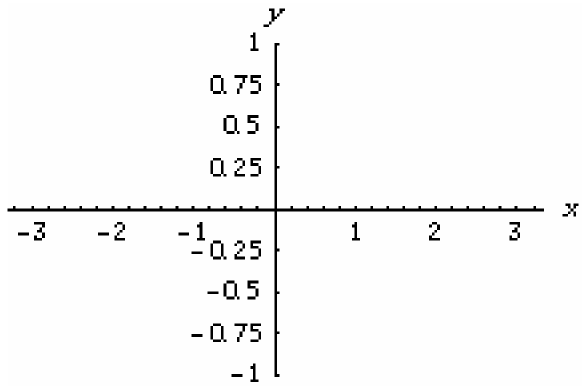
Assignment 21: Fourier Series (8.9) **Name____________________**
Please provide a handwritten response.

1. Execute **`?Sign`** and record the result below. Then execute **`f[x_] = -Sign[x]`** to define *f* as in Exercise 5, and use the **`Plot`** command to sketch the graph of *f* over $-\pi \le x \le \pi$; sketch the result on the axes at right.



2a. We can find the Fourier coefficients of *f* in at least two different ways in *Mathematica*. To apply the Euler–Fourier formulas directly, execute the following commands, noting the use of spaces between **`k`** and **`x`** to indicate multiplication:

```
a0 = (1/Pi)Integrate[f[x], {x, -Pi, Pi}]
a[k_] = (1/Pi)Integrate[f[x]Cos[k x], {x, -Pi, Pi}]
b[k_] = (1/Pi)Integrate[f[x]Sin[k x], {x, -Pi, Pi}]
```

Record this last result below, and explain why the first two results came out as they did.

2b. Now construct the partial sum **`F5`** of the Fourier series of *f* by executing

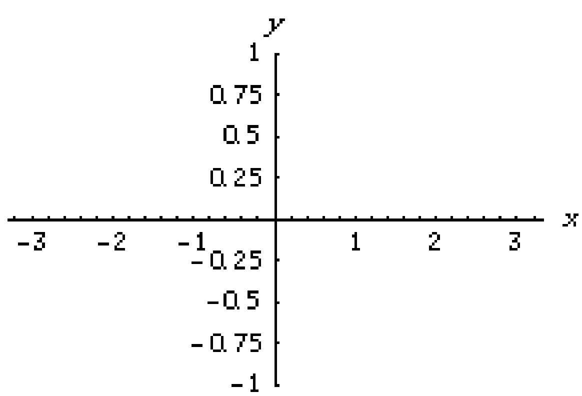
```
F5[x_] = a0/2 + Sum[a[k]Cos[k x] + b[k]Sin[k x], {k, 1, 5}]
```

Record the result below. Also graph **`f`** and **`F5`** together over $-\pi \le x \le \pi$ and sketch the result on your graph above.

2c. To measure how well this partial sum approximates *f* execute

```
Plot[f[x] - F5[x], {x, -Pi,
    Pi}, PlotRange->All]
```

and sketch the result on the axes at right. Roughly, what is the largest value, positive or negative, of the error in this approximation? (The **`PlotRange->All`** option is needed here to get the whole picture; what happens if you omit it?)



2d. Repeat parts **b** and **c** with `5` replaced by `50` and explain below why we might naturally expect our answer about the error in part **c** to become smaller. Does it?

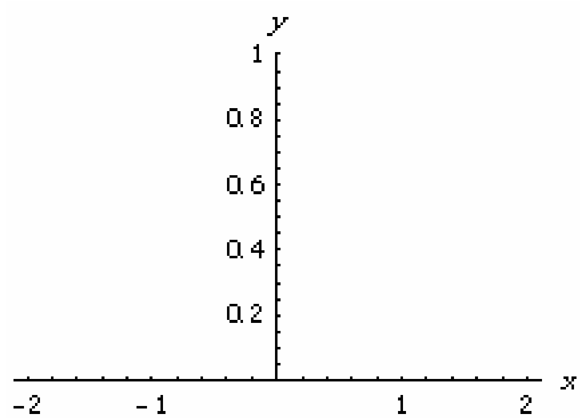
2e. Experiment with still larger values of n , as computer memory allows; are you able to find a partial sum of the Fourier series of f for which the maximum error in the approximation over $-\pi \le x \le \pi$ is smaller than your results so far? (When n is large it will be helpful to attach a semicolon to the end of the command in part **b** to suppress the output on the screen.)

2f. Read Writing Exercise 4; what might account for our rather surprising results in parts **c**–**e**?

3a. Execute `?Floor` and record the result below. Then execute

```
g[x_] = x - Floor[x]
```

and graph g over $-2 \le x \le 2$; sketch the result on the axes at right. Do the vertical lines have any significance?



3b. The period T of this function is not 2π ; what is it? Clear the variable `a0` and try to use the Euler–Fourier formulas to modify the first command in part **a** to define `a0` for g . Was this successful? Why?

3c. Actually *Mathematica* has built–in capacity to find many Fourier series, execute

```
Needs["Calculus`FourierTransform`"]
```

We can think of `g` as being the periodic function with period 1 which is equal to x over $0 \le x < 1$; execute

```
F4[x_] = FourierTrigSeries[x, {x, 0, 1}, 4]
```

Why is it that the constant term is nonzero but there are no cosine terms in the result?

4. What is the coefficient of $\cos\left(\frac{5\pi x}{3}\right)$ in the Fourier expansion of the function in Exercise 16?