Chapter 9 Supplement

Turbulent Flow

As discussed in Chapter 9, **turbulent** fluid flow is unsteady: the flow velocity at any point changes erratically. The flow cannot be described by simple streamlines, because the flow is chaotic and the fluid does not flow in neat layers. Vortices (regions of swirling fluid) appear and move through the fluid, stretching and twisting and interacting with one another as they go.

Onset of Turbulence A dimensionless quantity called the **Reynolds number**, named for Irish physicist Osborne Reynolds (1842–1912), is used as a rough guide to determine whether fluid flow is laminar or turbulent.

For flow through a cylindrical pipe, the Reynolds number is defined as

$$R = \frac{\rho v D}{\eta} \tag{9-S1}$$

where ρ is the density, v is the average fluid speed, D is the pipe diameter, and η is the viscosity. For a smooth pipe, the flow is laminar for R less than about 2300 and fully turbulent for R greater than about 4000. In the transition between those values, the flow can be partly laminar and partly turbulent. Surface roughness affects the onset of turbulence, so these numbers are only approximate.

The same expression [Eq. (9-S1)] is used to find the Reynolds number for a fluid flowing past a spherical object (or a spherical object moving through a fluid). In this case, v is the speed of the object relative to the undisturbed fluid far from the object and D is the diameter of the object. For R less than about 10, the flow is laminar and the drag force on the sphere is proportional to v. In transitional flow ($R \approx 10$ to 1000), eddies form, but the flow is still largely laminar. True turbulence occurs for R greater than about 1000; now the drag force is proportional to v^2. See the Chapter 4 Supplement on Air Resistance for more information about turbulent drag forces.

Application: Turbulent Blood Flow

Typical Reynolds numbers for blood flow in the body are

- Arterioles and venules, less than 1
- Arteries and veins, 100–500
- Aorta and venae cavae, about 3300

Thus, we expect some turbulence in the aorta and venae cavae.

Turbulence can arise any time the flow is constricted. In healthy individuals, sounds are produced by turbulent flow through the heart valves as they open and close. A blood pressure cuff wrapped around the upper arm constricts in the brachial artery; the sound of the ensuing turbulent flow can be heard through a stethoscope. No sound is heard when the pressure of the cuff is higher than the systolic pressure, because the flow is completely cut off. When the pressure in the cuff is less than the diastolic pressure, the sound stops because there is no longer a constriction. A cardiologist can use a stethoscope to detect *bruits*—sounds caused by turbulent blood flow past obstructions such as those caused by atherosclerosis.

Surface Tension

As discussed in Section 9.11, the surface of a liquid acts like a stretched membrane under tension. The **surface tension** of a liquid is the force per unit length with which the surface pulls on its edge. The direction of the force is tangent to the surface at its edge. The symbol for surface tension is γ, the lowercase Greek letter gamma. Its SI units are N/m, or equivalently, J/m^2. If the edge of the liquid is a straight line, then the forces are all in the same direction, and the force on an edge of length L due to surface tension has magnitude

$$F = \gamma L \qquad \textbf{(9-S2)}$$

Table 9.S1 compares surface tension with pressure.

Surface tension is caused by the cohesive forces that pull the molecules toward one another. These cohesive forces are strong enough to enable a liquid to sustain a *negative* absolute pressure: you can *pull* on a liquid under the right conditions. If not for the capability of sap to be pulled up the xylem by negative pressure, trees would be much more limited in height than they actually are (see Problem 3).

Inside a volume of water, the cohesive forces pull on a molecule in all directions since there are other molecules all around. However, a molecule at the surface of the water has neighbors on only one side. A result of the inward cohesive forces on molecules at the surface is that the water tends to minimize its surface area—the surface becomes like a stretched membrane under tension, compressing the fluid inside. A drop of water forms a spherical shape (ignoring gravity and other external forces) because a sphere has the smallest possible surface area for a given volume of fluid.

The surface tension is transmitted throughout the surface, just as in a stretched membrane. For any imaginary line in the surface, each side of the line pulls on the other side with a force per unit length equal to γ.

Water has a higher surface tension (γ = 0.073 N/m at 20˚C) than most other common liquids. Mercury is an exception (0.49 N/m at 20˚C). The surface tension of a liquid decreases with higher temperatures because the cohesive forces between molecules are weaker. For water, the surface tension at 100°C is about 20% less than at 20°C.

Bubbles A bubble is a spherical liquid surface enclosing air or another gas. The liquid surface has a surface tension that tries to contract the bubble, but the pressure of the enclosed gas pushes outward on the surface until equilibrium is attained. The pressure inside the bubble must be larger than the pressure outside so that there is a net outward force due to pressure (Fig. 9.S1). We call the difference the excess pressure $\Delta P = P_{\text{in}} - P_{\text{out}}$. The excess pressure depends both on the surface tension and the size of the bubble.

