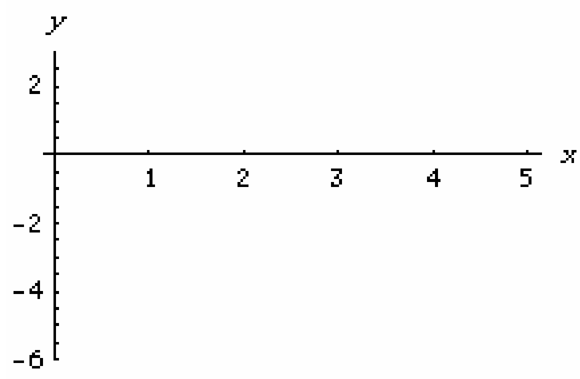
```
const = Solve[gensoln/.{x->1.5, y->1}, c]
```

to find our value of c , and record the result below.

To substitute this value of c in the general solution, execute

```
partsoln = gensoln/.const
```

and record the result below.



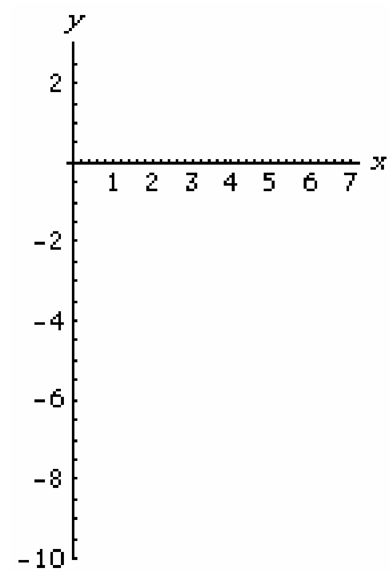
1d. As you can see, it would be impossible to solve this particular solution for y ; so, to graph this solution we will resort to the `ImplicitPlot` command as in Assignment 9. Execute

```
Needs["Graphics`ImplicitPlot`"]
```

We will use `ImplicitPlot` a bit differently here than we did before; in Assignment 9 we specified only the x–range for the graph, which allowed *Mathematica* to set the y–range automatically. We would like to do the same in this case, but our present example is more complicated than the earlier one was; it will be necessary here to specify the ranges for both x and y . Execute

```
ImplicitPlot[partsoln, {x, 0, 5}, {y, -6, 6}]
```

to graph the solution of our IVP over the viewing window $0 \le x \le 5$, $-6 \le y \le 6$. Sketch the result on the axes at right. Use a large dot to mark the point on the curve corresponding to the initial condition.



1e. Did the graph include the entire y–range $-6 \le y \le 6$? Why is this? Suppose we try to get a bigger view by changing `{x, 0, 5}` to `{x, -1, 5}` in the preceding command; do things get better or worse? Why?

1f. If there were no initial condition attached to our differential equation, then we could create a family of particular solutions by letting c range, say, from -5 to 5 ; all these solutions could then be graphed on the same axes, showing how the solutions vary with c . Execute

```
solnfamily = Table[gensoln/.c->i, {i, -5, 5}]
```

(you need not record the result!) followed by

```
ImplicitPlot[solnfamily, {x, 0, 7}, {y, -10, 3}]
```

and sketch the result on the axes at right. Can you get a better view using different viewing windows?