Assignment 7: Limits, Part III (1.7) **Name____________________**
Please provide a handwritten response.

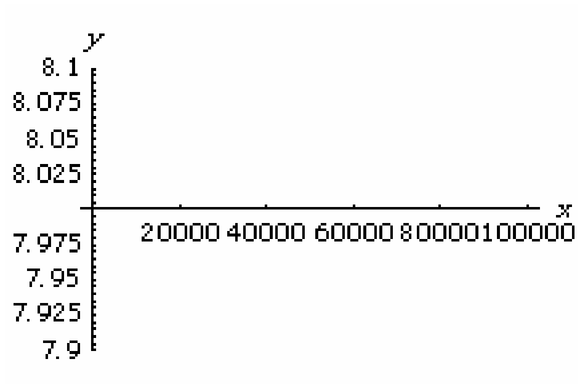
1a. Example 7.1 uses the function $f(x)=\dfrac{(x^3+4)^2-x^6}{x^3}$ to illustrate the dangers of loss of significance errors. Execute the command

```
f[x_] = ((x^3 + 4)^2 - x^6)/x^3
```

to define f and then execute the command

```
Plot[f[x], {x, 10000, 100000}]
```

to reproduce Figure 1.57a. Sketch the result on the axes at right. Does this graph give any indication of the value of $\lim_{x\to\infty} f(x)$? Explain.



1b. Next, execute the commands `f[1000.]` , `f[10000.]` , etc. to complete the table at right.

x	$f(x)$
1000	
10000	
100000	
1000000	
10000000	

1c. Now execute the command

```
Limit[f[x], x -> Infinity]
```

and record the result below. Is it likely that all of these results are correct? Which ones are not?

1d. Remark 7.1 points out that f can be rewritten as $f(x)=\dfrac{8x^3+16}{x^3}$; execute the commands `Clear[f]` and

```
f[x_] = (8x^3 + 16)/x^3
```

and then complete the table at right with this new (but equivalent) formula for f . Do you think these new results are more trustworthy?

x	$f(x)$
1000	
10000	
100000	
1000000	
10000000	

2. Scientific notation is used to write very large or very small numbers in a convenient form; for example, .000000000002673 would be written in scientific notation as 2.673×10^{-12} . In *Mathematica* execute the command **`2.673*10^(-12)`** and record the result below.

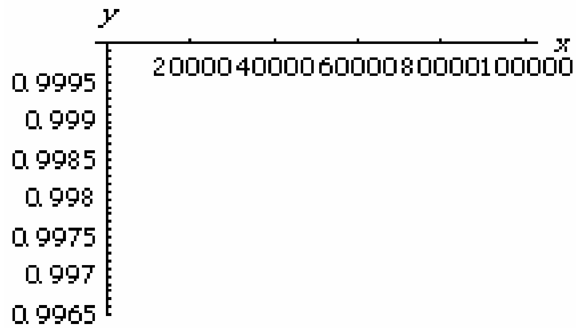
3a. In Exercise 3 we seek a value of x for which loss of significance occurs in $\lim\limits_{x\to\infty}\sqrt{x}\left(\sqrt{x+4}-\sqrt{x+2}\right)$. Define $g(x)=\sqrt{x}\left(\sqrt{x+4}-\sqrt{x+2}\right)$ by executing the command

```
g[x_] = Sqrt[x] (Sqrt[x + 4] - Sqrt[x + 2])
```

Then execute the command

```
Plot[g[x], {x, 0, 100000}]
```

and sketch the result on the axes at right. Based on this graph, what value would you give for $\lim\limits_{x\to\infty}\sqrt{x}\left(\sqrt{x+4}-\sqrt{x+2}\right)$?



3b. Next, execute the commands **`g[1.*10^13]`** , **`g[1.*10^14]`** , etc. to complete the table at right. Where does loss of significance occur?

x	$g(x)$
1×10^{13}	
1×10^{14}	
1×10^{15}	
1×10^{16}	
1×10^{17}	

3c. We can rewrite g to avoid loss of significance; you can check that multiplying $g(x)$ by $\dfrac{\sqrt{x+4}+\sqrt{x+2}}{\sqrt{x+4}+\sqrt{x+2}}$ gives $\dfrac{2\sqrt{x}}{\sqrt{x+4}+\sqrt{x+2}}$. Enter the commands **`Clear[g]`** and

```
g[x_] = 2Sqrt[x]/(Sqrt[x + 4] + Sqrt[x + 2])
```

Then complete the table at right just as in part **b**. Do these results seem more reliable?

x	$g(x)$
1×10^{13}	
1×10^{14}	
1×10^{15}	
1×10^{16}	
1×10^{17}	

3d. Finally, execute the command

```
Limit[g[x], x->Infinity]
```

and record the result below. Does it seem to be correct?

3e. Repeat parts **a** and **b** for Exercise 6 and record below a value of x at which loss of significance occurs.