

# Chapter 4 Supplement

## Air Resistance

A skydiver relies on a parachute to provide a large force of air resistance (called **drag**). Even with the parachute closed, drag is not negligible when the skydiver is falling rapidly. The **turbulent drag force** exerted on an object moving through air increases dramatically with speed; it is proportional to the *square* of the speed:

$$F_d = bv^2 \quad (4-S1)$$

where  $b$  is a constant that depends on the size and shape of the object. For a given shape,  $b$  is proportional to the cross-sectional area of the object (the area projected onto a flat surface perpendicular to the motion of the object through the fluid). The direction of the drag force is opposite to the direction of motion. (At low speeds, the drag force is viscous rather than turbulent and is proportional to  $v$  rather than to  $v^2$ . Viscous drag forces are discussed in Chapter 9.)

Since the drag force increases as the speed increases, a falling object approaches an equilibrium situation in which the drag force is equal in magnitude to the weight but opposite in direction. The velocity at which this equilibrium occurs is called the object's **terminal velocity**. The direction of the terminal velocity is always downward if there are no forces other than air resistance and gravity. The magnitude of the terminal velocity is called the terminal speed. As the velocity approaches the terminal velocity, the acceleration gets smaller and smaller. The acceleration is zero when the object falls straight down at its terminal velocity.

In Fig. 4.S1, a tennis ball and a sheet of paper are released from rest and fall through the air. The strobe photograph shows the positions of the two at equal 0.1 s time intervals. The ball has a terminal speed of about 30 m/s, so air resistance is negligible for the speeds shown in the photo. The displacement of the ball in equal time intervals increases linearly, showing that its acceleration is constant. The paper has a very large surface area for its small mass. As a result, its terminal speed is much smaller—less than 1 m/s. The displacement of the paper barely changes from one exposure to the next, showing that it is falling at nearly constant velocity and its acceleration is small.

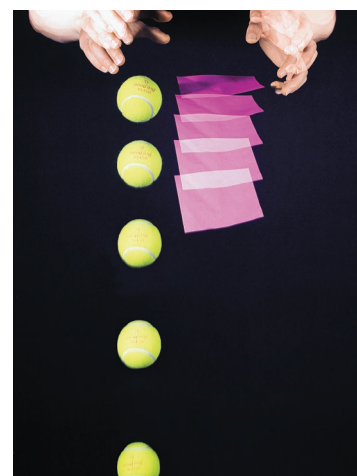
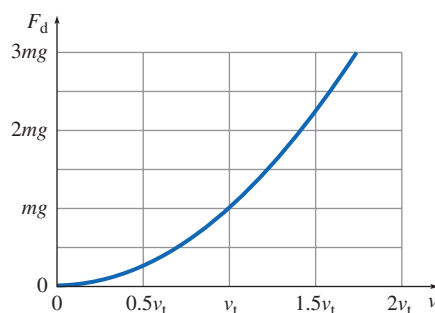
The net force on an object falling at its terminal velocity is zero, so the drag force is equal in magnitude to the weight (Fig. 4.S2). Therefore,  $F_d = mg = bv_t^2$ , where  $v_t$  is the terminal speed, and

$$v_t = \sqrt{\frac{mg}{b}} \quad (4-S2)$$

Using Eq. (4.S1) to eliminate  $b$  from Eq. (4.S2), the magnitude of the drag force at any speed  $v$  is

$$F_d = mg \frac{v^2}{v_t^2} \quad (4-S3)$$

Figure 4.S3 shows a graph of  $F_d$  versus  $v$ . Equation (4.S3) gives only the magnitude of the drag force. The direction is opposite to the object's velocity.



**Figure 4.S1** A stroboscopic photograph shows a tennis ball and a sheet of paper falling through air. The time between exposures is 0.1 s. Only 0.3 s after release, the paper is falling at a nearly constant velocity (notice the equal distance between the last three exposures) while the velocity of the ball continues to increase.



**Figure 4.S2** FBD for an object falling at its terminal velocity.

**Figure 4.S3** Graph of the magnitude of the drag force as a function of an object's speed. Note that the drag force is equal in magnitude to the object's weight ( $mg$ ) when  $v = v_t$ .

