5

Kinetic Energy, Work, and Power

FIGURE 5.1 A composite image of NASA satellite photographs taken at night. Photos were taken from November 1994 through March 1995.

Figure 5.1 is a composite image of satellite photographs taken at night, showing which parts of the world use the most energy for nighttime illumination. Not surprisingly, the United States, Western Europe, and Japan stand out. The amount of light emitted by a region during the night is a good measure of the amount of energy that region consumes.

In physics, energy has a fundamental significance: Practically no physical activity takes place without the expenditure or transformation of energy. Calculations involving the energy of a system are of primary importance in all of science and engineering. As we'll see in this chapter, problem-solving methods involving energy provide an alternative to working with Newton's laws and are often simpler and easier to use.

This chapter presents the concepts of kinetic energy, work, and power and introduces some techniques that use these ideas, such as the work–kinetic energy theorem, to solve several types of problems. Chapter 6 will introduce additional types of energy and expand the work–kinetic energy theorem to cover these; it will also discuss one of the great ideas in physics and, indeed, in all of science: the law of conservation of energy.

WHAT WE WILL LEARN

- Kinetic energy is the energy associated with the motion of an object.
- Work is energy transferred to an object or from an object as the result of the action of an external force. Positive work transfers energy to the object, and negative work transfers energy from the object.
- Work is the scalar product of the force vector and the displacement vector.
- The change in kinetic energy due to applied forces is equal to the work done by the forces.
- Power is the rate at which work is done.
- The power provided by a force acting on an object is the scalar product of the velocity vector for that object and the force vector.

5.1 Energy in Our Daily Lives

No physical quantity has a greater importance in our daily lives than energy. Energy consumption, energy efficiency, and energy "production" are of the utmost economic importance and are the focus of heated discussions about national policies and international agreements. (The word *production* is in quotes because energy is not produced but rather is converted from a less usable form to a more usable form.) Energy also has an important role in each individual's daily routine: energy intake through food calories and energy consumption through cellular processes, activities, work, and exercise. Weight loss or weight gain is ultimately due to an imbalance between energy intake and use.

Energy has many forms and requires several different approaches to cover completely. Thus, energy is a recurring theme throughout this book. We start in this chapter and the next by investigating forms of mechanical energy: kinetic energy and potential energy. But as we progress through the topics in this book you will see that other forms of energy are playing very important roles as well. Thermal energy is one of the central pillars of thermodynamics. Chemical energy is stored in chemical compounds, and chemical reactions can either consume energy from the environment (endothermic reactions) or yield usable energy to the surroundings (exothermic reactions). Our petroleum economy makes use of chemical energy and its conversion to mechanical energy and heat, which is another form of energy (or energy transfer).

In Chapter 31, we will see that electromagnetic radiation contains energy. This energy is the basis for one renewable form of energy—solar energy. Almost all other renewable energy sources on Earth can be traced back to solar energy. Solar energy is responsible for the wind that drives large wind turbines (Figure 5.2). The Sun's radiation is also responsible for evaporating water from the Earth's surface and moving it into the clouds, from which it falls down as rain and eventually joins rivers that can be dammed (Figure 5.3) to extract energy. Biomass, another renewable energy resource, depends on the ability of plants and animals to store solar energy during their metabolic and growth processes.

FIGURE 5.2 Wind farms harvest renewable energy.

(a) (b) (c)

FIGURE 5.3 Dams provide renewable electrical energy. (a) The Grand Coulee Dam on the Columbia River in Washington. (b) The Itaipú Dam on the Paraná River in Brazil and Paraguay. (c) The Three Gorges Dam on the Yangtze River in China.

(a)

(b)

FIGURE 5.4 (a) Solar farm with an adjustable array of mirrors; (b) solar panel.

In fact, the energy radiated onto the surface of Earth by the Sun exceeds the energy needs of the entire human population by a factor of more than 10,000. It is possible to convert solar energy directly into electrical energy by using photovoltaic cells (Figure 5.4b). Currently, intense research efforts are focused on increasing the efficiency and reliability of these photocells while reducing their cost. Versions of solar cells are already being used for some practical purposes, for example, in patio and garden lights. Experimental solar farms like the one in Figure 5.4a are in operation as well. The chapter on quantum physics (Chapter 36) will discuss in detail how photocells work. Problems with using solar energy are that it is not available at night, has seasonal variations, and is strongly reduced in cloudy conditions or bad weather. Depending on the installation and conversion methods used, present solar devices convert only 10–15% of solar energy into electrical energy; increasing this fraction is a key goal of research activity. Materials with a 30% or higher yield of electrical energy from solar energy have been developed in the laboratory but are still not deployed on an industrial scale. Using biomass to generate electricity, in comparison, has much lower efficiencies of solar energy capture, on the order of 1% or less.

The Earth itself contains useful energy in the form of heat, which we can harvest using geothermal power plants. Iceland satisfies about half of its energy needs from this resource. The world's largest geothermal power plant complex, the Geysers, is located in Northern California and provides approximately 60% of that region's electricity. Also, the currents, tides, and waves of Earth's oceans can be exploited to extract useful energy. These and other alternative energy resources are the focus of intensive research, and the near future will see impressive developments.

In Chapter 35, on relativity, we will see that energy and mass are not totally separate concepts but are related to each other via Einstein's famous formula $E = mc^2$. When we study nuclear physics (Chapter 40), we will find that splitting massive atomic nuclei (such as uranium or plutonium) liberates energy. Conventional nuclear power plants are based on this physical principle, called *nuclear fission.* We can also obtain useful energy by merging atomic nuclei with very small masses (hydrogen, for example) to form more massive nuclei, a process called *nuclear fusion.* The Sun and all other stars in the universe use nuclear fusion to generate energy.

The energy from nuclear fusion is thought by many to be the most likely means of satisfying the long-term energy needs of modern industrialized society. Perhaps the most likely approach to achieving progress toward controlled fusion reactions is the proposed international nuclear fusion reactor facility ITER ("the way" in Latin), which will be constructed in France. But there are other promising approaches to solving the problem of how to use nuclear fusion, for example, the National Ignition Facility (NIF) opened in May 2009 at Lawrence Livermore National Laboratory in California. We will discuss these technologies in greater detail in Chapter 40.

Related to energy are work and power. We all use these words informally, but this chapter will explain how these quantities relate to energy in precise physical and mathematical terms.

You can see that energy occupies an essential place in our lives. One of the goals of this book is to give you a solid grounding in the fundamentals of energy science. Then you will be able to participate in some of the most important policy discussions of our time in an informed manner.

A final question remains: What is energy? In many textbooks, energy is defined as the ability to do work. However, this definition only shifts the mystery without giving a deeper explanation. And the truth is that there is no deeper explanation. In his famous *Feynman Lectures on Physics,* the Nobel laureate and physics folk hero Richard Feynman wrote in 1963: "It is important to realize that in physics today, we have no knowledge of what energy *is.* We do not have a picture that energy comes in little blobs of a definite amount. It is not that way. However, there are formulas for calculating some numerical quantity, and when we add it all together it gives '28'—always the same number. It is an abstract thing in that it does not tell us the mechanism or the *reasons* for the various formulas." Five decades later, this has not changed. The concept of energy and, in particular, the law of energy conservation (see Chapter 6), are extremely useful tools for figuring out the behavior of systems. But no one has yet given an explanation as to the true nature of energy.

5.2 Kinetic Energy

The first kind of energy we'll consider is the energy associated with the motion of a moving object: **kinetic energy.** Kinetic energy is defined as one-half the product of a moving object's mass and the square of its speed:

$$K = \tfrac{1}{2}mv^2. \quad (5.1)$$

Note that, by definition, kinetic energy is always positive or equal to zero, and it is only zero for an object at rest. Also note that kinetic energy, like all forms of energy, is a scalar, not a vector, quantity. Because it is the product of mass (kg) and speed squared (m/s·m/s), the units of kinetic energy are kg m^2/s^2. Because energy is such an important quantity, it has its own SI unit, the **joule (J).** The SI force unit, the newton, is 1 N = 1 kg m/s^2, and we can make a useful conversion:

$$\text{Energy unit:} \quad 1\text{ J} = 1\text{ N m} = 1\text{ kg m}^2/\text{s}^2. \quad (5.2)$$

Let's look at a few sample energy values to get a feeling for the size of the joule. A car of mass 1310 kg being driven at the speed limit of 55 mph (24.6 m/s) has a kinetic energy of

$$K_{\text{car}} = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(1310\text{ kg})(24.6\text{ m/s})^2 = 4.0\cdot 10^5\text{ J}.$$

The mass of the Earth is $6.0\cdot 10^{24}$ kg, and it orbits the Sun with a speed of $3.0\cdot 10^4$ m/s. The kinetic energy associated with this motion is $2.7\cdot 10^{33}$ J. A person of mass 64.8 kg jogging at 3.50 m/s has a kinetic energy of 397 J, and a baseball (mass of "5 ounces avoirdupois" = 0.142 kg) thrown at 80. mph (35.8 m/s) has a kinetic energy of 91 J. On the atomic scale, the average kinetic energy of an air molecule is $6.1\cdot 10^{-21}$ J, as we will see in Chapter 19. The typical magnitudes of kinetic energies of some moving objects are presented in Figure 5.5. You can see from these examples that the range of energies involved in physical processes is tremendously large.

Some other frequently used energy units are the electron-volt (eV), the food calorie (Cal), and the mega-ton of TNT (Mt):

$$1\text{ eV} = 1.602\cdot 10^{-19}\text{ J}$$
$$1\text{ Cal} = 4186\text{ J}$$
$$1\text{ Mt} = 4.18\cdot 10^{15}\text{ J}.$$

On the atomic scale, 1 electron-volt (eV) is the kinetic energy that an electron gains when accelerated by an electric potential of 1 volt. The energy content of the food we eat is usually (and mistakenly) given in terms of calories but should be given in food calories. As we'll see when we study thermodynamics, 1 food calorie is equal to 1 kilocalorie; a nice round number to remember is that about 10 MJ (~2500 food calories) of energy is stored in the food we eat each day. On a larger scale, 1 Mt is the energy released by exploding 1 million metric tons of the explosive TNT, an energy release achieved only by nuclear weapons or by catastrophic natural events such as the impact of a large asteroid. For comparison, in 2007,

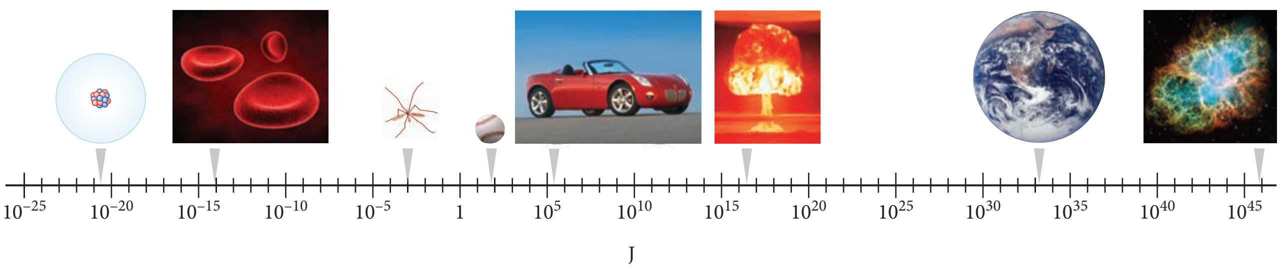


FIGURE 5.5 Range of kinetic energies displayed on a logarithmic scale. The kinetic energies (left to right) of an air molecule, a red blood cell traveling through the aorta, a mosquito in flight, a thrown baseball, a moving car, and the Earth orbiting the Sun are compared with the energy released from a 15-Mt nuclear explosion and by a supernova, which emits particles with a total kinetic energy of approximately 10^{46} J.

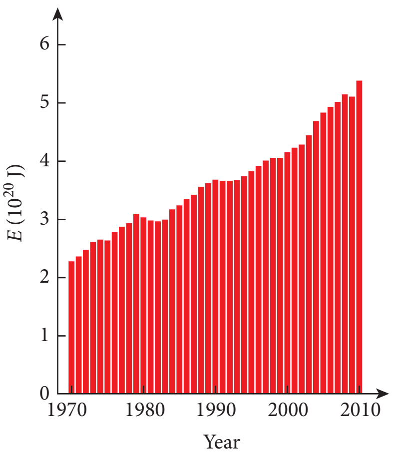


FIGURE 5.6 Total annual global energy consumption by humans from 1970 to 2010. (Data sources: US Energy Information Agency and yearbook.enerdata.net).

the annual energy consumption by all humans on Earth reached $5 \cdot 10^{20}$ J (see Figure 5.6). (All of these concepts will be discussed further in subsequent chapters.)

For motion in more than one dimension, we can write the total kinetic energy as the sum of the kinetic energies associated with the components of velocity in each spatial direction. To show this, we start with the definition of kinetic energy (equation 5.1) and then use $v^2 = v_x^2 + v_y^2 + v_z^2$:

$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}m\left(v_x^2 + v_y^2 + v_z^2\right) = \tfrac{1}{2}mv_x^2 + \tfrac{1}{2}mv_y^2 + \tfrac{1}{2}mv_z^2. \quad (5.3)$$

(*Note:* Kinetic energy is a scalar, so these components are not added like vectors but simply by taking their algebraic sum.) Thus, we can think of kinetic energy as the sum of the kinetic energies associated with the motion in the x-direction, y-direction, and z-direction. This concept is particularly useful for ideal projectile problems, where the motion consists of free fall in the vertical direction (y-direction) and motion with constant velocity in the horizontal direction (x-direction).

EXAMPLE 5.1 Falling Vase

PROBLEM

A crystal vase (mass = 2.40 kg) is dropped from a height of 1.30 m and falls to the floor, as shown in Figure 5.7. What is its kinetic energy just before impact? (Neglect air resistance for now.)

