Assignment 30: Surface Area; Triple Integrals (13.4–7) **Name____________________**
Please provide a handwritten response.

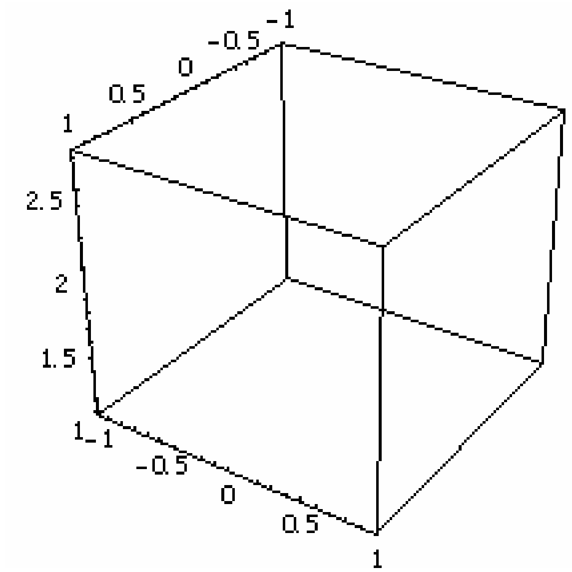
1a. To graph the portion of $z = f(x, y) = e^{x^2+y^2}$ inside $x^2 + y^2 = 1$, execute

```
f[x_, y_] = Exp[x^2 + y^2]
```

and then use `Plot3D` as we have before to draw the graph of *f* over $-1 \le x \le 1$, $-1 \le y \le 1$. Next execute `Needs["Graphics`ParametricPlot3D`"]` followed by

```
CylindricalPlot3D[f[r Cos[t], r Sin[t]], {r, 0, 1}, {t, 0, 2Pi},
        ViewPoint->{3, 2, 2}]
```

to draw the graph of $z = f(r\cos\theta, r\sin\theta)$ in cylindrical cöordinates over $r \le 1$, $0 \le \theta \le 2\pi$, and sketch the result in the box at right. Which method gave the better graph? Why?



1b. By formula (4.3) the surface area is given by

$$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \sqrt{[f_x(x, y)]^2 + [f_y(x, y)]^2 + 1}\,dy\,dx \, .$$

First define the integrand above by executing

```
int[x_,y_] = Sqrt[D[f[x, y], x]^2
                    + D[f[x, y], y]^2 + 1]
```

and then try to find the surface area by executing

```
Integrate[int[x, y],{x,-1,1},{y,-Sqrt[1 - x^2],Sqrt[1 - x^2]}]
```

Was `Integrate` successful here?

1c. Does converting to polar cöordinates help? Execute

```
Integrate[r int[r Cos[t], r Sin[t]], {t, 0, 2Pi}, {r, 0, 1}]
```

and record the result below.

1d. Try parts **b** and **c** again, replacing `Integrate` with `NIntegrate`, and record the results below. Does this work better?

2. To evaluate the triple integral $\int_0^2 \int_0^{4-2x} \int_0^{4-2x-z} 6xy\, dy dz dx$ from Example 5.3, execute

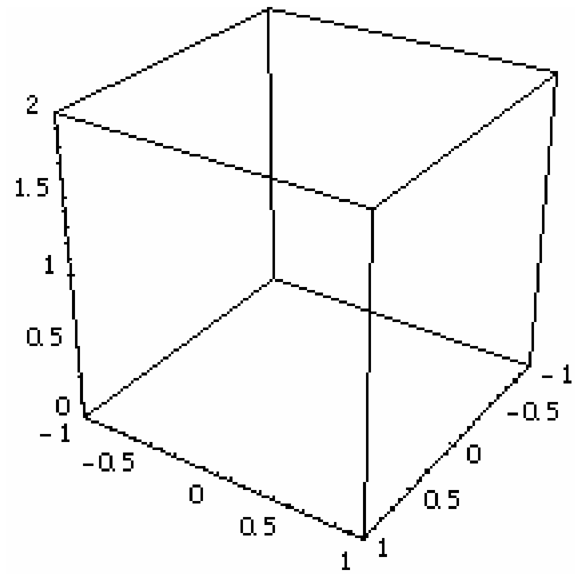
```
Integrate[6x y, {x, 0, 2}, {z, 0, 4 - 2x}, {y, 0, 4 - 2x - z}]
```

Do you get the correct value?

3a. To draw a picture of the region Q between $z = \sqrt{x^2 + y^2}$ and $z = \sqrt{4 - x^2 - y^2}$ execute

```
cone = CylindricalPlot3D[r, {r, 0, Sqrt[2]}, {t, 0, 2Pi},
                         ViewPoint->{3, 2, 2}]
```

followed by a similar command to draw the graph of $z = \sqrt{4 - r^2}$ over the same ranges for t and θ. Use **Show** to combine the two graphs and sketch the result in the box at right. How would you describe in words the shape of this region?



3b. The triple integral $\iiint_Q z e^{\sqrt{x^2+y^2}}\, dV$ would be written $\int_0^{2\pi} \int_0^1 \int_r^{\sqrt{4-r^2}} rze^r\, dz dr d\theta$ when converted to cylindrical cöordinates. Give both an exact and a decimal value for this integral.

4a. We can draw the "roof" of the solid in Exercise 43, Section 13.7 using another command from the package that was loaded in Question **1a**. Because $z = \rho\cos\phi$ in spherical cöordinates, the equation $x^2 + y^2 + z^2 = 4z$ is equivalent to $\rho = 4\cos\phi$; execute (still using **t** for θ)

```
SphericalPlot3D[4Cos[phi], {phi, 0, Pi/4}, {t, 0, 2Pi},
                ViewPoint->{3, 2, 2}]
```

and describe the result below.

4b. How would you describe the "floor" of the solid?

4c. Set up an integral giving the volume of the solid, and then evaluate it, recording below both the exact and approximate decimal values; use **rho** or some other convenient label to represent ρ in *Mathematica*.