**Table 4.S1** Some Typical Terminal Speeds

Object	Terminal Speed (m/s)
Feather	0.5
Snowflake	1
Raindrop	7
Skydiver (open parachute)	5–9
Basketball	20
Baseball	40
Skydiver (spread-eagle)	50–60
Skydiver (dive)	100
Bullet	100

The terminal speed of an object depends on its size, shape, and mass (see Table 4.S1). A skydiver with the parachute closed will reach a terminal speed of about 50 m/s ( $\approx 100$  mi/h) in the spread-eagle position, or as much as 100 m/s ( $\approx 200$  mi/h) in a dive. When the parachute is opened, the drag force increases dramatically—the larger surface area of the parachute means that more air has to be pushed out of the way. The terminal speed with the parachute open is typically about 9 m/s (20 mi/h). When the parachute is opened, the skydiver is initially moving *faster* than the new terminal speed. For  $v > v_t$ , the drag force is larger in magnitude than the weight, making the acceleration *upward*. The skydiver slows down, approaching the new terminal speed. Note that the terminal speed is not the maximum possible speed; it is the speed that the falling object approaches, regardless of initial conditions, when the only forces acting are drag and gravity.

### Example 4.S1

#### Skydivers Falling Freely

Two skydivers have identical parachutes. Their masses (including the parachutes) are 62.0 kg and 82.0 kg. Which of the skydivers has the larger terminal speed? What is the ratio of their terminal speeds?

**Strategy** With identical parachutes, we expect the same amount of drag at a given speed. The more massive skydiver must fall faster in order for the magnitude of the drag force to equal his weight, so the 82.0 kg skydiver should have a larger terminal speed. For the ratio of the terminal speeds, we first find how the terminal speed depends on mass, all other things being equal. Then we work by proportions.

**Solution** At terminal speed  $v_t$ , the drag force must be equal in magnitude to the weight.

$$mg = F_d = bv_t^2$$

Since the parachutes are identical, we expect the constant  $b$  to be the same for the two divers. Therefore,

$$v_t \propto \sqrt{m}$$

The more massive skydiver has a larger terminal speed—he must move faster in order for the drag force to equal his larger weight. The ratio of the terminal speeds is

$$\frac{v_{t2}}{v_{t1}} = \sqrt{\frac{m_2}{m_1}} = \sqrt{\frac{82.0}{62.0}} = 1.15$$

The terminal speed of the 82.0 kg diver is 1.15 times that of the less massive skydiver, or 15% faster.

**Discussion** The 82.0 kg skydiver is 32% more massive:

$$\frac{82.0 \text{ kg}}{62.0 \text{ kg}} = 1.32$$

but his terminal speed is only 15% greater. That is because the drag force is proportional to the *square* of the speed. It only takes a 15% greater speed to make the drag force 32% greater:

$$(1.15)^2 = 1.32$$

#### Practice Problem 4.S1 Air Resistance at Terminal Speed

A pilot has bailed out of her airplane at a height of 2000 m above the surface of the Earth. The mass of the pilot plus the parachute is 112 kg. What is the force of air resistance when the pilot reaches terminal speed?

### Example 4.S2

#### Dropping the Ball

A basketball is dropped off a tall building. (a) What is the initial acceleration of the ball, just after it is released? (b) What is the acceleration of the ball when it is falling at its terminal speed? (c) What is the acceleration of the ball when falling at half its terminal speed?

**Strategy** We choose the positive  $y$ -axis to point upward as usual. The ball is dropped from rest, so *initially* the only force acting is gravity—the drag force is zero when the velocity is zero. Once the ball is moving, air drag contributes to the net force on the basketball.

**Solution** (a) The initial acceleration is the free fall acceleration ( $\vec{a} = \vec{g}$ ) since the drag force is zero.

(b) Once the ball reaches terminal velocity, the drag force is equal in magnitude to the weight of the ball, but acts upward. The net force on the ball is zero, so the acceleration is zero. At terminal velocity,  $\vec{a} = 0$ .