SOLUTION

Once we know the velocity of the vase just before impact, we can put it into the equation defining kinetic energy. To obtain this velocity, we recall the kinematics of free-falling objects. In this case, it is most straightforward to use the relationship between the initial and final velocities and heights that we derived in Chapter 2 for free-fall motion:

$$v_y^2 = v_{y0}^2 - 2g(y - y_0).$$

(Remember that the y-axis must be pointing up to use this equation.) Because the vase is released from rest, the initial velocity components are $v_{x0} = v_{y0} = 0$. Because there is no acceleration in the x-direction, the x-component of velocity remains zero during the fall of the vase: $v_x = 0$. Therefore, we have

$$v^2 = v_x^2 + v_y^2 = 0 + v_y^2 = v_y^2.$$

We then obtain

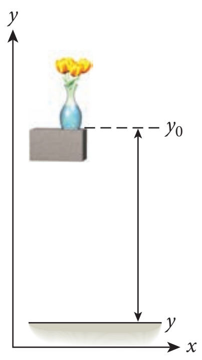
$$v^2 = v_y^2 = 2g(y_0 - y).$$

We use this result in equation 5.1:

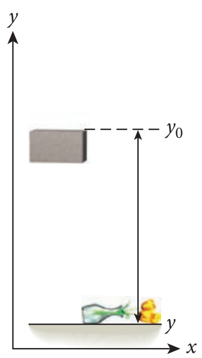
$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}m\big(2g(y_0 - y)\big) = mg(y_0 - y).$$

Inserting the numbers given in the problem statement gives us the answer:

$$K = (2.40 \text{ kg})(9.81 \text{ m/s}^2)(1.30 \text{ m}) = 30.6 \text{ J}.$$



(a)



(b)

FIGURE 5.7 (a) A vase is released from rest at a height of y_0. (b) The vase falls to the floor, which has a height of y.

5.3 Work

In Example 5.1, the vase started out with zero kinetic energy, just before it was released. After falling a distance of 1.30 m, it had acquired a kinetic energy of 30.6 J. The greater the height from which the vase is released, the greater the speed the vase will attain (ignoring air resistance), and therefore the greater its kinetic energy becomes. In fact, as we found in Example 5.1, the kinetic energy of the vase depends linearly on the height from which it falls: $K = mg(y_0 - y)$.

The gravitational force, $\vec{F}_g = -mg\hat{y}$, accelerates the vase and therefore gives it its kinetic energy. We can see from the equation we used in Example 5.1 that the kinetic energy also depends linearly on the magnitude of the gravitational force. Doubling the mass of the vase would double the gravitational force acting on it and thus double its kinetic energy.

Because the speed of an object can be increased or decreased by accelerating or decelerating it, respectively, its kinetic energy also changes in this process. For the vase, we have just seen that the force of gravity is responsible for this change. We account for a change in the kinetic energy of an object caused by a force with the concept of work, W.

Definition

Work is the energy transferred to or from an object as the result of the action of a force. Positive work is a transfer of energy to the object, and negative work is a transfer of energy from the object.

The vase gained kinetic energy from positive work done by the gravitational force and so $W_g = mg(y_0 - y)$.

Note that this definition is not restricted to kinetic energy. The relationship between work and energy described in this definition holds in general for different forms of energy besides kinetic energy. This definition of work is not exactly the same as the meaning attached to the word *work* in everyday language. The work being considered in this chapter is mechanical work in connection with energy transfer. However, the work, physical as well as mental, that we commonly speak of does not necessarily involve the transfer of energy.

5.4 Work Done by a Constant Force

Suppose we let the vase of Example 5.1 slide, from rest, along an inclined plane that has an angle θ with respect to the horizontal (Figure 5.8). For now, we neglect the friction force, but we will come back to it later. As we showed in Chapter 4, in the absence of friction, the acceleration along the plane is given by $a = g \sin\theta = g \cos\alpha$. (Here the angle $\alpha = 90° - \theta$ is the angle between the gravitational force vector and the displacement vector; see Figure 5.8.)

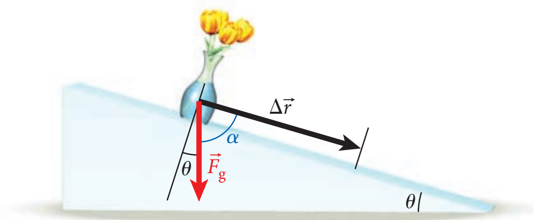


FIGURE 5.8 Vase sliding without friction on an inclined plane.

We can determine the kinetic energy the vase has in this situation as a function of the displacement, $\Delta\vec{r}$. Most conveniently, we can perform this calculation by using the relationship between the squares of initial and final velocities, the displacement, and the acceleration, which we obtained for one-dimensional motion in Chapter 2:

$$v^2 = v_0^2 + 2a\Delta r.$$

We set $v_0 = 0$ because we are again assuming that the vase is released from rest, that is, with zero kinetic energy. Then we use the expression for the acceleration, $a = g\cos\alpha$, that we just obtained. Now we have

$$v^2 = (2g\cos\alpha)\Delta r \Rightarrow K = \tfrac{1}{2}mv^2 = mg\Delta r\cos\alpha.$$

The kinetic energy transferred to the vase was the result of positive work done by the gravitational force, and so

$$\Delta K = mg\Delta r\cos\alpha = W_g. \qquad (5.4)$$

Let's look at two limiting cases for equation 5.4:

- For $\alpha = 0$, both the gravitational force and the displacement are in the negative y-direction. Thus, these vectors are parallel, and we have the result we already derived for the case of the vase falling under the influence of gravity, $W_g = mg\Delta r$.
- For $\alpha = 90°$, the gravitational force is still in the negative y-direction, but the vase cannot move in the negative y-direction because it is sitting on the horizontal surface of the plane. Hence, there is no change in the kinetic energy of the vase, and there is no work done by the gravitational force on the vase; that is, $W_g = 0$. The work done on the vase by the gravitational force is also zero if the vase moves at a constant speed along the surface of the plane.

Self-Test Opportunity 5.1

Draw the free-body diagram for the vase that is sliding down the inclined plane.

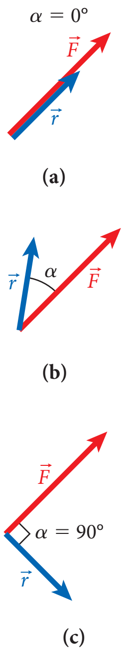
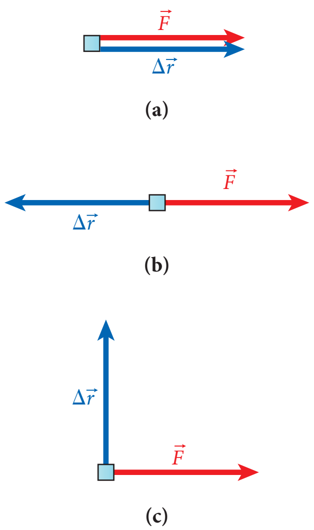


FIGURE 5.9 (a) $\vec{F}$ is parallel to $\vec{r}$ and $W = |\vec{F}||\vec{r}|$. (b) The angle between $\vec{F}$ and $\vec{r}$ is α and $W = |\vec{F}||\vec{r}|\cos\alpha$. (c) $\vec{F}$ is perpendicular to $\vec{r}$ and $W = 0$.

Concept Check 5.1

Consider an object undergoing a displacement $\Delta\vec{r}$ and experiencing a force $\vec{F}$. In which of the three cases shown below is the work done by the force on the object zero?



Because $mg = |\vec{F}_g|$ and $\Delta r = |\Delta\vec{r}|$, we can write the work done on the vase as $W = |\vec{F}||\Delta\vec{r}|\cos\alpha$. From the two limiting cases we have just discussed, we gain confidence that we can use the equation we have just derived for motion on an inclined plane as the definition of the work done by a constant force:

$$W = |\vec{F}||\Delta\vec{r}|\cos\alpha, \quad \text{where } \alpha \text{ is the angle between } \vec{F} \text{ and } \Delta\vec{r}.$$

This equation for the work done by a constant force acting over some spatial displacement holds for all constant force vectors, arbitrary displacement vectors, and angles between the two. Figure 5.9 shows three cases for the work done by a force $\vec{F}$ acting over a displacement $\vec{r}$. In Figure 5.9a, the maximum work is done because $\alpha = 0$ and $\vec{F}$ and $\vec{r}$ are in the same direction. In Figure 5.9b, $\vec{F}$ is at an arbitrary angle α with respect to $\vec{r}$. In Figure 5.9c, no work is done because $\vec{F}$ is perpendicular to $\vec{r}$.

Using a scalar product (see Section 1.6), we can write the work done by a constant force as

$$W = \vec{F} \bullet \Delta\vec{r}. \tag{5.5}$$

This equation is the main result of this section. It says that the work done by a constant force $\vec{F}$ in displacing an object by $\Delta\vec{r}$ is the scalar product of the two vectors. In particular, if the displacement is perpendicular to the force, the scalar product is zero, and no work is done.

Note that we can use any force vector and any displacement vector in equation 5.5. If there is more than one force acting on an object, the equation holds for any of the individual forces, and it holds for the net force. The mathematical reason for this generalization lies in the distributive property of the scalar product. To verify this statement, we can look at a constant net force that is the sum of individual constant forces, $\vec{F}_{\text{net}} = \sum_i \vec{F}_i$. According to equation 5.5, the work done by this net force is

$$W_{\text{net}} = \vec{F}_{\text{net}} \bullet \Delta\vec{r} = \left(\sum_i \vec{F}_i\right) \bullet \Delta\vec{r} = \sum_i \left(\vec{F}_i \bullet \Delta\vec{r}\right) = \sum_i W_i.$$

In other words, the net work done by the net force is equal to the sum of the work done by the individual forces. We have demonstrated this additive property of work only for constant forces, but it is also valid for variable forces (or, strictly speaking, only for conservative forces, which we'll encounter in Chapter 6). But to repeat the main point: Equation 5.5 is valid for each individual force as well as the net force. We will typically consider the net force when calculating the work done on an object, but we will omit the index "net" to simplify the notation.

One-Dimensional Case

In all cases of motion in one dimension, the work done to produce the motion is given by

$$\begin{aligned} W &= \vec{F} \bullet \Delta\vec{r} \\ &= \pm F_x \cdot |\Delta\vec{r}| = F_x \Delta x \\ &= F_x (x - x_0). \end{aligned} \tag{5.6}$$

The force $\vec{F}$ and displacement $\Delta\vec{r}$ can point in the same direction, $\alpha = 0 \Rightarrow \cos\alpha = 1$, resulting in positive work, or they can point in opposite directions, $\alpha = 180° \Rightarrow \cos\alpha = -1$, resulting in negative work.

Work–Kinetic Energy Theorem

The relationship between kinetic energy of an object and the work done by the force(s) acting on it, called the **work–kinetic energy theorem,** is expressed formally as

$$\Delta K \equiv K - K_0 = W. \tag{5.7}$$

Here, K is the kinetic energy that an object has after work W has been done on it and K_0 is the kinetic energy before the work is done. The definitions of W and K are such that equation 5.7 is equivalent to Newton's Second Law. To see this equivalence, consider a constant force acting in one dimension on an object of mass m. Newton's Second Law is then $F_x = ma_x$, and the (also constant!) acceleration, a_x, of the object is related to the difference in the squares of its initial and final velocities via $v_x^2 - v_{x0}^2 = 2a_x(x - x_0)$, which is one of the five kinematical equations we derived in Chapter 2. Multiplication of both sides of this equation by $\frac{1}{2}m$ yields

$$\tfrac{1}{2}mv_x^2 - \tfrac{1}{2}mv_{x0}^2 = ma_x(x - x_0) = F_x\Delta x = W. \tag{5.8}$$

Thus, we see that, for this one-dimensional case, the work–kinetic energy theorem is equivalent to Newton's Second Law.

Because of the equivalence we have just established, if more than one force is acting on an object, we can use the net force to calculate the work done. Alternatively, and more commonly in energy problems, if more than one force is acting on an object, we can calculate the work done by each force, and then W in equation 5.7 represents their sum.

The work–kinetic energy theorem specifies that the change in kinetic energy of an object is equal to the work done on the object by the forces acting on it. We can rewrite equation 5.7 to solve for K or K_0:

$$K = K_0 + W$$

or

$$K_0 = K - W.$$

By definition, the kinetic energy cannot be less than zero; so, if an object has $K_0 = 0$, the work–kinetic energy theorem implies that $K = K_0 + W = W \geq 0$.

While we have only verified the work–kinetic energy theorem for a constant force, it is also valid for variable forces, as we will see below. Is it valid for all kinds of forces? The short answer is no! Friction forces are one kind of force that violate the work–kinetic energy theorem. We will discuss this point further in Chapter 6.

Self-Test Opportunity 5.2

Show the equivalence between Newton's Second Law and the work–kinetic energy theorem for the case of a constant force acting in three-dimensional space.

Work Done by the Gravitational Force

With the work–kinetic energy theorem at our disposal, we can now take another look at the problem of an object falling under the influence of the gravitational force, as in Example 5.1. On the way down, the work done by the gravitational force on the object is

$$W_g = +mgh, \tag{5.9}$$

where $h = |y - y_0| = |\Delta\vec{r}| > 0$. The displacement $\Delta\vec{r}$ and the force of gravity $\vec{F}_g$ point in the same direction, resulting in a positive scalar product and therefore positive work. This situation is illustrated in Figure 5.10a. Since the work is positive, the gravitational force increases the kinetic energy of the object.

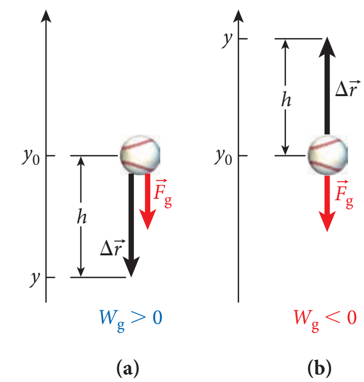


FIGURE 5.10 Work done by the gravitational force. (a) The object during free fall. (b) Tossing an object upward.

