Chapter 12 Supplement

Attenuation

Attenuation is the decrease in intensity of a sound wave as it propagates away from its source. Even in an idealized medium, geometrical attenuation occurs because the total power carried by the wave is spread over an increasingly large area. In a real medium, other processes contribute to the decrease in a wave's intensity. **Absorption** is the dissipation of some of the wave's energy (converting it into thermal energy). Absorption is caused by viscosity, heat conduction, and molecular relaxation, in which some of the translational kinetic energy of the molecules is converted into rotational and vibrational energy. The sound wave can also be **scattered** in directions other than the forward direction of propagation, due to inhomogeneity in the medium.

In the idealized case of a plane wave, for which there is no spreading of the wave, attenuation due to absorption and scattering is an exponential function of distance:

$$I = I_0 e^{-2\alpha x} \tag{12-S1}$$

The **attenuation coefficient** α depends on properties of the medium such as the density, viscosity, thermal conductivity, and speed of sound. In air, it depends on temperature and relative humidity. The reduction in sound intensity level with distance is approximately (8.7 dB)α. For example, if the attenuation coefficient for 4.0 MHz ultrasound in soft tissue is $\alpha = 0.40\ \text{cm}^{-1}$, then the reduction in intensity level with distance is $(8.7\ \text{dB})(0.40\ \text{cm}^{-1}) = 3.5\ \text{dB/cm}$.

The attenuation coefficient has a strong frequency dependence. For example, α is proportional to the square of the frequency in a pure monatomic gas. In other media, the relationship is more complicated. In air at 20°C and 40% relative humidity, $\alpha = 0.54\ \text{km}^{-1}$ at 1 kHz and $\alpha = 22\ \text{km}^{-1}$, about 40 times larger, at 10 kHz.

Application: Ultrasonic imaging

The frequencies typically used in ultrasonic imaging range from 2 to 15 MHz. The attenuation coefficient in soft tissue is approximately proportional to frequency throughout this range. A rough value for soft tissue is

$$\alpha \approx (0.1\ \text{cm}^{-1}\cdot\text{MHz}^{-1})f \tag{12-S2}$$

Thus, at 2 MHz, $\alpha \approx (0.1\ \text{cm}^{-1}\cdot\text{MHz}^{-1})(2\ \text{MHz}) \approx 0.2\ \text{cm}^{-1}$, whereas at 15 MHz, $\alpha \approx (0.1\ \text{cm}^{-1}\cdot\text{MHz}^{-1})(15\ \text{MHz}) \approx 1.5\ \text{cm}^{-1}$. As a result of the exponential dependence on distance, the frequency chosen makes a dramatic difference on how far the ultrasound can penetrate. For example, the reduction in intensity at a depth of 4 cm is:

$$I/I_0 = e^{-2\alpha x} = e^{-2(0.2\ \text{cm}^{-1})(4\ \text{cm})} = 0.02 \text{ at } 2\ \text{MHz} \tag{12-S3}$$

$$I/I_0 = e^{-2\alpha x} = e^{-2(1.5\ \text{cm}^{-1})(4\ \text{cm})} = 6 \times 10^{-6} \text{ at } 15\ \text{MHz} \tag{12-S4}$$

Since $\lambda \propto f^{-1}$, higher frequencies are desirable for finer resolution of details, but attenuation imposes a limit on how high a frequency is practical.

Supersonic Flight

The pressure variation shown in Fig. 12.S1 is the sonic boom caused by an airplane flying at a constant velocity faster than sound. The variation is called an *N*-wave (because the pressure graph is shaped like the letter N). The initial, sudden rise in pressure is the shock wave formed at the nose of the plane. Then the pressure decreases linearly,

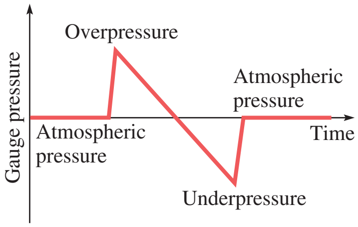


Figure 12.S1 The variation in pressure at a point on the ground as a supersonic plane flies overhead.

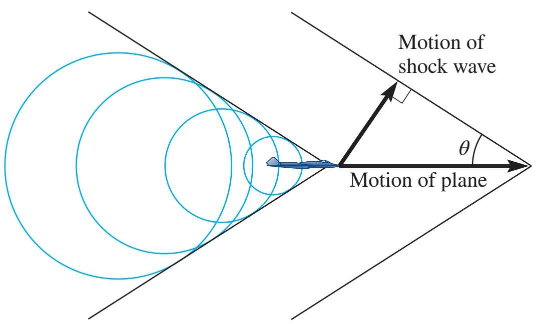
becoming less than atmospheric, until it abruptly returns to atmospheric as the tail wave passes. The duration of the sonic boom ranges from about 0.1 s for small military planes up to about 0.5 s for the Space Shuttle.

The shock cone spreads out as it moves away from the plane; the higher the plane, the lower the intensity of the sonic boom. The largest intensity boom occurs directly under the flight path, but the boom is heard throughout the cone area. The sudden onset and release of pressure can even do physical damage to something under its path, breaking a window or shaking a building. It is not that the magnitude of the pressure change is so large, but the change happens in a very short time interval. The overpressure at ground level for supersonic planes flying at typical operating altitudes ranges from 50 to 100 Pa. Aircraft flying at low altitudes can cause overpressures of as much as 7 kPa.

The speed of a supersonic plane is often given as a *Mach number*, named for Austrian physicist Ernst Mach (1838–1916). The Mach number is the ratio of the speed of the plane to the speed of sound. A plane flying at Mach 3.3 where the air temperature is 11°C is moving at 3.3×338 m/s = 1100 m/s with respect to the air.

Problems

1. A 6.0 mm thick sheet of cork is applied to a wall for soundproofing. The attenuation coefficient at 500 Hz is 0.20 mm^{-1}. By what percentage does the intensity of a 500 Hz sound wave change as it passes through the cork?
2. The intensity of ultrasound is $I = 0.45I_0$ after it passes through 1.0 cm of soft tissue. What is the intensity after passing through 3.0 cm of tissue?
3. The attenuation coefficient for 4.0 MHz ultrasound in muscle is 0.52 cm^{-1}. What is the rate (in dB/cm) that the intensity level of 4.0 MHz ultrasound decreases as it passes through muscle?
4. A supersonic plane moves at speed v_{plane}; the speed of sound is v_{sound}. The conical shock wave makes an angle θ with the direction of motion of the plane, as shown in the figure. Show that $\sin \theta = v_{sound}/v_{plane}$. [*Hint*: Consider how far and in what direction the plane and the shock wave move during a time interval Δt.]
5. A plane is flying at supersonic speed at an elevation where the speed of sound is 322 m/s. The shock wave cone forms with an angle $\theta = 22.0°$ with the direction of motion of the plane (see Problem 4). (a) What is the Mach number for this plane? (b) How fast is the plane traveling?
6. ✦ An airplane is flying 1.0 km directly over your position on the ground at Mach 2.0. How far from that overhead position will the airplane have moved along its horizontal flight path when you hear the sonic boom? [*Hint*: See Problem 4.]
7. A wind tunnel is used to simulate the flight of a plane. Air at 20°C is blown past the model plane at very high speeds. If a shock cone angle of $\theta = 40.0°$ develops, how fast is the air moving? [*Hint*: See Problem 4.]



Problems 4–7

Answers to Problems

12.S1 90.9% decrease
12.S3 4.5 dB/cm
12.S5 (a) 2.67 (b) 860 m/s
12.S7 534 m/s