Assignment 20: Infinite Series (8.2–7) **Name____________________**
Please provide a handwritten response.

1a. To find the partial sum S_{10} of the infinite series $\sum_{k=1}^{\infty} \frac{1}{k^{0.9}}$ execute

```
Sum[1./k^(0.9), {k, 1, 10}]
```

and record the result in the table. By changing the `10` to `100` , etc. complete the second column of the table.

n	$S_n = \sum_{k=1}^{n} \frac{1}{k^{0.9}}$	$S_n = \sum_{k=1}^{n} \frac{5}{k^{1.1}}$
10		
100		
1000		
10000		
100000		

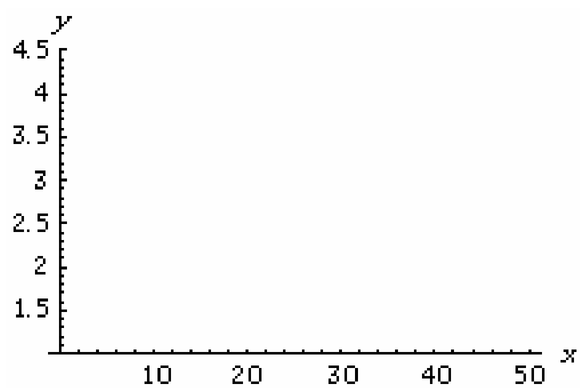
1b. Likewise modify the command in part **a** to find the partial sums of the infinite series $\sum_{k=1}^{\infty} \frac{5}{k^{1.1}}$ and complete the third column. Notice that in each row, the entry in the second column is smaller than that in the third; can this be the case for all n ? Why?

1c. Add one more row to the bottom of the table corresponding to $n = 10^8$ and fill it in; are the results consistent with your answer to part **b**?

2a. The text explains that the harmonic series $\sum_{k=1}^{\infty} \frac{1}{k}$ diverges. To get an idea of how quickly or slowly it does so, execute `s[n_] = Sum[1./k, {k, 1, n}];` followed by

```
psums = Table[{n, s[n]}, {n, 1, 50}];
```

to construct a list called `psums` of ordered pairs $\left(n, \sum_{k=1}^{n} \frac{1}{k}\right)$ where the "y–value" is the n^{th} partial sum of the harmonic series. Then execute `ListPlot[psums]` and roughly sketch the result on the axes at right.

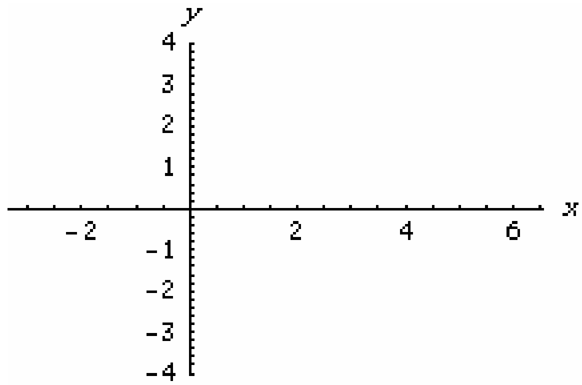


2b. Execute `Clear[psums]` and repeat the last two commands in part **a** with `50` replaced by `500` ; would you say that the partial sums are approaching ∞ quickly?

3a. To find the Taylor polynomial with $c = \frac{\pi}{2}$ and $n = 4$ for $\cos x$ (Exercise 18, Section 8.7), execute

```
Series[Cos[x], {x, Pi/2, 4}]
```

and record the result below; what do you think the final term means?



3b. We can remove this final term using **`Normal`**; execute **`tp[x_] = Normal[%]`** and enter the result below.

3c. Now plot the cosine function and the Taylor polynomial over $-\pi \le x \le 2\pi$ by executing

```
Plot[{Cos[x], tp[x]}, {x, -Pi, 2Pi}]
```

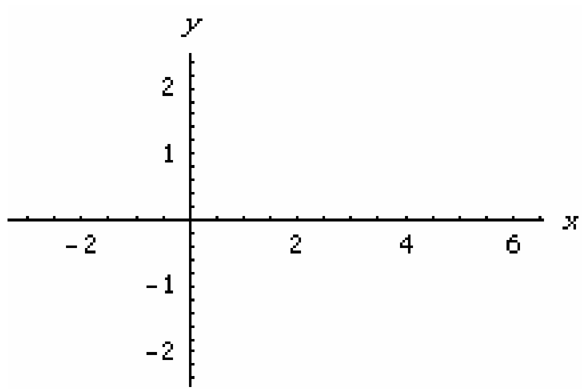
Sketch the result on the axes at right, labeling the graphs; on roughly what interval are the two graphs indistinguishable on your computer screen?

3d. Change the **`4`** in part **a** to **`8`** and then execute **`Clear[tp]`** followed by the commands in parts **a**–**c** once again. For the new Taylor polynomial, sketch its graph with labeling on your graph above, and answer the question in part **c** again.

3e. To measure the error in this Taylor approximation, execute

```
Plot[Cos[x] - tp[x], {x, -Pi, 2Pi}, PlotRange->All]
```

and sketch the result on the axes at right. How large (positive or negative) does the error become, and for what value(s) of *x* is the error greatest?



3f. By increasing *n* still further while keeping everything else the same, can we reduce the maximum error in part **e** to less than 0.1? Experiment to find how large a value of *n* is needed.

3g. Try to answer part **f** with $\cos x$ changed to $\tan^{-1} x$ (denoted **`ArcTan[x]`**), *c* to 0 and the interval to $-1.5 \le x \le 1.5$. Can you find *n* large enough? Why?