We can reverse this situation and toss the object vertically upward, making it a projectile and giving it an initial kinetic energy. This kinetic energy will decrease until the projectile reaches the top of its trajectory. During this time, the displacement vector $\Delta\vec{r}$ points up, in the opposite direction to the force of gravity (Figure 5.10b). Thus, the work done by the gravitational force during the object's upward motion is

$$W_g = -mgh. \tag{5.10}$$

Therefore, the work done by the gravitational force reduces the kinetic energy of the object during its upward motion. This conclusion is consistent with the general formula for work done by a constant force, $W = \vec{F} \bullet \Delta\vec{r}$, because the displacement (pointing upward) of the object and the gravitational force (pointing downward) are in opposite directions.

Work Done in Lifting and Lowering an Object

Now let's consider the situation in which a vertical external force is applied to an object—for example, by attaching the object to a rope and lifting it up or lowering it down. The work–kinetic energy theorem now has to include the work done by the gravitational force, W_g, and the work done by the external force, W_F:

$$K - K_0 = W_g + W_F.$$

For the case where the object is at rest both initially, $K_0 = 0$, and finally, $K = 0$, we have

$$W_F = -W_g.$$

The work done by force in lifting or lowering the object is then

$$W_F = -W_g = mgh \text{ (for lifting) or } W_F = -W_g = -mgh \text{ (for lowering).} \tag{5.11}$$

EXAMPLE 5.2 Weightlifting

In the sport of weightlifting, the task is to pick up a very large mass, lift it over your head, and hold it there at rest for a moment. This action is an example of doing work by lifting and lowering a mass.

PROBLEM 1

The German lifter Ronny Weller won the silver medal at the Olympic Games in Sydney, Australia, in 2000. He lifted 257.5 kg in the "clean and jerk" competition. Assuming he lifted the mass to a height of 1.83 m and held it there, what was the work he did in this process?

SOLUTION 1

This problem is an application of equation 5.11 for the work done against the gravitational force. The work Weller did was

$$W = mgh = (257.5 \text{ kg})(9.81 \text{ m/s}^2)(1.83 \text{ m}) = 4.62 \text{ kJ}.$$

PROBLEM 2

Once Weller successfully completed the lift and was holding the mass with outstretched arms above his head, what was the work done by him in lowering the weight slowly (with negligible kinetic energy) back down to the ground?

SOLUTION 2

This calculation is the same as that in Solution 1 except the sign of the displacement changes. Thus, the answer is that –4.62 kJ of work is done in bringing the weight back down—exactly the opposite of what we obtained for Problem 1!

Now is a good time to remember that we are dealing with strictly mechanical work. Every lifter knows that you can feel the muscles "burn" just as much when holding the mass overhead or lowering the mass (in a controlled way) as when lifting it. (In Olympic competitions, by the way, the weightlifters just drop the mass after the successful lift.) However, this physiological effect is not mechanical work, which is what we are presently interested in. Instead it is the conversion of chemical energy, stored in different molecules such as sugars, into the energy needed to contract the muscles.

You may think that Olympic weightlifting is not the best example to consider, because the force used to lift the mass is not constant. This is true, but as we discussed previously, the work–kinetic energy theorem applies to nonconstant forces. Additionally, even when a crane lifts a mass very slowly and with constant speed, the lifting force is still not exactly constant, because a slight initial acceleration is needed to get the mass from zero speed to a finite value and a deceleration occurs at the end of the lifting process.

Lifting with Pulleys

When we studied pulleys and ropes in Chapter 4, we learned that pulleys act as force multipliers. For example, with the setup shown in Figure 5.11, the force needed to lift a pallet of bricks of mass m by pulling on the rope is only half the gravitational force, $T = \frac{1}{2}mg$. How does the work done in pulling up the pallet of bricks with ropes and pulleys compare to the work of lifting it without such mechanical aids?

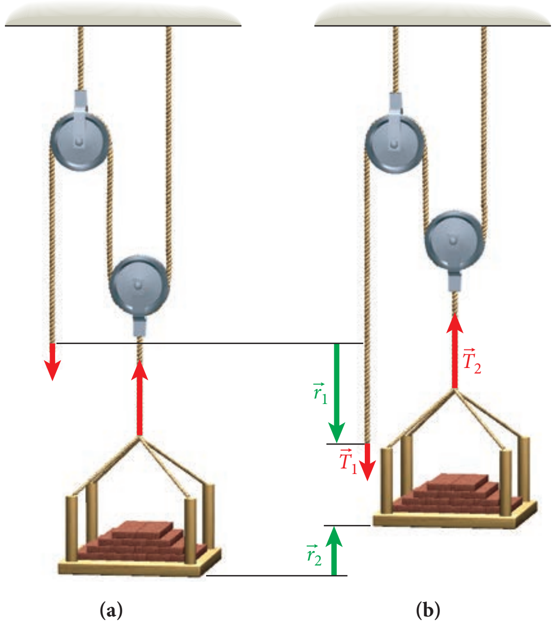


FIGURE 5.11 Forces and displacements for the process of lifting a pallet of bricks at a work site with the aid of a rope-and-pulley mechanism. (a) Pallet in the initial position. (b) Pallet in the final position.

Figure 5.11 shows the initial and final positions of the pallet of bricks and the ropes and pulleys used to lift it. Lifting it without mechanical aids would require the force $\vec{T}_2$, as indicated, whose magnitude is given by $T_2 = mg$. The work done by force $\vec{T}_2$ in this case is $W_2 = \vec{T}_2 \bullet \vec{r}_2 = T_2 r_2 = mgr_2$. Pulling on the rope with force $\vec{T}_1$ of magnitude $T_1 = \frac{1}{2}T_2 = \frac{1}{2}mg$ accomplishes the same thing. However, the displacement is then twice as long, $r_1 = 2r_2$, as you can see by examining Figure 5.11. Thus, the work done in this case is $W_1 = \vec{T}_1 \bullet \vec{r}_1 = (\frac{1}{2}T_2)(2r_2) = mgr_2 = W_2$.

The same amount of work is done in both cases. It is necessary to compensate for the reduced force by pulling the rope through a longer distance. This result is general for the use of pulleys or lever arms or any other mechanical force multiplier: The total work done is the same as it would be if the mechanical aid were not used. Any reduction in the force is always going to be compensated for by a proportional lengthening of the displacement.

We end this section with a solved problem, which we solve with kinetic energy concepts of Section 5.2. As a double-check step we use the concepts of work for a constant force, which we developed in the present section, showing that both paths lead to the same answer.

Concept Check 5.2

If you lift an object a distance h with the aid of a rope and n pulleys and do work W_h in the process, how much work will be required to lift the same object a distance $2h$?

a) W_h

b) $2W_h$

c) $0.5W_h$

d) nW_h

e) $2W_h/n$

SOLVED PROBLEM 5.1 Shot Put

PROBLEM

Shot put competitions use metal balls with a mass of 16 lb (7.26 kg). A competitor throws the shot at an angle of 43.3° and releases it from a height of 1.82 m above where it lands, and it lands a horizontal distance of 17.7 m from the point of release. What is the kinetic energy of the shot as it leaves the thrower's hand?

SOLUTION

THINK We are given the horizontal distance, $x_s = 17.7$ m, the height of release, $y_0 = 1.82$ m, and the angle of the initial velocity, $\theta_0 = 43.3°$, but not the initial speed, v_0. If we can figure out the initial speed from the given data, then calculating the initial kinetic energy will be straightforward because we also know the mass of the shot: $m = 7.26$ kg.

Because the shot is very heavy, air resistance can be safely ignored. This situation is an excellent realization of ideal projectile motion. After the shot leaves the thrower's hand, the

– *Continued*

only force on the shot is the force of gravity, and the shot will follow a parabolic trajectory until it lands on the ground. Thus, we'll solve this problem by application of the rules about ideal projectile motion.

SKETCH The trajectory of the shot is shown in Figure 5.12.

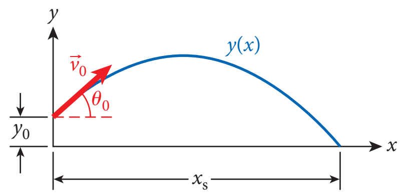


FIGURE 5.12 Parabolic trajectory of a thrown shot.

RESEARCH The initial kinetic energy K of the shot of mass m is given by

$$K = \tfrac{1}{2}mv_0^2.$$

Now we need to decide how to obtain v_0. We are given the distance, x_s, to where the shot hits the ground, but this is *not* equal to the range, R (for which we obtained a formula in Chapter 3), because the range formula assumes that the heights of the start and end of the trajectory are equal. Here the initial height of the shot is y_0, and the final height is zero. Therefore, we have to use the full expression for the trajectory of an ideal projectile from Chapter 3:

$$y = y_0 + x\tan\theta_0 - \frac{x^2 g}{2v_0^2\cos^2\theta_0}.$$

This equation describes the y-component of the trajectory as a function of the x-component.

In this problem, we know that $y(x = x_s) = 0$, that is, that the shot touches the ground at $x = x_s$. Substituting for x when $y = 0$ in the equation for the trajectory results in

$$0 = y_0 + x_s\tan\theta_0 - x_s^2\frac{g}{2v_0^2\cos^2\theta_0}.$$

SIMPLIFY We solve this equation for v_0^2:

$$y_0 + x_s\tan\theta_0 = \frac{x_s^2 g}{2v_0^2\cos^2\theta_0} \Rightarrow$$

$$2v_0^2\cos^2\theta_0 = \frac{x_s^2 g}{y_0 + x_s\tan\theta_0} \Rightarrow$$

$$v_0^2 = \frac{x_s^2 g}{2\cos^2\theta_0(y_0 + x_s\tan\theta_0)}.$$

Now, substituting for v_0^2 in the expression for the initial kinetic energy gives us

$$K = \tfrac{1}{2}mv_0^2 = \frac{mx_s^2 g}{4\cos^2\theta_0(y_0 + x_s\tan\theta_0)}.$$

CALCULATE Putting in the given numerical values, we get

$$K = \frac{(7.26\text{ kg})(17.7\text{ m})^2(9.81\text{ m/s}^2)}{4(\cos^2 43.3°)[1.82\text{ m} + (17.7\text{ m})(\tan 43.3°)]} = 569.295\text{ J}.$$

ROUND All of the numerical values given for this problem had three significant figures, so we report our answer as

$$K = 569\text{ J}.$$

DOUBLE-CHECK Since we have an expression for the initial speed, $v_0^2 = x_s^2 g/[2\cos^2\theta_0(y_0 + x_s\tan\theta_0)]$, we can find the horizontal and vertical components of the initial velocity vector:

$$v_{x0} = v_0\cos\theta_0 = 9.11\text{ m/s}$$
$$v_{y0} = v_0\sin\theta_0 = 8.59\text{ m/s}.$$

As we discussed in Section 5.2, we can split up the total kinetic energy in ideal projectile motion into contributions from the motion in horizontal and vertical directions (see equation 5.3). The kinetic energy due to the motion in the x-direction remains constant. The kinetic energy due to the motion in the y-direction is initially

$$\tfrac{1}{2}mv_{y0}^2 = 268\text{ J}.$$

At the top of the shot's trajectory, the vertical velocity component is zero, as in all projectile motion. This also means that the kinetic energy associated with the vertical motion is zero at this point. All 268 J of the initial kinetic energy due to the y-component of the motion has been used up to do work against the force of gravity. This work is (refer to equation 5.10) $-268\text{ J} = -mgh$, where $h = y_{max} - y_0$ is the maximum height of the trajectory. We thus find the value of h:

$$h = \frac{268\text{ J}}{mg} = \frac{268\text{ J}}{(7.26\text{ kg})(9.81\text{ m/s}^2)} = 3.76\text{ m}.$$

Let's use known concepts of projectile motion to find the maximum height for the initial velocity we have determined. In Section 3.4, the maximum height H of an object in projectile motion was shown to be

$$H = y_0 + \frac{v_{y0}^2}{2g}.$$

Putting in the numbers gives $v_{y0}^2/2g = 3.76$ m. This value is the same as that obtained by applying energy considerations.

5.5 Work Done by a Variable Force

Suppose the force acting on an object is not constant. What is the work done by such a force? In a case of motion in one dimension with a variable x-component of force, $F_x(x)$, the work is

$$W = \int_{x_0}^{x} F_x(x')dx'. \quad (5.12)$$

(The integrand has x' as a dummy variable to distinguish it from the integral limits.) Equation 5.12 shows that the work W is the area under the curve of $F_x(x)$ (see Figure 5.13 in the following derivation).

DERIVATION 5.1

If you have already taken integral calculus, you can skip this section. If equation 5.12 is your first exposure to integrals, the following derivation is a useful introduction. We'll derive the one-dimensional case and use our result for the constant force as a starting point.

In the case of a constant force, we can think of the work as the area under the horizontal line that plots the value of the constant force in the interval between x_0 and x. For a variable force, the work is the area under the curve $F_x(x)$, but that area is no longer a simple rectangle. In the case of a variable force, we need to divide the interval from x_0 to x into many small equal intervals. Then we approximate the area under the curve $F_x(x)$ by a series of rectangles and add their areas to approximate the work. As you can see from Figure 5.13a, the area of the rectangle between x_i and x_{i+1} is given by $F_x(x_i)(x_{i+1} - x_i) = F_x(x_i)\Delta x$. We obtain an approximation for the work by summing over all rectangles:

$$W \approx \sum_i W_i = \sum_i F_x(x_i)\Delta x.$$

Now we space the points x_i closer and closer by using more and more of them. This method makes Δx smaller and causes the total area of the series of rectangles to be a better approximation of the area under the curve $F_x(x)$, as shown in Figure 5.13b. In the limit as $\Delta x \to 0$, the sum approaches the exact expression for the work:

$$W = \lim_{\Delta x \to 0} \left(\sum_i F_x(x_i)\Delta x \right).$$

This limit of the sum of the areas is exactly how the integral is defined:

$$W = \int_{x_0}^{x} F_x(x')dx'.$$

We have derived this result for the case of one-dimensional motion. The derivation of the three-dimensional case proceeds along similar lines but is more involved in terms of algebra.

