Assignment 29: Double Integrals (13.1–3) **Name____________________**
Please provide a handwritten response.

1. To evaluate the double integral $\int_0^1 \int_0^{y^2} \frac{3}{4+y^3}\,dxdy$ in Exercise 22, Section 13.1 execute

```
f[x_, y_] = 3/(4 + y^3)
```

followed by

```
Integrate[f[x, y], {y, 0, 1}, {x, 0, y^2}]
```

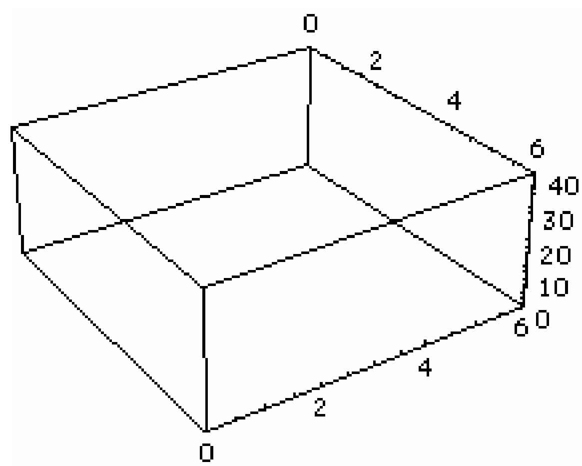
(Note the order in which the variables of integration and their limits appear.) Use **N** to obtain a decimal answer and record it below.

2a. Example 1.1 calculates Riemann sums for the volume under the graph of $f(x,y) = x^2 \sin\frac{\pi y}{6}$ on the rectangle $R = \{(x,y) \mid 0 \le x \le 6, 0 \le y \le 6\}$. Clear **f** and execute

```
f[x_, y_] = x^2 Sin[Pi y/6]
```

Because Figure 13.5a shows the graph from our normal viewpoint, we will look at the graph from a different point of view. Execute the following command to graph $f(x,y)$ and sketch the result in the box at right. If you were the observer in Figure 13.5a, in what direction would you look to see the observer in this present graph?

```
Plot3D[f[x, y], {x, 0, 6}, {y, 0, 6}, ViewPoint->{3, -2, 2}]
```



2b. We can find Riemann sums for the volume using not only 4 or 9 squares, but also 16, 25, etc. If we partition R into n^2 squares, then $\Delta A_i = \frac{36}{n^2}$ for all i. It will be more convenient here to label the center of each square as (u_i, v_j), $1 \le i, j \le n$ rather than using only the index i as in the Example; specifically,
$u_i = \frac{3}{n} + (i-1)\frac{6}{n}$, $1 \le i \le n$ and
$v_j = \frac{3}{n} + (j-1)\frac{6}{n}$, $1 \le j \le n$. Thus
$V \approx \sum_{j=1}^{n}\sum_{i=1}^{n} f(u_i, v_j)\Delta A_i = \frac{36}{n^2}\sum_{j=1}^{n}\sum_{i=1}^{n} u_i^2 \sin\frac{\pi v_j}{6}$. To carry this out in *Mathematica* execute **n = 2** followed by

```
u[i_] = (3/n) + (i - 1)(6/n)
v[j_] = (3/n) + (j - 1)(6/n)
```

```
(36/n^2)Sum[f[u[i], v[j]], {i, 1, n}, {j, 1, n}]
```

Our value $n = 2$ corresponds to $2^2 = 4$ squares total in the partition, as in Figure 13.5b; did we get the same result as in the Example?

2c. Clear variables, increase *n* to 3 (corresponding to $3^2 = 9$ squares) and calculate the Riemann sum; does the result agree with that in the text?

2d. Repeat this process to fill out the table at right, making use of the **`N`** command as needed. Do your results seem to corroborate those in the text?

2e. How large must *n* be taken to make the Riemann sum less than 275.03?

n	$\sum_{j=1}^{n}\sum_{i=1}^{n} f(u_i, v_j)\Delta A_i$
2	
3	
10	
50	

2f. The exact value of the volume is given by $\int_0^6\int_0^6 f(x, y)\,dx\,dy$; execute

```
Integrate[f[x, y], {y, 0, 6}, {x, 0, 6}]
```

Apply **`N`** to the result; does it agree with the value given in the Example?

3a. To study $\int_{-2}^{2}\int_{0}^{\sqrt{4-x^2}} \sin(x^2 + y^2)\,dy\,dx$ (see Exercise 28, Section 13.3) clear **`f`** and execute

```
f[x_, y_] = Sin[x^2 + y^2]
```

Use **`Plot3D`** to draw the graph of $f(x, y) = \sin(x^2 + y^2)$ over $-2 \le x \le 2$, $-2 \le y \le 2$ using the options **`ViewPoint->{3, 2, 2}`** (as usual) and **`PlotPoints->50`** . How would you describe the resulting surface? What part of it corresponds to the given integral?

3b. Use **`Integrate`** to evaluate the integral, apply **`N`** to the output and record the result below.

3c. Now transform the integral to polar cöordinates by executing

```
Integrate[r f[r Cos[t], r Sin[t]], {t, 0, Pi}, {r, 0, 2}]
```

and again apply **`N`** to the result. Which method do you think was easier for *Mathematica*?