For a bubble with a single surface, such as an air bubble within a liquid or a drop of water in air, the excess pressure is

$$\Delta P = \frac{2\gamma}{r} \qquad (9\text{-S3})$$

Table 9.S1 Comparison Between Pressure and Surface Tension

Property	Pressure	Surface tension
Units	Force per unit area (N/m^2)	Force per unit length (N/m)
External force exerted on	Boundary of the fluid volume (objects touching the surface)	Boundary of the fluid surface (objects touching an edge of the surface)
Expansion/contraction	Outward (tends to expand the fluid)	Inward (tends to contract the fluid)

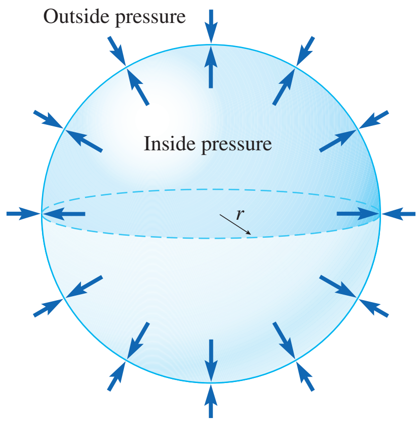

Figure 9.S1 In a soap bubble, the air pressure inside is higher than the pressure outside.

where r is the radius of the bubble. A soap bubble in air has both an inner surface and an outer surface. Because each surface has surface tension, the excess pressure is twice as large:

$$\Delta P = \frac{4\gamma}{r} \tag{9-S4}$$

Note that the excess pressure is inversely proportional to the radius of the bubble. A small bubble has a larger excess pressure than a large bubble. This is an example of a more general rule: surface tension is most significant when dealing with small objects, and is negligible in large enough objects.

Capillarity When a thin glass tube, open at both ends, is placed with one end under the surface of some water, the water rises up the tube. In equilibrium there is a column of water in the tube of height h above the surface of the water outside the tube (Fig. 9.S2a). How is this possible?

Adhesive forces enable the glass to pull upward on the water. A concave **meniscus** (curved surface) forms at the top of the column of water. The adhesive forces between glass and water are strong, so the contact angle θ is small, and the radius of curvature of the meniscus is roughly equal to the radius of the tube (r). Then the water surface is approximately hemispherical.

Just in a bubble, there is a pressure difference across the surface. The net force on the water surface due to surface tension is $2\pi\gamma r$, upward, so the net force due to the pressure difference must be equal to that, but directed downward. Therefore, the pressure difference is $\Delta P = 2\gamma/r$, with the pressure higher on top. The surface tension has reduced the pressure just below the meniscus to $P_{atm} - 2\gamma/r$, where P_{atm} is atmospheric

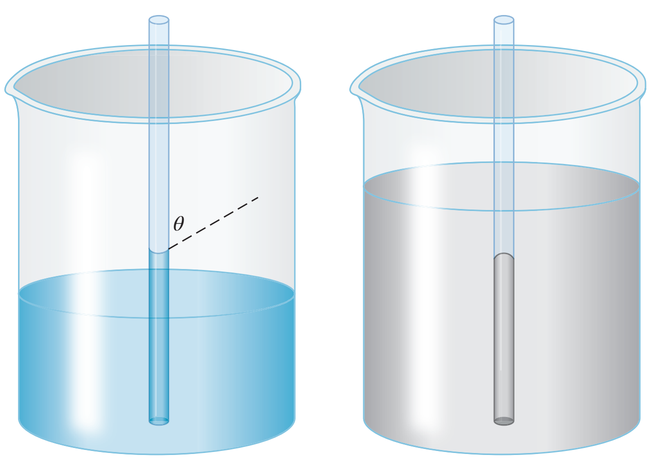

Figure 9.S2 Capillary tube in (a) water and (b) mercury

pressure. The water must, therefore, rise a height h in the capillary so that the pressure is P_{atm} at the water level outside:

$$P_{\text{atm}} - \frac{2\gamma}{r} = P_{\text{atm}} - \rho g h \tag{9-S5}$$

or

$$h = \frac{2\gamma}{\rho g r} \tag{9-S6}$$

For a liquid such as mercury, which doesn't wet the glass, the meniscus is convex. The pressure is therefore higher *below* the surface, and the mercury level in the tube is *lower* than the level of the mercury outside the tube (Fig. 9.S2b).

Problems

(see also the problems at the end of Chapter 9 in the text)

1. A rain drop of diameter 0.60 mm falls at 11 m/s. Do you expect the flow of air around the drop to be laminar or turbulent? Explain.
2. When turning off a faucet, you can usually hear that the sound of the flowing water gets louder just before the flow stops. What makes it get louder?
3. *What makes sap rise in trees?* One possibility is capillary action. Model the xylem vessels in the tree as thin glass tubes with radii between 0.02 mm and 0.20 mm, and assume the sap is water. What would be the maximum height that sap could rise in trees due to capillary action alone? (This would be the maximum height of the tree if there were no "pumping.") Is it plausible that capillary action is primarily responsible for the flow of sap?
4. ✦ If the adhesive forces between a liquid and the surface of a capillary tube are not strong, then the meniscus will have a radius larger than the radius of the tube. (a) Suppose that the meniscus makes a contact angle of θ with the surface of the capillary tube of radius r, as in Fig. 9.S2a. The force on the liquid surface due to the tube at any point is directed tangent to the meniscus. Find the total force on the liquid surface due to the tube. [*Hint*: Sum the vertical components.] (b) How high will the liquid rise in a capillary tube of radius r? (c) In the case of strong adhesion such as glass-water, the contact angle approaches zero. Show that your expression agrees with Eq. (9-S6) in this case.

Answers to Problems

1. $R = \rho v D/\eta = 220$, so we expect predominantly laminar flow.

3. For 0.02 mm radius, $h = 2\gamma/(\rho g r) = 0.7$ m. Since trees get much taller than that, it is not plausible that capillary action is primarily responsible.