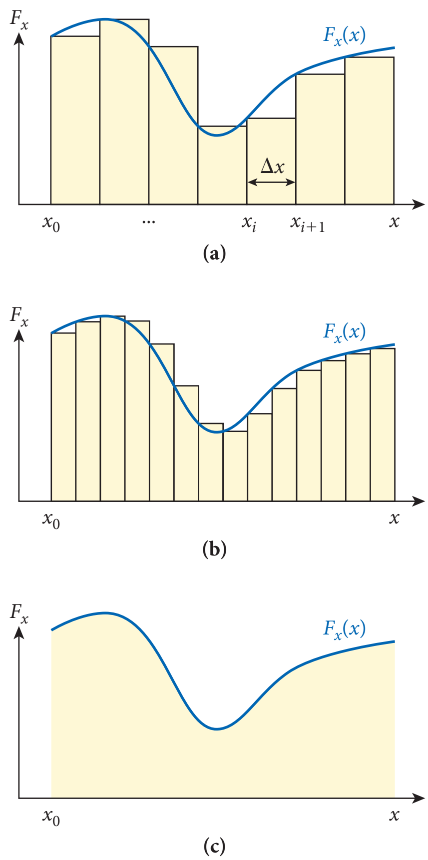


FIGURE 5.13 (a) A series of rectangles approximates the area under the curve obtained by plotting the force as a function of the displacement; (b) a better approximation using rectangles of smaller width; (c) the exact area under the curve.

As promised earlier, we can verify that the work–kinetic energy theorem (equation 5.7) is valid when the force is variable. We show this result for one-dimensional motion for simplicity, but the work–kinetic energy theorem also holds for variable forces and displacements in more than one dimension. We assume a variable force in the x-direction, $F_x(x)$, as in equation 5.12, which we can express as

$$F_x(x) = ma,$$

using Newton's Second Law. We use the chain rule of calculus to obtain

$$a=\frac{dv}{dt}=\frac{dv}{dx}\frac{dx}{dt}.$$

We can then use equation 5.12 and integrate over the displacement to get the work done:

$$W=\int_{x_0}^{x}F_x(x')dx'=\int_{x_0}^{x}ma\,dx'=\int_{x_0}^{x}m\frac{dv}{dx'}\frac{dx'}{dt}dx'.$$

We now change the variable of integration from displacement (x) to velocity (v):

$$W=\int_{x_0}^{x}m\frac{dx'}{dt}\frac{dv}{dx'}dx'=\int_{v_0}^{v}mv'dv'=m\int_{v_0}^{v}v'dv',$$

where v' is a dummy variable of integration. We carry out the integration and obtain the promised result:

$$W=m\int_{v_0}^{v}v'dv'=m\left[\frac{v'^2}{2}\right]_{v_0}^{v}=\frac{1}{2}mv^2-\frac{1}{2}mv_0^2=K-K_0=\Delta K.$$

5.6 Spring Force

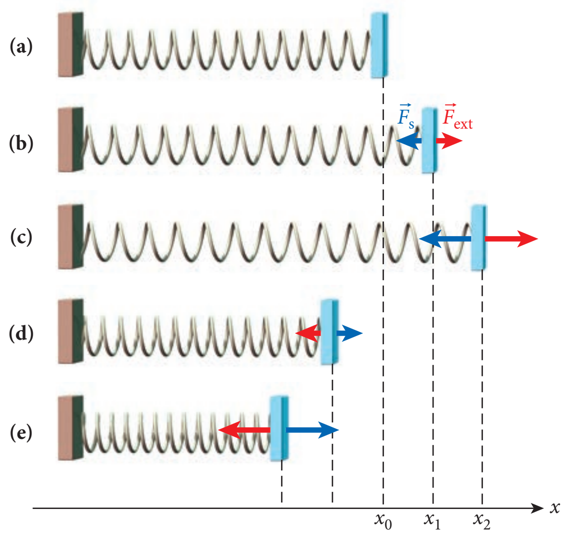


FIGURE 5.14 Spring force. The spring is in its equilibrium position in (a), is stretched in (b) and (c), and is compressed in (d) and (e). In each nonequilibrium case, the external force acting on the end of the spring is shown as a red arrow, and the spring force as a blue arrow.

Let's examine the force that is needed to stretch or compress a spring. We start with a spring that is neither stretched nor compressed from its normal length and take the end of the spring in this condition to be located at the equilibrium position, x_0, as shown in Figure 5.14a. If we pull the end of this spring a bit toward the right using an external force, $\vec{F}_{ext}$, the spring gets longer. In the stretching process, the spring generates a force directed to the left, that is, pointing toward the equilibrium position, and increasing in magnitude with increasing length of the spring. This force is conventionally called the **spring force,** $\vec{F}_s$.

Pulling with an external force of a given magnitude stretches the spring to a certain displacement from equilibrium, at which point the spring force is equal in magnitude to the external force (Figure 5.14b). Doubling this external force doubles the displacement from equilibrium (Figure 5.14c). Conversely, pushing with an external force toward the left compresses the spring from its equilibrium length, and the resulting spring force points to the right, again toward the equilibrium position (Figure 5.14d). Doubling the amount of compression (Figure 5.14e) also doubles the spring force, just as with stretching.

We can summarize these observations by noting that the magnitude of the spring force is proportional to the magnitude of the displacement of the end of the spring from its equilibrium position, and that the spring force always points toward the equilibrium position and thus is in the direction opposite to the displacement vector:

$$\vec{F}_s=-k(\vec{x}-\vec{x}_0). \tag{5.13}$$

As usual, this vector equation can be written in terms of components; in particular, for the x-component, we can write

$$F_s=-k\left(x-x_0\right). \tag{5.14}$$

The constant k is by definition always positive. The negative sign in front of k indicates that the spring force is always directed opposite to the direction of the displacement from the equilibrium position. We can choose the equilibrium position to be $x_0=0$, allowing us to write

$$F_s=-kx. \tag{5.15}$$

This simple force law is called **Hooke's Law,** after the British physicist Robert Hooke (1635–1703), a contemporary of Newton and the Curator of Experiments for the Royal Society.

Note that for a displacement $x > 0$, the spring force points in the negative direction, and $F_s < 0$. The converse is also true; if $x < 0$, then $F_s > 0$. Thus, in all cases, the spring force points toward the equilibrium position, $x = 0$. At exactly the equilibrium position, the spring force is zero, $F_s(x = 0) = 0$. As a reminder from Chapter 4, zero force is one of the defining conditions for equilibrium. The proportionality constant, k, that appears in Hooke's Law is called the **spring constant** and has units of $\mathrm{N/m} = \mathrm{kg/s^2}$. The spring force is an important example of a **restoring force:** It always acts to restore the end of the spring to its equilibrium position.

Linear restoring forces that follow Hooke's Law can be found in many systems in nature. Examples are the forces on an atom that has moved slightly out of equilibrium in a crystal lattice, the forces due to shape deformations in atomic nuclei, and any other force that leads to oscillations in a physical system (discussed in further detail in Chapters 14 through 16). In Chapter 6, we will see that we can usually approximate the force in many physical situations by a force that follows Hooke's Law.

Of course, Hooke's Law is not valid for all spring displacements. Everyone who has played with a spring knows that if it is stretched too much, it will deform and then not return to its equilibrium length when released. If stretched even further, it will eventually break into two parts. Every spring has an elastic limit—a maximum deformation—below which Hooke's Law is still valid; however, where exactly this limit lies depends on the material characteristics of the particular spring. For our considerations in this chapter, we assume that springs are always inside the elastic limit.

EXAMPLE 5.3 Spring Constant

PROBLEM 1

A spring has a length of 15.4 cm and is hanging vertically from a support point above it (Figure 5.15a). A weight with a mass of 0.200 kg is attached to the spring, causing it to extend to a length of 28.6 cm (Figure 5.15b). What is the value of the spring constant?

SOLUTION 1

We place the origin of our coordinate system at the top of the spring, with the positive direction upward, as is customary. Then, $x_0 = -15.4$ cm and $x = -28.6$ cm. According to Hooke's Law, the spring force is

$$F_s = -k(x - x_0).$$

Also, we know the force exerted on the spring was provided by the weight of the 0.200-kg mass: $F = -mg = -(0.200 \text{ kg})(9.81 \text{ m/s}^2) = -1.962$ N. Again, the negative sign indicates the direction. Now we can solve the force equation for the spring constant:

$$k = -\frac{F_s}{x - x_0} = -\frac{1.962 \text{ N}}{(-0.286 \text{ m}) - (-0.154 \text{ m})} = 14.9 \text{ N/m}.$$

Note that we would have obtained exactly the same result if we had put the origin of the coordinate system at another point or if we had elected to designate the downward direction as positive.

PROBLEM 2

How much force is needed to hold the weight at a position 4.6 cm above −28.6 cm (Figure 5.15c)?

SOLUTION 2

At first sight, this problem might appear to require a complicated calculation. However, remember that the mass has stretched the spring to a new equilibrium position. To move the mass from that position takes an external force. If the external force moves the mass up 4.6 cm, then it has to be exactly equal in magnitude and opposite in direction to the spring force resulting from a displacement of 4.6 cm. Thus, all we have to do to find the external force is to use Hooke's Law for the spring force (choosing new equilibrium position to be at $x_0 = 0$):

$$F_{ext} + F_s = 0 \Rightarrow F_{ext} = -F_s = kx = (0.046 \text{ m})(14.9 \text{ N/m}) = 0.68 \text{ N}.$$

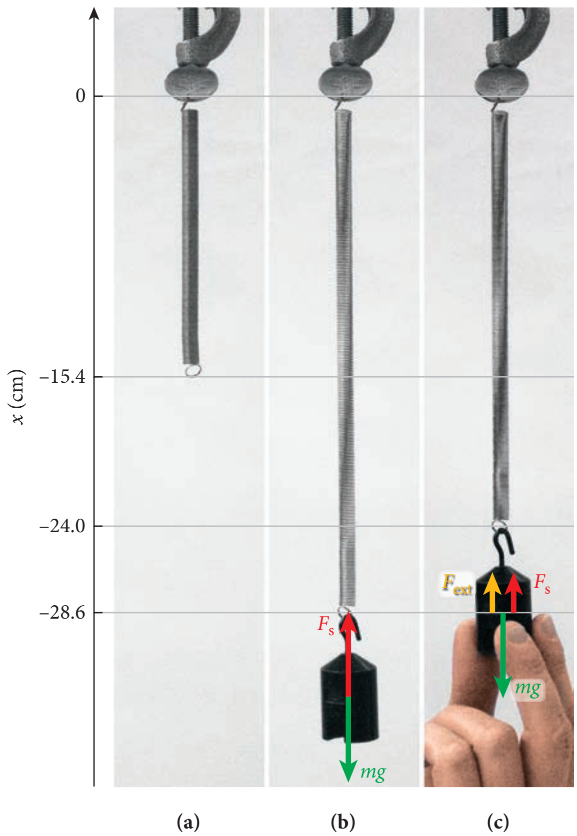


FIGURE 5.15 Mass on a spring. (a) The spring without any mass attached. (b) The spring with the mass hanging freely. (c) The mass pushed upward by an external force.

Self-Test Opportunity 5.3

A block is hanging vertically from a spring at the equilibrium displacement. The block is then pulled down a bit and released from rest. Draw the free-body diagram for the block in each of the following cases:

a) The block is at the equilibrium displacement.

b) The block is at its highest vertical point.

c) The block is at its lowest vertical point.

At this point, it is worthwhile to generalize the observations made in Example 5.3: Adding a constant force—for example, by suspending a mass from the spring—only shifts the equilibrium position. (This generalization is true for all forces that depend linearly on displacement.) Moving the mass, up or down, away from the new equilibrium position then results in a force that is linearly proportional to the displacement from the new equilibrium position. Adding another mass will only cause an additional shift to a new equilibrium position. Of course, adding more mass cannot be continued without limit. At some point, the addition of more and more mass will overstretch the spring. Then the spring will not return to its original length once the mass is removed, and Hooke's Law is no longer valid.

Work Done by the Spring Force

The displacement of a spring is a case of motion in one spatial dimension. Thus, we can apply the one-dimensional integral of equation 5.12 to find the work done by the spring force in moving from x_0 to x. The result is

$$W_s = \int_{x_0}^{x} F_s(x')dx' = \int_{x_0}^{x} (-kx')dx' = -k\int_{x_0}^{x} x'dx'.$$

The work done by the spring force is then

$$W_s = -k\int_{x_0}^{x} x'dx' = -\tfrac{1}{2}kx^2 + \tfrac{1}{2}kx_0^2. \tag{5.16}$$

If we set $x_0 = 0$ and start at the equilibrium position, as we did in arriving at Hooke's Law (equation 5.15), the second term on the right side in equation 5.16 becomes zero and we obtain

$$W_s = -\tfrac{1}{2}kx^2. \tag{5.17}$$

Note that because the spring constant is always positive, the work done by the spring force is always negative for displacements from equilibrium. Equation 5.16 shows that the work done by the spring force is positive if the starting spring displacement is farther from equilibrium than the ending displacement. External work of magnitude $\frac{1}{2}kx^2$ will stretch or compress it out of its equilibrium position.

SOLVED PROBLEM 5.2 Compressing a Spring

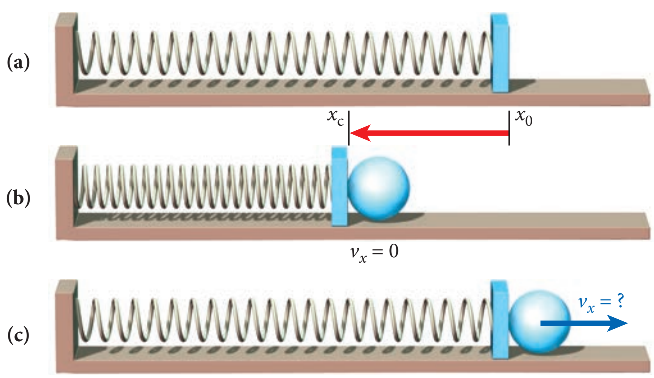


FIGURE 5.16 (a) Spring in its equilibrium position; (b) compressing the spring; (c) relaxing the compression and accelerating the steel ball.

