**Assignment 9: Implicit Differentiation (2.8)** **Name____________________**
**Please provide a handwritten response.**

**1a.** The implicit function $x^2y^2 - 2x = 4 - 4y$ from Example 8.2 can be entered into *Mathematica* by executing

```
eqn = x^2*y[x]^2 - 2x == 4 - 4y[x]
```

Record the result below. (Be careful! The <u>double</u> equal sign **==** is used within the equation itself, whereas the <u>single</u> equal sign **=** is used to assign the label **eqn** to the entire equation. Also, whereas *x* is simply entered as **x** in the command above, *y* must be entered as **y[x]** , to make it clear to *Mathematica* that *y* is to be considered as a function of *x* in this equation.)

**1b.** We can "take the derivative of both sides with respect to *x*" by executing the command **deriv = D[eqn, x]** ; record the result below. (We will use the "differentiation operator" **D** to find more derivatives later.)

**1c.** We can solve this equation for the desired $y'(x)$ by executing

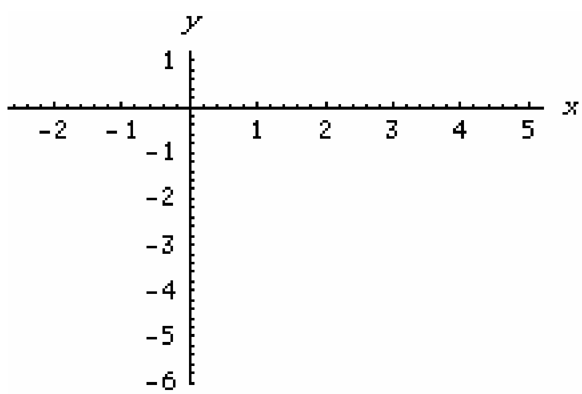
```
yprime = Solve[deriv, y'[x]]
```

Record the result below; does it agree with Example 8.2 so far?

**2.** *Mathematica* can draw the graph of our equation, but first we must provide some extra capability by loading in a "package". Execute the command

```
Needs["Graphics`ImplicitPlot`"]
```

to load in the **ImplicitPlot** package. (Be careful — the **`** character is <u>not</u> the single–quote, but rather is located to the left of the "1" on your keyboard; however, the **"** character <u>is</u> the double–quote.) Now execute




```
curve = ImplicitPlot[x^2*y^2 - 2x == 4 - 4y, {x, -5, 5}]
```

and sketch the result on the axes. (Notice that here we must refer to *y* simply as **y** , not as **y[x]** as we did above; that's just how *Mathematica* works.) Does your graph look like Figure 2.41 so far?

**3a.** In Example 8.2 a tangent line is drawn to the graph at the point $(2,-2)$; using *Mathematica*, however, we are free to use any value of *x* we wish, for example $x = 2.235$. Execute the command

```
eqn/. x->2.235
```

and record the result below. How was **`eqn`** changed by the replacement **`/. x->2.235`** ?

Now execute the command

```
Solve[eqn/. x->2.235, y[2.235]]
```

and record the result below. How many points on this curve satisfy $x = 2.235$ ? Mark them with dots on the curve you drew in Question **2**, and label their cöordinates clearly.

**3b.** One of the *y*–values you found in part **a** is $-1.76271$; based on your graph in Question **2**, would you expect $y'$ to be positive or negative at the point $(2.235, -1.76271)$ ? About how large would you expect $y'$ to be? Why?

**3c.** Execute the command

```
yprime/.{x->2.235, y[x]->-1.76271}
```

to replace `x` and `y[x]` in **`yprime`** with the appropriate values, which will give the exact value of $y'$ at the point $(2.235, -1.76271)$. Record the result below.

**3d.** Since we found that $y' = 0.873528$ in part **c**, an equation of the tangent line to our curve at the point $(2.235, -1.76271)$ is given by $y = 0.873528(x - 2.235) - 1.76271$; execute

```
t[x_] = 0.873528(x - 2.235) - 1.76271
```

and then graph the tangent line by executing

```
tanline = Plot[t[x], {x, -2.5, 5}]
```

**3e.** Finally, we can use the **`Show`** command to draw the curve and the tangent line together. Execute

```
Show[curve, tanline]
```

and sketch the result on the axes at right.