Chapter 10 Supplement

Period of a Physical Pendulum

To find the period of a physical pendulum, we first find the net torque acting on the physical pendulum and then use the rotational form of Newton's second law. Taking torques about the rotation axis, only gravity gives a nonzero torque. If the pendulum has mass m and the distance from the axis to the center of mass is d, then the torque is

$$\tau = F_{\perp}r = -(mg \sin \theta)d \qquad (10\text{-S1})$$

where θ is the angle indicated in Fig. 10.S1 and $F_{\perp} = mg \sin \theta$ is the component of the weight perpendicular to the line from the axis to the center of mass. The other component of the gravitational force, $mg \cos \theta$, passes through the axis of rotation, so it does not contribute to the torque. In Eq. (10-S1), both τ and θ are positive if they are counterclockwise; the negative sign says that they always have opposite sign, since the torque always acts to bring θ closer to zero. Assuming small amplitudes, $\sin \theta \approx \theta$ (in radians) and the torque is

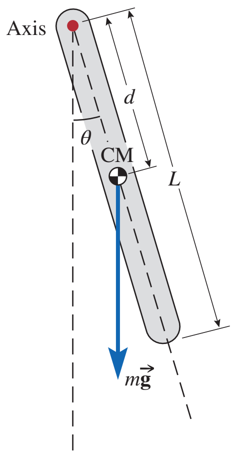

Figure 10.S1 A physical pendulum.

$$\tau \approx -mgd\theta \qquad (10\text{-S2})$$

Thus, for small-amplitude oscillations the restoring torque is proportional to the displacement angle θ, just as the restoring force was proportional to the displacement for the simple pendulum and the mass on a spring.

The net torque is equal to the rotational inertia times the angular acceleration:

$$\tau = I\alpha \approx -mgd\theta \qquad (10\text{-S4})$$

and the angular acceleration is

$$\alpha \approx -\frac{mgd}{I}\theta \qquad (10\text{-S5})$$

Since the angular acceleration is a negative constant times the angular displacement from equilibrium, we indeed have SHM. Equation (10-S5) is analogous to the equation for the linear acceleration of an oscillating spring

$$\alpha_x = -\omega^2 x \qquad (10\text{-S6})$$

if

$$\frac{mgd}{I} = \omega^2 \qquad (10\text{-S7})$$

Note that ω in Eqs. (10-S6) and (10-S7) represents the angular frequency, not the angular speed. Therefore, the angular frequency of a physical pendulum for small-amplitude oscillations is

$$\omega = 2\pi f = \sqrt{\frac{mgd}{I}} \qquad \mathbf{(10\text{-S8})}$$

and the period is

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{mgd}} \qquad \mathbf{(10\text{-S9})}$$

Problems

1. A pendulum consists of a uniform rod of length 140 cm suspended from one end. What is its period of oscillation?
2. A pendulum consists of a uniform rod of length 60 cm suspended from one end. It is swinging with an amplitude (maximum displacement angle) of 0.16 rad. What are its maximum angular speed and maximum angular acceleration?

Answers to Problems

1. 1.9 s