A massless spring located on a smooth horizontal surface is compressed by a force of 63.5 N, which results in a displacement of 4.35 cm from the initial equilibrium position. As shown in Figure 5.16, a steel ball of mass 0.075 kg is then placed in front of the spring and the spring is released.

PROBLEM

What is the speed of the steel ball when it is shot off by the spring, that is, right after it loses contact with the spring? (Assume there is no friction between the surface and the steel ball; the steel ball will then simply slide across the surface and will not roll.)

SOLUTION

THINK If we compress a spring with an external force, we do work against the spring force. Releasing the spring by withdrawing the external force enables the spring to do work on the steel ball, which acquires kinetic energy in this process. Calculating the initial work done against the spring force enables us to figure out the kinetic energy that the steel ball will have and thus will lead us to the speed of the ball.

SKETCH We draw a free-body diagram at the instant before the external force is removed (see Figure 5.17). At this instant, the steel ball is at rest in equilibrium, because the external force and the spring force exactly balance each other. Note that the diagram also includes the support surface and shows two more forces acting on the ball: the force of gravity, $\vec{F}_g$, and the normal force from the support surface, $\vec{N}$. These two forces cancel each other out and thus do not enter into our calculations, but it is worthwhile to note the complete set of forces that act on the ball.

We set the x-coordinate of the ball at its left edge, which is where the ball touches the spring. This is the physically relevant location, because it measures the elongation of the spring from its equilibrium position.

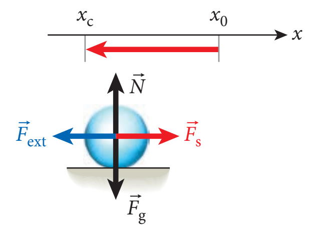


FIGURE 5.17 Free-body diagram of the steel ball before the external force is removed.

RESEARCH The motion of the steel ball starts once the external force is removed. Without the blue arrow in Figure 5.17, the spring force is the only unbalanced force in the situation, and it accelerates the ball. This acceleration is not constant over time (as is the case for free-fall motion, for example), but rather changes in time. However, the beauty of applying energy considerations is that we do not need to know the acceleration to calculate the final speed.

As usual, we are free to choose the origin of the coordinate system, and we put it at x_0, the equilibrium position of the spring. This implies that we set $x_0 = 0$. The relation between the x-component of the spring force at the moment of release and the initial compression of the spring x_c is

$$F_s(x_c) = -kx_c.$$

Because $F_s(x_c) = -F_{ext}$, we find

$$kx_c = F_{ext}.$$

The magnitude of this external force, as well as the value of the displacement, was given, and so we can calculate the value of the spring constant from this equation. Note that with our choice of the coordinate system, $F_{ext} < 0$, because its vector arrow points in the negative x-direction. In addition, $x_c < 0$, because the displacement from equilibrium is in the negative direction.

We can now calculate the work W needed to compress this spring. Since the force that the ball exerts on the spring is always equal and opposite to the force that the spring exerts on the ball, the definition of work allows us to set

$$W = -W_s = \tfrac{1}{2}kx_c^2.$$

According to the work–kinetic energy theorem, this work is related to the change in the kinetic energy of the steel ball via

$$K = K_0 + W = 0 + W = \tfrac{1}{2}kx_c^2.$$

Finally, the ball's kinetic energy is, by definition,

$$K = \tfrac{1}{2}mv_x^2,$$

which allows us to determine the ball's speed.

SIMPLIFY We solve the equation for the kinetic energy for the speed, v_x, and then use the $K = \tfrac{1}{2}kx_c^2$ to obtain

$$v_x = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(\tfrac{1}{2}kx_c^2)}{m}} = \sqrt{\frac{kx_c^2}{m}} = \sqrt{\frac{F_{ext}x_c}{m}}.$$

(In the third step, we canceled out the factors 2 and $\tfrac{1}{2}$, and in the fourth step, we used $kx_c = F_{ext}$.)

CALCULATE Now we are ready to insert the numbers: $x_c = -0.0435$ m, $m = 0.075$ kg, and $F_{ext} = -63.5$ N. Our result is

$$v_x = \sqrt{\frac{(-63.5\text{ N})(-0.0435\text{ m})}{0.075\text{ kg}}} = 6.06877\text{ m/s}.$$

Note that we choose the positive root for the x-component of the ball's velocity. By examining Figure 5.16, you can see that this is the appropriate choice, because the ball will move in the positive x-direction after the spring is released.

ROUND Rounding to the two-digit accuracy to which the mass was specified, we state our result as

$$v_x = 6.1\text{ m/s}.$$

DOUBLE-CHECK We are limited in the checking we can perform to verify that our answer makes sense until we study motion under the influence of the spring force in more detail in Chapter 14. However, our answer passes the minimum requirements in that it has the proper units and the order of magnitude seems in line with typical velocities for balls propelled from spring-loaded toy guns.

Concept Check 5.3

If you compress a spring a distance h from its equilibrium position and do work W_h in the process, how much work will be required to compress the same spring a distance $2h$?

a) W_h

b) $2W_h$

c) $0.5W_h$

d) $4W_h$

e) $0.25W_h$

5.7 Power

We can now readily calculate the amount of work required to accelerate a 1550-kg (3420-lb) car from a standing start to a speed of 26.8 m/s (60.0 mph). The work done is simply the difference between the final and initial kinetic energies. The initial kinetic energy is zero, and the final kinetic energy is

$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(1550\text{ kg})(26.8\text{ m/s})^2 = 557\text{ kJ},$$

which is also the amount of work required. However, the work requirement is not that interesting to most of us—we'd be more interested in how quickly the car is able to reach 60 mph. That is, we'd like to know the rate at which the car can do this work.

Power is the rate at which work is done. Mathematically, this means that the power, P, is the time derivative of the work, W:

$$P = \frac{dW}{dt}. \tag{5.18}$$

It is also useful to define the average power, $\overline{P}$ as

$$\overline{P} = \frac{W}{\Delta t}. \tag{5.19}$$

The SI unit of power is the **watt** (W). [Beware of confusing the symbol for work, *W (italicized),* and the abbreviation for the unit of power, W (nonitalicized).]

$$1\text{ W} = 1\text{ J/s} = 1\text{ kg m}^2/\text{s}^3. \tag{5.20}$$

Conversely, one joule is also one watt times one second. This relationship is reflected in a very common unit of energy (not power!), the **kilowatt-hour** (kWh):

$$1\text{ kWh} = (1000\text{ W})(3600\text{ s}) = 3.6\cdot 10^6\text{ J} = 3.6\text{ MJ}.$$

The unit kWh appears on utility bills and quantifies the amount of electrical energy that has been consumed. Kilowatt-hours can be used to measure any kind of energy. Thus, the kinetic energy of the 1550-kg car moving with a speed of 26.8 m/s, which we calculated as 557 kJ, can be expressed with equal validity as

$$(557{,}000\text{ J})(1\text{ kWh}/3.6\cdot 10^6\text{ J}) = 0.155\text{ kWh}.$$

The two most common non-SI power units are the horsepower (hp) and the foot-pound per second (ft lb/s): 1 hp = 550 ft lb/s = 746 W.

It is instructive to look at a logarithmic scale of power consumption of different devices. This is done in the lower row of Figure 5.18. Wristwatches consume about a μW of power, and a green laser pointer is rated up to 5 mW. Among the items in your home a blow dryer has one of the highest power consumption rates at 1 to 2 kW. Cars have engines rated at approximately 100 kW, and a large plane (Boeing 747, Airbus 380) uses on the order of 100 MW to become airborne. The average power consumption of the entire United States is approximately 3.5 TW. Since there are a little more than 300 million people living in the

Concept Check 5.4

Is each of the following statements true or false?

a) Work cannot be done in the absence of motion.

b) More power is required to lift a box slowly than to lift a box quickly.

c) A force is required to do work.

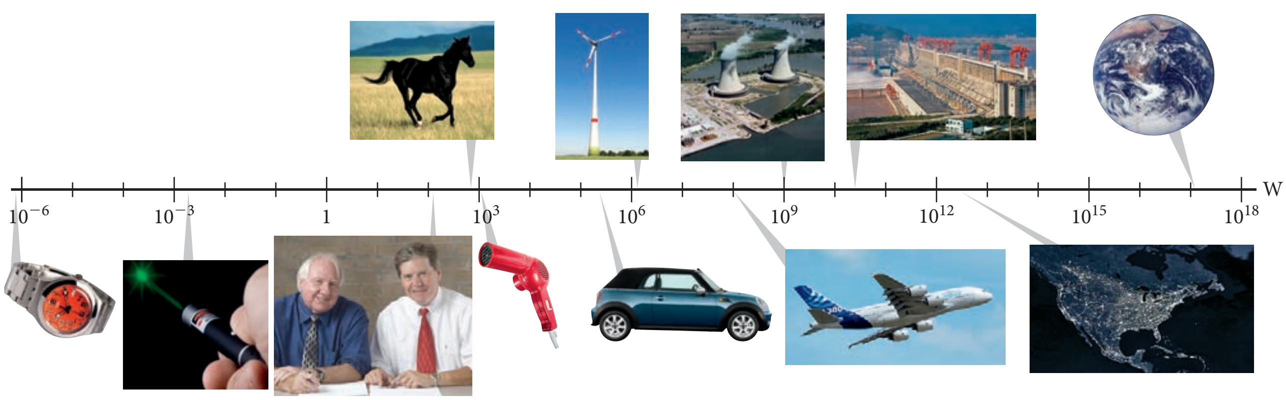


FIGURE 5.18 Logarithmic display of some power sources (upper row of pictures) and power consumption (lower row).

United States, this means an average power use of approximately 10 kW per person. For comparison, the average power a human needs to stay alive is approximately 100 W (= 10 MJ of food intake per day divided by the number of seconds in a day).

The upper row of Figure 5.18 lists power sources. On the left we mark 1 horsepower. A typical wind turbine generates on the order of 1 MW, a typical nuclear power plant 1 GW. The largest power producer in the world is the Chinese Three Gorges Dam, completed in 2012, with a peak power production of more than 22 GW. The entire power that the Sun radiates onto the surface of Earth is 175 PW, more than 10,000 times the present combined power consumption of all humans.

EXAMPLE 5.4 Accelerating a Car

PROBLEM

Returning to the example of an accelerating car, let's assume that the car, of mass 1550 kg, can reach a speed of 60 mph (26.8 m/s) in 7.1 s. What is the average power needed to accomplish this?

SOLUTION

We already found that the car's kinetic energy at 60 mph is

$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(1550 \text{ kg})(26.8 \text{ m/s})^2 = 557 \text{ kJ}.$$

The work to get the car to the speed of 60 mph is then

$$W = \Delta K = K - K_0 = 557 \text{ kJ}.$$

The average power needed to get to 60 mph in 7.1 s is therefore

$$\overline{P} = \frac{W}{\Delta t} = \frac{5.57 \cdot 10^5 \text{ J}}{7.1 \text{ s}} = 78.4 \text{ kW} = 105 \text{ hp}.$$

If you own a car with a mass of at least 1550 kg that has an engine with 105 hp, you know that it cannot possibly reach 60 mph in 7.1 s. An engine with at least 180 hp is needed to accelerate a car of mass 1550 kg (including the driver, of course) to 60 mph in that time interval.

Our calculation in Example 5.4 is not quite correct for several reasons. First, not all of the power output of the engine is available to do useful work such as accelerating the car. Second, friction and air resistance forces act on a moving car, but were ignored in Example 5.4. Chapter 6 will address work and energy in the presence of friction forces (rolling friction and air resistance in this case). Finally, a car's rated horsepower is a peak specification, realizable only at the most beneficial rpm-domain of the engine. As you accelerate the car from rest, this peak output of the engine is not maintainable as you shift through the gears.

The average mass, power, and fuel efficiency (for city driving) of mid-sized cars sold in the United States from 1975 to 2010 are shown in Figure 5.19. The mass of a car is important in city driving because of the many instances of acceleration in stop-and-go conditions. We can combine the work–kinetic energy theorem (equation 5.17) and the definition of average power (equation 5.19) to get

$$\overline{P} = \frac{W}{\Delta t} = \frac{\Delta K}{\Delta t} = \frac{\tfrac{1}{2}mv^2}{\Delta t} = \frac{mv^2}{2\Delta t}. \quad (5.21)$$

You can see that the average power required to accelerate a car from rest to a speed v in a given time interval, Δt, is proportional to the mass of the car. The energy consumed by the car is equal to the average power times the time interval. Thus, the larger the mass of a car, the more energy is required to accelerate it in a given amount of time.