(c) When the ball is falling at half its terminal speed, the drag force is significant, but it is smaller than the weight. The net force is down and, therefore, the acceleration is still downward, though with a smaller magnitude. The magnitude of the drag force at any speed is given by

$$F_d = mg \frac{v^2}{v_t^2}$$

The drag force acts in the direction opposite to the velocity. The ball is falling straight down, so the drag force is upward. The net vertical force is

$$\sum F_y = F_d - mg = mg \frac{v^2}{v_t^2} - mg = mg \left( \frac{v^2}{v_t^2} - 1 \right)$$

Now we apply Newton's second law:

$$\sum F_y = ma_y$$

Solving for the acceleration yields

$$a_y = g \left( \frac{v^2}{v_t^2} - 1 \right)$$

At a time when the speed is half the terminal speed,

$$v = \frac{1}{2}v_t \quad \text{and} \quad \frac{v^2}{v_t^2} = \frac{1}{4}$$

$$a_y = g \left( \frac{1}{4} - 1 \right) = -\frac{3}{4}g$$

so that the acceleration of the ball is

$$\vec{a} = \frac{3}{4}\vec{g}$$

where  $\vec{a}$  and  $\vec{g}$  both point downward.

**Discussion** How do we know when air resistance is negligible? If we know the approximate terminal speed of an object, then air resistance is negligible as long as its speed moving through the air is small compared to the terminal speed.

#### Practice Problem 4.S2 Acceleration Graph Sketch

Sketch a qualitative graph of  $v_y(t)$  for the basketball using a  $y$ -axis that is positive pointing upward. [Hint: At first, air resistance is negligible. After a long time, the basketball is in equilibrium. Figure out what the beginning and end of the graph look like and then connect them smoothly.]

### Conceptual Questions

1. What is meant by the terminal speed of a falling object? Can an object ever move through air faster than the object's terminal speed? If so, give an example.
2. In free fall, neglecting air resistance, the  $x$ - and  $y$ -components of the motion are independent. Are the components independent when air resistance is significant? Why or why not? [Hint: The drag force has a magnitude determined by the *speed* of the projectile and a direction opposite to the velocity.]
3. Two balls are falling straight down at the same speed. Explain why the drag force is larger on the larger ball (the one with a larger cross-sectional area).
4. Explain why the drag force on a falling object increases as the object's speed increases.

### Problems

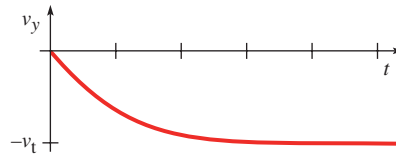
1. The terminal speed of a ping-pong ball is 9.0 m/s. By neglecting air resistance, estimate how long it takes for a ping-pong ball falling from rest to reach half of this speed.
2. The terminal speed of a golf ball is 40 m/s. (a) By neglecting air resistance, estimate how long it takes for a golf ball falling from rest to reach half of this speed. (b) Estimate how far the ball falls during this time.
3. A paratrooper with a fully loaded pack has a mass of 120 kg. The force due to air resistance on him when falling with an unopened parachute has magnitude  $F_d = bv^2$ , where  $b = 0.14 \text{ N}\cdot\text{s}^2/\text{m}^2$ . (a) If he is falling with an unopened parachute at 64 m/s, what is the force of air resistance acting on him? (b) What is his acceleration? (c) What is his terminal speed?

4. A bobcat weighing 72 N jumps out of a tree. (a) What is the drag force on the bobcat when it falls at its terminal velocity? (b) What is the drag force on the bobcat when it falls at 75% of its terminal velocity? (c) What is the acceleration of the bobcat when it falls at its terminal velocity? (d) What is the acceleration when it falls at 75% of its terminal velocity?
5. How far must something fall for its speed to get close to its terminal speed? (a) Use dimensional analysis to come up with an estimate based on the object's terminal speed  $v_t$  and the gravitational field strength  $g$ . Estimate this distance for (b) a raindrop ( $v_t \approx 7$  m/s) and (c) a skydiver in a dive ( $v_t \approx 100$  m/s).

### Answers to Practice Problems

4.S1 1.10 kN

4.S2



### Answers to Problems

4.S1 0.46 s

4.S3 (a) 570 N upward (b)  $5.0 \text{ m/s}^2$  downward (c) 92 m/s

4.S5 (a)  $d = \frac{v_t^2}{g}$  (b) 5 m (c) 1 km