Following the 1973 oil embargo, the average mass of mid-sized cars decreased from 2100 kg to 1500 kg between 1975 and 1982. During that same period, the average power decreased from 160 hp to 110 hp, and the fuel

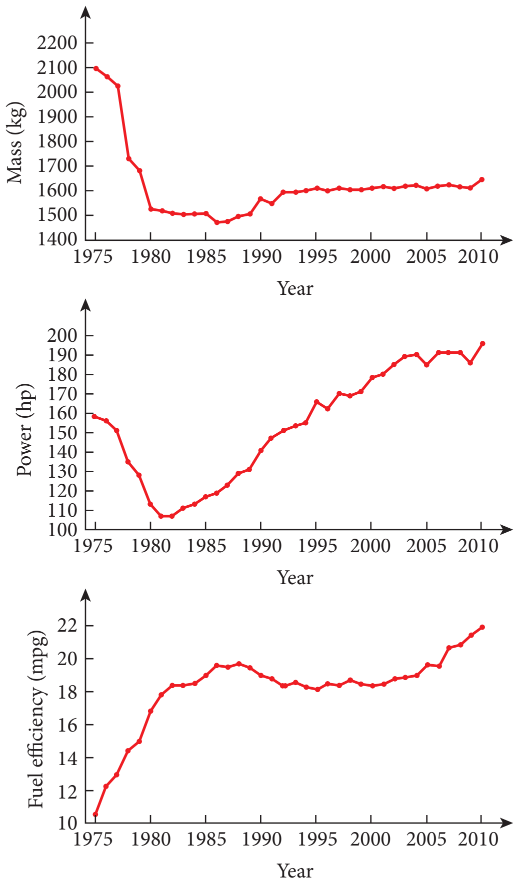


FIGURE 5.19 The mass, power, and fuel efficiency of mid-sized cars sold in the United States from 1975 to 2010. The fuel efficiency is that for typical city driving.

efficiency increased from 10 to 18 mpg. From 1982 to 2010, however, the average mass and fuel efficiency of mid-sized cars stayed roughly constant, while the power increased steadily. Apparently, buyers of mid-sized cars in the United States have valued increased power over increased efficiency.

SOLVED PROBLEM 5.3 Wind Power

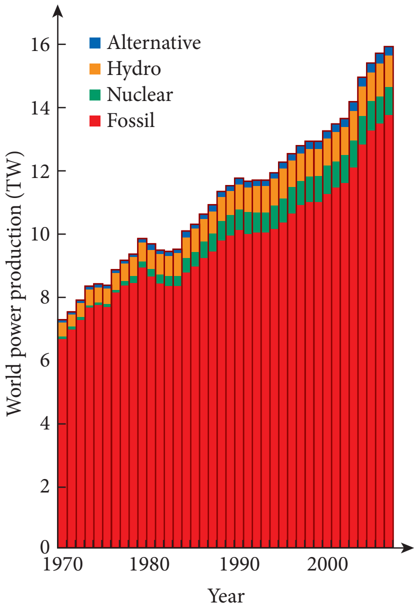


FIGURE 5.20 Worldwide power production as a function of time for different power sources.

The total power consumption of all humans combined is approximately 16 TW ($1.6 \cdot 10^{13}$ W), and it is expected to double during the next 15 to 20 years. Almost 90% of the power produced comes from fossil fuels; see Figure 5.20. Since the burning of fossil fuels is currently adding more than 10 billion tons of carbon dioxide to Earth's atmosphere per year, it is not clear how much longer this mode of power generation is sustainable. Other sources of power, such as wind, have to be considered. Some huge wind farms have been constructed (see Figure 5.21), and many more are under development.

PROBLEM

How much average power is contained in wind blowing at 10.0 m/s across the rotor of a large wind turbine, such as the Enercon E-126, which has a hub height of 135 m and a rotor radius of 63 m?

SOLUTION

THINK Since the wind speed is given, we can calculate the kinetic energy of the amount of air blowing across the rotor's surface. If we can calculate how much air moves across the rotor per unit of time, then we can calculate the power as the ratio of the kinetic energy of the air to the time interval.

SKETCH The rotor surface is a circle, and we can assume that the wind blows perpendicular to it, because the turbines in wind farms are oriented so that that is the case. Indicated in the sketch (Figure 5.22) is the cylindrical volume of air moving across the rotor per time interval.

RESEARCH Earlier in this chapter, we learned that the kinetic energy is given by $E = \frac{1}{2}mv^2$; here, m is the mass of air, and v is the wind speed. A very handy rule of thumb is that 1 m^3 of air has a mass of 1.20 kg at sea level and room temperature. The average power is given by $P = W/\Delta t$, and the work is related to the change in kinetic energy through the work–kinetic energy theorem $W = \Delta K$.

FIGURE 5.21 Large-scale wind farm producing power.

We can thus write, for the average power of the wind moving across the rotor of the wind turbine,

$$P = \frac{W}{\Delta t} = \frac{\Delta K}{\Delta t} = \frac{\Delta\left(\frac{1}{2}mv^2\right)}{\Delta t} = \frac{1}{2}v^2\frac{\Delta m}{\Delta t}.$$

In the last step, we have assumed that the wind speed is constant and does not change.

What is Δm? We know that density is mass/volume, and so we can write $\Delta m = \rho\Delta V$, where $\rho = 1.20$ kg/m^3 is the air density and ΔV is the volume of air moved across the rotor per unit of time. Here ΔV is a cylinder with length $l = v\Delta t$ and base area A = area of the rotor (see Figure 5.22), v is again the wind speed, and the area is the area of a circle, $A = \pi R^2$.

SIMPLIFY Now we are ready to insert our expressions for Δm and ΔV into our equation for the average power:

$$P = \frac{1}{2}v^2\frac{\Delta m}{\Delta t} = \frac{1}{2}v^2\frac{\rho\Delta V}{\Delta t} = \frac{1}{2}v^2\frac{\rho A l}{\Delta t} = \frac{1}{2}v^2\frac{\rho(\pi R^2)(v\Delta t)}{\Delta t} = \frac{1}{2}v^3\rho\pi R^2.$$

We see that the average wind power is proportional to the cube of the wind speed!

CALCULATE Inserting the given numbers for the rotor's radius, the wind speed, and the air density yields

$$P = \tfrac{1}{2}(10.0\text{ m/s})^3(1.2\text{ kg/m}^3)\pi(63\text{ m})^2 = 7.481389\cdot 10^6\text{ kg m}^2/\text{s}^3$$

ROUND Since the rotor's radius was given to only two significant figures, we round the final result to the same number of significant figures. So our answer is 7.5 MW.

DOUBLE-CHECK In stating our final result, we replaced kg m^2/s^3 with W, which is correct by definition and also means that our answer is dimensionally correct. Although 7.5 MW seems to be a very large amount of power, enough to satisfy the average electricity needs of approximately 5000 households, this is a gigantic wind turbine. If you look it up on the Internet, you will see that it is rated close to our result. In Example 13.7 we will return to wind turbines and determine what maximum fraction of this total power contained in the wind one can extract.

Obviously, since the wind power is proportional to the cube of the wind speed, it matters *a lot* where wind turbines are located. Figure 5.23 shows a map of average wind speed in the United States. You can see that winds up to 10 m/s are realistic in the central part of the country, from North Dakota to the Texas panhandle, which makes these states prime locations for wind farms.

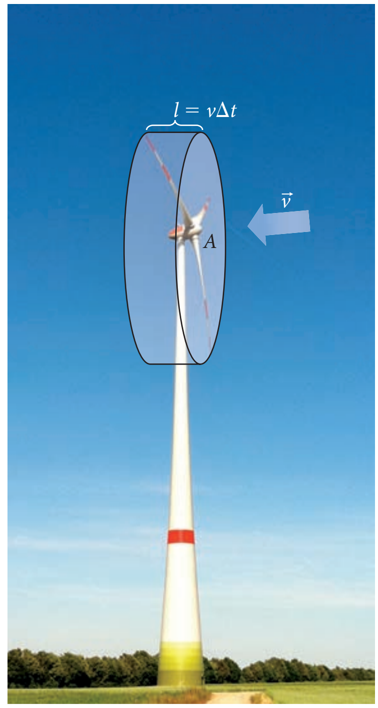


FIGURE 5.22 Sketch for finding the wind power for a large wind turbine.

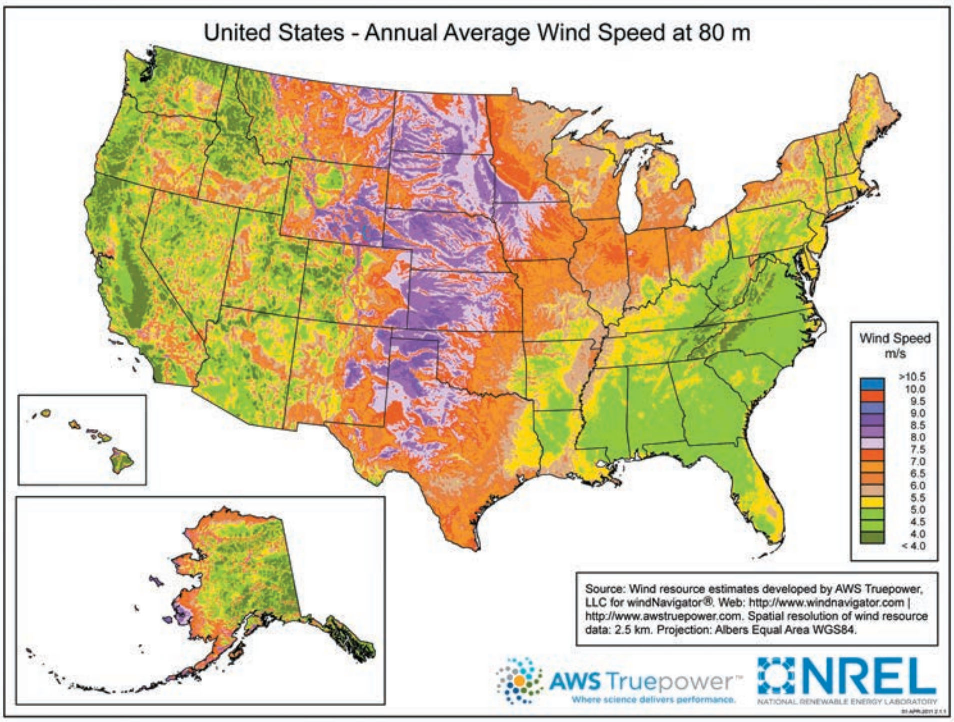


FIGURE 5.23 Map of the United States, showing average wind speed at an altitude of 80 m above ground.

Power, Force, and Velocity

For a constant force, the work is given by $W = \vec{F} \bullet \Delta\vec{r}$ and the differential work as $dW = \vec{F} \bullet d\vec{r}$. In this case, the time derivative is

$$P = \frac{dW}{dt} = \frac{\vec{F} \bullet d\vec{r}}{dt} = \vec{F} \bullet \vec{v} = Fv\cos\alpha, \tag{5.22}$$

where α is the angle between the force vector and the velocity vector. Therefore, the power is the scalar product of the force vector and the velocity vector. While we show this only for a constant force, equation 5.22 also holds for a nonconstant force.

SOLVED PROBLEM 5.4 Riding a Bicycle

PROBLEM

A bicyclist coasts down a 4.2° slope at a steady speed of 5.1 m/s. Assuming a total mass of 82.2 kg (bicycle plus rider), what power output must the cyclist expend to pedal up the same slope at the same speed?

– *Continued*

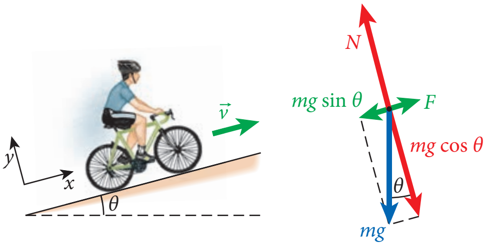


FIGURE 5.24 Sketch of the bicycle moving up the slope (left) and the free-body diagram (right).

SOLUTION

THINK If you ride your bike on a horizontal surface and stop pedaling, you slow down to a stop. The net force that causes you to stop is the combination of friction in the mechanical components of the bike and air resistance. In the problem statement, we learned that the bicycle rolls down the hill at a constant speed, which means that the net force acting on it is zero (Newton's First Law!). The force directed along the slope is $mg\sin\theta$ (see Figure 5.24). For the net force to be zero, this force has to be balanced by the forces of friction and air resistance, which act opposite to the direction of motion, or up the slope. So in this case the forces of friction and air resistance have exactly the same magnitude as the component of the gravitational force along the slope. But if the cyclist is pedaling up the same slope, gravity and the forces of air resistance and friction point in the same direction. Thus, we can calculate the total work done against all forces in this case (and only in this case!) by simply calculating the work done against gravity and then multiplying by a factor of 2.

SKETCH Figure 5.24 shows a sketch of the situation where the cyclist is pedaling against gravity.

RESEARCH We can calculate the work done against gravity, and then multiply it by 2. The component of the gravitational force along the slope is $mg\sin\theta$. F is the force exerted by the bicyclist and Power = Fv. Using Newton's Second Law, we have

$$\sum F_x = ma_x = 0 \Rightarrow F - mg\sin\theta = 0 \Rightarrow F = mg\sin\theta.$$

SIMPLIFY We find the total power the cyclist has to expend by inserting the expression for the force and using the factor of 2, as discussed above: $P = 2Fv = 2(mg\sin\theta)v$.

CALCULATE Inserting the given numbers, we get

$$P = 2(82.2\text{ kg})(9.81\text{ m/s}^2)\sin(4.2°)(5.1\text{ m/s}) = 602.391\text{ W}.$$

ROUND The slope and the speed are given to two significant figures, which is what we round our final answer to: $P = 0.60$ kW.

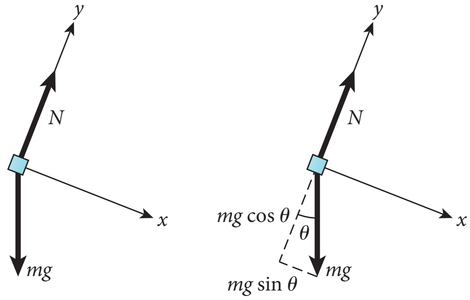
DOUBLE-CHECK Our result is $P = 0.60$ kW (1 hp/0.746 kW) = 0.81 hp. Thus, going up a 4.2° slope at 5.1 m/s requires approximately 0.8 hp, which is what a good cyclist can expend for quite some time. (But it's hard!)

WHAT WE HAVE LEARNED | EXAM STUDY GUIDE

- Kinetic energy is the energy associated with the motion of an object, $K = \frac{1}{2}mv^2$.
- The SI unit of work and energy is the joule: 1 J = 1 kg m^2/s^2.
- Work is the energy transferred to an object or transferred from an object as the result of the action of a force. Positive work is a transfer of energy to the object, and negative work is a transfer of energy from the object.
- Work done by a constant force is $W = |\vec{F}||\Delta\vec{r}|\cos\alpha$, where α is the angle between $\vec{F}$ and $\Delta\vec{r}$.
- Work done by a variable force in one dimension is $$W = \int_{x_0}^{x} F_x(x')dx'.$$
- Work done by the gravitational force in the process of lifting an object is $W_g = -mgh < 0$, where $h = |y - y_0|$; the work done by the gravitational force in lowering an object is $W_g = +mgh > 0$.
- The spring force is given by Hooke's Law: $F_s = -kx$.
- Work done by the spring force is $$W = -k\int_{x_0}^{x} x'dx' = -\tfrac{1}{2}kx^2 + \tfrac{1}{2}kx_0^2.$$
- The work–kinetic energy theorem is $\Delta K \equiv K - K_0 = W$.
- Power, P, is the time derivative of the work, W: $P = \dfrac{dW}{dt}$.
- The average power, $\bar{P}$, is $\bar{P} = \dfrac{W}{\Delta t}$.
- The SI unit of power is the watt (W): 1 W = 1 J/s.
- The relationship between power, force, and velocity is $P = \dfrac{dW}{dt} = \dfrac{\vec{F}\bullet d\vec{r}}{dt} = \vec{F}\bullet\vec{v} = Fv\cos\alpha$, where α is the angle between the force vector and the velocity vector.

ANSWERS TO SELF-TEST OPPORTUNITIES

5.1



5.2 $\vec{F} = m\vec{a}$ can be rewritten as

$F_x = ma_x$

$F_y = ma_y$

$F_z = ma_z$

for each component

$v_x^2 - v_{x0}^2 = 2a_x(x - x_0)$

$v_y^2 - v_{y0}^2 = 2a_y(y - y_0)$

$v_z^2 - v_{z0}^2 = 2a_z(z - z_0)$

multiply by $\frac{1}{2}m$

$\frac{1}{2}mv_x^2 - \frac{1}{2}mv_{x0}^2 = ma_x(x - x_0)$

$\frac{1}{2}mv_y^2 - \frac{1}{2}mv_{y0}^2 = ma_y(y - y_0)$

$\frac{1}{2}mv_z^2 - \frac{1}{2}mv_{z0}^2 = ma_z(z - z_0)$

add the three equations

$$\tfrac{1}{2}m\left(v_x^2 + v_y^2 + v_z^2\right) - \tfrac{1}{2}m\left(v_{x0}^2 + v_{y0}^2 + v_{z0}^2\right) = ma_x(x - x_0) + ma_y(y - y_0) + ma_z(z - z_0)$$

$$K = \tfrac{1}{2}m\left(v_x^2 + v_y^2 + v_z^2\right) = \tfrac{1}{2}mv^2$$

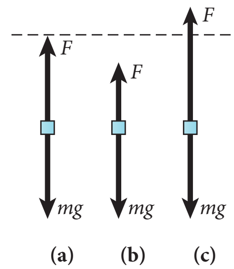
$$K_0 = \tfrac{1}{2}m\left(v_{x0}^2 + v_{y0}^2 + v_{z0}^2\right) = \tfrac{1}{2}mv_0^2$$

$$\Delta\vec{r} = (x - x_0)\hat{x} + (y - y_0)\hat{y} + (z - z_0)\hat{z}$$

$$\vec{F} = ma_x\hat{x} + ma_y\hat{y} + ma_z\hat{z}$$

$$K - K_0 = \Delta K = \vec{F} \bullet \Delta\vec{r} = W$$

5.3



PROBLEM-SOLVING GUIDELINES: KINETIC ENERGY, WORK, AND POWER

1. In all problems involving energy, the first step is to clearly identify the system and the changes in its conditions. If an object undergoes a displacement, check that the displacement is always measured from the same point on the object, such as the front edge or the center of the object. If the speed of the object changes, identify the initial and final speeds at specific points. A diagram is often helpful to show the position and the speed of the object at two different times of interest.

2. Be careful to identify the force that is doing work. Also note whether forces doing work are constant forces or variable forces, because they need to be treated differently.

3. You can calculate the sum of the work done by individual forces acting on an object or the work done by the net force acting on an object; the result should be the same. (You can use this as a way to check your calculations.)

4. Remember that the direction of the restoring force exerted by a spring is always opposite the direction of the displacement of the spring from its equilibrium point.

5. The formula for power, $P = \vec{F} \bullet \vec{v}$, is very useful, if information on the velocity is available. When using the more general definition of power, be sure to distinguish between the average power, $\overline{P} = \dfrac{W}{\Delta t}$, and the instantaneous value of the power, $P = \dfrac{dW}{dt}$.

MULTIPLE-CHOICE QUESTIONS

5.1 Which of the following is a correct unit of energy?
a) kg m/s^2
b) kg m^2/s
c) kg m^2/s^2
d) kg^2 m/s^2
e) kg^2 m^2/s^2

5.2 An 800-N box is pushed up an inclined plane that is 4.0 m long. It requires 3200 J of work to get the box to the top of the plane, which is 2.0 m above the base. What is the magnitude of the average friction force on the box? (Assume the box starts at rest and ends at rest.)
a) zero
b) not zero but less than 400 N
c) greater than 400 N
d) 400 N
e) 800 N

5.3 An engine pumps water continuously through a hose. If the speed with which the water passes through the hose nozzle is v and if k is the mass per unit length of the water jet as it leaves the nozzle, what is the power being imparted to the water?
a) $\frac{1}{2}kv^3$
b) $\frac{1}{2}kv^2$
c) $\frac{1}{2}kv$
d) $\frac{1}{2}v^2/k$
e) $\frac{1}{2}v^3/k$

5.4 A 1500-kg car accelerates from 0 to 25 m/s in 7.0 s. What is the average power delivered by the engine (1 hp = 746 W)?
a) 60 hp
b) 70 hp
c) 80 hp
d) 90 hp
e) 180 hp

5.5 Which of the following is a correct unit of power?
a) kg m/s^2
b) N
c) J
d) m/s^2
e) W

5.6 How much work is done when a 75.0-kg person climbs a flight of stairs 10.0 m high at constant speed?
a) $7.36 \cdot 10^5$ J
b) 750 J
c) 75 J
d) 7500 J
e) 7360 J

5.7 How much work do movers do (horizontally) in pushing a 150-kg crate 12.3 m across a floor at constant speed if the coefficient of friction is 0.70?
a) 1300 J
b) 1845 J
c) $1.3 \cdot 10^4$ J
d) $1.8 \cdot 10^4$ J
e) 130 J

5.8 Eight books, each 4.6 cm thick and of mass 1.8 kg, lie on a flat table. How much work is required to stack them on top of one another?
a) 141 J
b) 23 J
c) 230 J
d) 0.81 J
e) 14 J

5.9 A particle moves parallel to the x-axis. The net force on the particle increases with x according to the formula $F_x = (120 \text{ N/m})x$, where the force is in newtons when x is in meters. How much work does this force do on the particle as it moves from $x = 0$ to $x = 0.50$ m?
a) 7.5 J
b) 15 J
c) 30 J
d) 60 J
e) 120 J

5.10 A skydiver is subject to two forces: gravity and air resistance. Falling vertically, she reaches a constant terminal speed at some time after jumping from a plane. Since she is moving at a constant velocity from that time until her chute opens, we conclude from the work–kinetic energy theorem that, over that time interval,
a) the work done by gravity is zero.
b) the work done by air resistance is zero.
c) the work done by gravity equals the negative of the work done by air resistance.
d) the work done by gravity equals the work done by air resistance.
e) her kinetic energy increases.

5.11 Jack is holding a box that has a mass of m kg. He walks a distance of d m at a constant speed of v m/s. How much work, in joules, has Jack done on the box?
a) mgd
b) $-mgd$
c) $\frac{1}{2}mv^2$
d) $-\frac{1}{2}mv^2$
e) zero

5.12 If negative work is being done by an object, which one of the following statements is true?
a) An object is moving in the negative x-direction.
b) An object has negative kinetic energy.
c) Energy is being transferred from an object.
d) Energy is being transferred to an object.

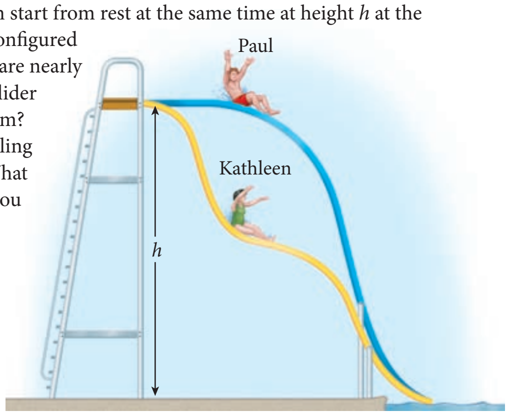
5.13 The work–kinetic energy theorem is equivalent to
a) Newton's First Law.
b) Newton's Second Law.
c) Newton's Third Law.
d) Newton's Fourth Law.
e) none of Newton's laws.

5.14 Kathleen climbs a flight of stairs. What can we say about the work done by gravity on her?
a) Gravity does negative work on her.
b) Gravity does positive work on her.
c) Gravity does no work on her.
d) We can't tell what work gravity does on her.

CONCEPTUAL QUESTIONS

5.15 If the net work done on a particle is zero, what can be said about the particle's speed?

5.16 Paul and Kathleen start from rest at the same time at height h at the top of two differently configured water slides. The slides are nearly frictionless. a) Which slider arrives first at the bottom? b) Which slider is traveling faster at the bottom? What physical principle did you use to answer this?



5.17 Does the Earth do any work on the Moon as the Moon moves in its orbit?

5.18 A car, of mass m, traveling at a speed v_1 can brake to a stop within a distance d. If the car speeds up by a factor of 2, so that $v_2 = 2v_1$, by what factor is its stopping distance increased, assuming that the braking force F is approximately independent of the car's speed?

EXERCISES

A blue problem number indicates a worked-out solution is available in the Student Solutions Manual. One • and two •• indicate increasing level of problem difficulty.

Section 5.2

5.19 The damage done by a projectile on impact is correlated with its kinetic energy. Calculate and compare the kinetic energies of these three projectiles:

a) a 10.0-kg stone at 30.0 m/s

b) a 100.0-g baseball at 60.0 m/s

c) a 20.0-g bullet at 300. m/s

5.20 A limo is moving at a speed of 100. km/h. If the mass of the limo, including passengers, is 1900. kg, what is its kinetic energy?

5.21 Two railroad cars, each of mass 7000. kg and traveling at 90.0 km/h, collide head on and come to rest. How much mechanical energy is lost in this collision?

5.22 Think about the answers to these questions next time you are driving a car:

a) What is the kinetic energy of a 1500.-kg car moving at 15.0 m/s?

b) If the car changed its speed to 30.0 m/s, how would the value of its kinetic energy change?

5.23 A 200.-kg moving tiger has a kinetic energy of 14,400 J. What is the speed of the tiger?

•5.24 Two cars are moving. The first car has twice the mass of the second car but only half as much kinetic energy. When both cars increase their speed by 5.00 m/s, they then have the same kinetic energy. Calculate the original speeds of the two cars.

•5.25 What is the kinetic energy of an ideal projectile of mass 20.1 kg at the apex (highest point) of its trajectory, if it was launched with an initial speed of 27.3 m/s and at an initial angle of 46.9° with respect to the horizontal?

Section 5.4

5.26 A force of 5.00 N acts over a distance of 12.0 m in the direction of the force. Find the work done.

5.27 Two baseballs are thrown off the top of a building that is 7.25 m high. Both are thrown with initial speed of 63.5 mph. Ball 1 is thrown horizontally, and ball 2 is thrown straight down. What is the difference in the speeds of the two balls when they touch the ground? (Neglect air resistance.)

5.28 A 95.0-kg refrigerator rests on the floor. How much work is required to move it at constant speed for 4.00 m along the floor against a friction force of 180. N?

5.29 A hammerhead of mass $m = 2.00$ kg is allowed to fall onto a nail from a height $h = 0.400$ m. Calculate the maximum amount of work it could do on the nail.

5.30 You push your couch a distance of 4.00 m across the living room floor with a horizontal force of 200.0 N. The force of friction is 150.0 N. What is the work done by you, by the friction force, by gravity, and by the net force?

•5.31 Suppose you pull a sled with a rope that makes an angle of 30.0° to the horizontal. How much work do you do if you pull with 25.0 N of force and the sled moves 25.0 m?

•5.32 A father pulls his son, whose mass is 25.0 kg and who is sitting on a swing with ropes of length 3.00 m, backward until the ropes make an angle of 33.6° with respect to the vertical. He then releases his son from rest. What is the speed of the son at the bottom of the swinging motion?

•5.33 A constant force, $\vec{F} = (4.79, -3.79, 2.09)$ N, acts on an object of mass 18.0 kg, causing a displacement of that object by $\vec{r} = (4.25, 3.69, -2.45)$ m. What is the total work done by this force?

•5.34 A mother pulls her daughter, whose mass is 20.0 kg and who is sitting on a swing with ropes of length 3.50 m, backward until the ropes make an angle of 35.0° with respect to the vertical. She then releases her daughter from rest. What is the speed of the daughter when the ropes make an angle of 15.0° with respect to the vertical?

•5.35 A ski jumper glides down a 30.0° slope for 80.0 ft before taking off from a negligibly short horizontal ramp. If the jumper's takeoff speed is 45.0 ft/s, what is the coefficient of kinetic friction between skis and slope? Would the value of the coefficient of friction be different if expressed in SI units? If yes, by how much would it differ?

•5.36 At sea level, a nitrogen molecule in the air has an average kinetic energy of $6.2 \cdot 10^{-21}$ J. Its mass is $4.7 \cdot 10^{-26}$ kg. If the molecule could shoot straight up without colliding with other molecules, how high would it rise? What percentage of the Earth's radius is this height? What is the molecule's initial speed? (Assume that you can use $g = 9.81$ m/s^2; although we'll see in Chapter 12 that this assumption may not be justified for this situation.)

••5.37 A bullet moving at a speed of 153 m/s passes through a plank of wood. After passing through the plank, its speed is 130. m/s. Another bullet, of the same mass and size but moving at 92.0 m/s, passes through an identical plank. What will this second bullet's speed be after passing through the plank? Assume that the resistance offered by the plank is independent of the speed of the bullet.

Section 5.5

•5.38 A particle of mass m is subjected to a force acting in the x-direction. $F_x = (3.00 + 0.500x)$ N. Find the work done by the force as the particle moves from $x = 0.00$ to $x = 4.00$ m.

•5.39 A force has the dependence $F_x(x) = -kx^4$ on the displacement x, where the constant $k = 20.3$ N/m^4. How much work does it take to change the displacement, working against the force, from 0.730 m to 1.35 m?

•5.40 A body of mass m moves along a trajectory $\vec{r}(t)$ in three-dimensional space with constant kinetic energy. What geometric relationship has to exist between the body's velocity vector, $\vec{v}(t)$, and its acceleration vector, $\vec{a}(t)$, in order to accomplish this?

•5.41 A force given by $\vec{F}(x) = 5x^3\hat{x}$ (in N/m^3) acts on a 1.00-kg mass moving on a frictionless surface. The mass moves from $x = 2.00$ m to $x = 6.00$ m.

a) How much work is done by the force?

b) If the mass has a speed of 2.00 m/s at $x = 2.00$ m, what is its speed at $x = 6.00$ m?

Section 5.6

5.42 An ideal spring has the spring constant $k = 440.$ N/m. Calculate the distance this spring must be stretched from its equilibrium position for 25.0 J of work to be done.

5.43 A spring is stretched 5.00 cm from its equilibrium position. If this stretching requires 30.0 J of work, what is the spring constant?

5.44 A spring with spring constant k is initially compressed a distance x_0 from its equilibrium length. After returning to its equilibrium position, the spring is then stretched a distance x_0 from that position. What is the ratio of the work that needs to be done on the spring in the stretching to the work done in the compressing?

•5.45 A spring with a spring constant of 238.5 N/m is compressed by 0.231 m. Then a steel ball bearing of mass 0.0413 kg is put against the end of the spring, and the spring is released. What is the speed of the ball bearing right after it loses contact with the spring? (The ball bearing will come off the spring exactly as the spring returns to its equilibrium position. Assume that the mass of the spring can be neglected.)

Section 5.7

5.46 A horse draws a sled horizontally across a snow-covered field. The coefficient of friction between the sled and the snow is 0.195, and the mass of the sled, including the load, is 202.3 kg. If the horse moves the sled at a constant speed of 1.785 m/s, what is the power needed to accomplish this?

5.47 A horse draws a sled horizontally on snow at constant speed. The horse can produce a power of 1.060 hp. The coefficient of friction between the sled and the snow is 0.115, and the mass of the sled, including the load, is 204.7 kg. What is the speed with which the sled moves across the snow?

5.48 While a boat is being towed at a speed of 12.0 m/s, the tension in the towline is 6.00 kN. What is the power supplied to the boat through the towline?

5.49 A car of mass 1214.5 kg is moving at a speed of 62.5 mph when it misses a curve in the road and hits a bridge piling. If the car comes to rest in 0.236 s, how much average power (in watts) is expended in this interval?

5.50 An engine expends 40.0 hp in moving a car along a level track at a speed of 15.0 m/s. How large is the total force acting on the car in the direction opposite to the motion of the car?

•5.51 A car of mass 942.4 kg accelerates from rest with a constant power output of 140.5 hp. Neglecting air resistance, what is the speed of the car after 4.55 s?

•5.52 A bicyclist coasts down a 7.0° slope at a steady speed of 5.0 m/s. Assuming a total mass of 75 kg (bicycle plus rider), what must the cyclist's power output be to pedal up the same slope at the same speed?

•5.53 A small blimp is used for advertising purposes at a football game. It has a mass of 93.5 kg and is attached by a towrope to a truck on the ground. The towrope makes an angle of 53.3° downward from the horizontal, and the blimp hovers at a constant height of 19.5 m above the ground. The truck moves on a straight line for 840.5 m on the level surface of the stadium parking lot at a constant velocity of 8.90 m/s. If the drag coefficient (K in $F = Kv^2$) is 0.500 kg/m, how much work is done by the truck in pulling the blimp (assuming there is no wind)?

••5.54 A car of mass m accelerates from rest along a level straight track, not at a constant acceleration but with constant engine power, P. Assume that air resistance is negligible.

a) Find the car's velocity as a function of time.

b) A second car starts from rest alongside the first car on the same track, but maintains a constant acceleration. Which car takes the initial lead? Does the other car overtake it? If yes, write a formula for the distance from the starting point at which this happens.

c) You are in a drag race, on a straight level track, with an opponent whose car maintains a constant acceleration of 12.0 m/s^2. Both cars have identical masses of 1000. kg. The cars start together from rest. Air resistance is assumed to be negligible. Calculate the minimum power your engine needs for you to win the race, assuming the power output is constant and the distance to the finish line is 0.250 mi.

Additional Exercises

5.55 At the 2004 Olympic Games in Athens, Greece, the Iranian athlete Hossein Rezazadeh won the super-heavyweight class gold medal in weightlifting. He lifted 472.5 kg (1042 lb) combined in his two best lifts in the competition. Assuming that he lifted the weights a height of 196.7 cm, what work did he do?

5.56 How much work is done against gravity in lifting a 6.00-kg weight through a distance of 20.0 cm?

5.57 A certain tractor is capable of pulling with a steady force of 14.0 kN while moving at a speed of 3.00 m/s. How much power in kilowatts and in horsepower is the tractor delivering under these conditions?

5.58 A shot-putter accelerates a 7.30-kg shot from rest to 14.0 m/s. If this motion takes 2.00 s, what average power was supplied?

5.59 An advertisement claims that a certain 1200.-kg car can accelerate from rest to a speed of 25.0 m/s in 8.00 s. What average power must the motor supply in order to cause this acceleration? Ignore losses due to friction.

5.60 A car of mass $m = 1250$ kg is traveling at a speed of $v_0 = 105$ km/h (29.2 m/s). Calculate the work that must be done by the brakes to completely stop the car.

5.61 An arrow of mass $m = 88.0$ g (0.0880 kg) is fired from a bow. The bowstring exerts an average force of $F = 110.$ N on the arrow over a distance $d = 78.0$ cm (0.780 m). Calculate the speed of the arrow as it leaves the bow.

5.62 The mass of a physics textbook is 3.40 kg. You pick the book up off a table and lift it 0.470 m at a constant speed of 0.270 m/s.

a) What is the work done by gravity on the book?

b) What is the power you supplied to accomplish this task?

5.63 A sled, with mass m, is given a shove up a frictionless incline, which makes a 28.0° angle with the horizontal. Eventually, the sled comes to a stop at a height of 1.35 m above where it started. Calculate its initial speed.

5.64 A man throws a rock of mass $m = 0.325$ kg straight up into the air. In this process, his arm does a total amount of work $W_{net} = 115$ J on the rock. Calculate the maximum distance, h, above the man's throwing hand that the rock will travel. Neglect air resistance.

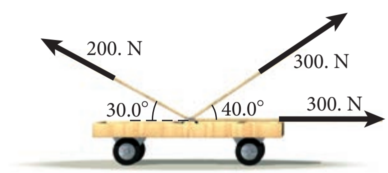
5.65 A car does the work $W_{car} = 7.00 \cdot 10^4$ J in traveling a distance $x = 2.80$ km at constant speed. Calculate the average force F (from all sources) acting on the car in this process.

•5.66 A softball, of mass $m = 0.250$ kg, is pitched at a speed $v_0 = 26.4$ m/s. Due to air resistance, by the time it reaches home plate, it has slowed by 10.0%. The distance between the plate and the pitcher is $d = 15.0$ m. Calculate the average force of air resistance, F_{air}, that is exerted on the ball during its movement from the pitcher to the plate.

•5.67 A flatbed truck is loaded with a stack of sacks of cement whose combined mass is 1143.5 kg. The coefficient of static friction between the bed of the truck and the bottom sack in the stack is 0.372, and the sacks are not tied down but held in place by the force of friction between the bed and the bottom sack. The truck accelerates uniformly from rest to 56.6 mph in 22.9 s. The stack of sacks is 1 m from the end of the truck bed. Does the stack slide on the truck bed? The coefficient of kinetic friction between the bottom sack and the truck bed is 0.257. What is the work done on the stack by the force of friction between the stack and the bed of the truck?

•5.68 A driver notices that her 1000.-kg car slows from $v_0 = 90.0$ km/h (25.0 m/s) to $v = 70.0$ km/h (19.4 m/s) in $t = 6.00$ s moving on level ground in neutral gear. Calculate the power needed to keep the car moving at a constant speed, $v_{ave} = 80.0$ km/h (22.2 m/s). Assume that energy is lost at a constant rate during the deceleration.

•5.69 The 125-kg cart in the figure starts from rest and rolls with negligible friction. It is pulled by three ropes as shown. It moves 100. m horizontally. Find the final velocity of the cart.



$F_1 = 300.$ N at 0°
$F_2 = 300.$ N at 40.0°
$F_3 = 200.$ N at 150.°

•5.70 Calculate the power required to propel a 1000.0-kg car at 25.0 m/s up a straight slope inclined 5.00° above the horizontal. Neglect friction and air resistance.

•5.71 A grandfather pulls his granddaughter, whose mass is 21.0 kg and who is sitting on a swing with ropes of length 2.50 m, backward and releases her from rest. The speed of the granddaughter at the bottom of the swinging motion is 3.00 m/s. What is the angle (in degrees, measured relative to the vertical) from which she is released?

•**5.72** A 65-kg hiker climbs to the second base camp on Nanga Parbat in Pakistan, at an altitude of 3900 m, starting from the first base camp at 2200 m. The climb is made in 5.0 h. Calculate (a) the work done against gravity, (b) the average power output, and (c) the rate of energy input required, assuming the energy conversion efficiency of the human body is 15%.

5.73 An x-component of a force has the dependence $F_x(x) = -cx^3$ on the displacement x, where the constant $c = 19.1$ N/m^3. How much work does it take to oppose this force and change the displacement from 0.810 m to 1.39 m?

5.74 A massless spring lying on a smooth horizontal surface is compressed by a force of 63.5 N, which results in a displacement of 4.35 cm from the initial equilibrium position. How much work will it take to compress the spring from 4.35 cm to 8.15 cm?

•**5.75** A car is traveling at a constant speed of 26.8 m/s. It has a drag coefficient $c_d = 0.333$ and a cross-sectional area of 3.25 m^2. How much power is required just to overcome air resistance and keep the car traveling at this constant speed? Assume the density of air is 1.15 kg/m^3.

MULTI-VERSION EXERCISES

5.76 A variable force is given by $F(x) = Ax^6$, where $A = 11.45$ N/m^6. This force acts on an object of mass 2.735 kg that moves on a frictionless surface. Starting from rest, the object moves from $x = 1.093$ m to $x = 4.429$ m. How much does the kinetic energy of the object change?

5.77 A variable force is given by $F(x) = Ax^6$, where $A = 13.75$ N/m^6. This force acts on an object of mass 3.433 kg that moves on a frictionless surface. Starting from rest, the object moves from $x = 1.105$ m to a new position, x. The object gains $5.662 \cdot 10^3$ J of kinetic energy. What is the new position x?

5.78 A variable force is given by $F(x) = Ax^6$, where $A = 16.05$ N/m^6. This force acts on an object of mass 3.127 kg that moves on a frictionless surface. Starting from rest, the object moves from a position x_0 to a new position, $x = 3.313$ m. The object gains $1.00396 \cdot 10^4$ J of kinetic energy. What is the initial position x_0?

5.79 Santa's reindeer pull his sleigh through the snow at a speed of 3.333 m/s. The mass of the sleigh, including Santa and the presents, is 537.3 kg. Assuming that the coefficient of kinetic friction between the runners of the sleigh and the snow is 0.1337, what is the total power (in hp) that the reindeer are providing?

5.80 Santa's reindeer pull his sleigh through the snow at a speed of 2.561 m/s. The mass of the sleigh, including Santa and the presents, is 540.3 kg. Assuming that the reindeer can provide a total power of 2.666 hp, what is the coefficient of friction between the runners of the sleigh and the snow?

5.81 Santa's reindeer pull his sleigh through the snow at a speed of 2.791 m/s. Assuming that the reindeer can provide a total power of 3.182 hp and the coefficient of friction between the runners of the sleigh and the snow is 0.1595, what is the mass of the sleigh, including Santa and the presents?

5.82 A horizontal spring with spring constant $k = 15.19$ N/m is compressed 23.11 cm from its equilibrium position. A hockey puck with mass $m = 170.0$ g is placed against the end of the spring. The spring is released, and the puck slides on horizontal ice, with a coefficient of kinetic friction of 0.02221 between the puck and the ice. How far does the hockey puck travel on the ice after it leaves the spring?

5.83 A horizontal spring with spring constant $k = 17.49$ N/m is compressed 23.31 cm from its equilibrium position. A hockey puck with mass $m = 170.0$ g is placed against the end of the spring. The spring is released, and the puck slides on horizontal ice a distance of 12.13 m after it leaves the spring. What is the coefficient of kinetic friction between the puck and the ice?

5.84 A load of bricks at a construction site has a mass of 75.0 kg. A crane raises this load from the ground to a height of 45.0 m in 52.0 s at a low constant speed. What is the average power of the crane?

5.85 A load of bricks at a construction site has a mass of 75.0 kg. A crane with 725 W of power raises this load from the ground to a height of 45.0 m at a low constant speed. How long does it take to raise the load?

5.86 A load of bricks at a construction site has a mass of 75.0 kg. A crane with 815 W of power raises this load from the ground to a certain height in 52.0 s at a low constant speed. What is the final height of the load?