Chapter 27 Supplement

Radii of the Bohr Orbits

An electron of mass m_e in a circular orbit of radius r at speed v has rotational inertia $I = m_e r^2$ and angular momentum $L = I\omega$:

$$L = I\omega = m_e r^2 \omega = m_e v r \qquad \text{(27-S1)}$$

since $\omega = v/r$. Then the Bohr condition on angular momentum becomes

$$m_e v r_n = n\hbar \quad (n = 1, 2, 3, \ldots) \qquad \text{(27-S2)}$$

where r_n is the radius of the orbit with angular momentum $n\hbar$ Using Newton's second law $(\Sigma\vec{\mathbf{F}} = m\vec{\mathbf{a}})$ applied to an electron held in circular orbit by the Coulomb force (see Problem 1), Bohr showed that the only orbital radii that satisfy Eq. (27-S2) are

$$r_n = \frac{n^2\hbar^2}{m_e k Z e^2} \quad (n = 1, 2, 3, \ldots) \qquad \text{(27-S3)}$$

Problems

1. ✦ A single electron orbits a nucleus with charge $+Ze$ at constant speed in a circle of radius r. (a) Using Coulomb's law, write an expression for the magnitude of the electric force on the electron in terms of r, Z, the elementary charge e, and the Coulomb constant k. (b) Apply Newton's second law to the electron and use it to show that the electron's speed is

$$v = \sqrt{\frac{kZe^2}{m_e r}}$$

[*Hint*: The electron is in uniform circular motion.] (c) Use the Bohr assumption about the electron's angular momentum, Eq. (27-S2), to show that the radius of the n^{th} Bohr orbit is

$$r_n = \frac{n^2\hbar^2}{m_e k Z e^2}$$

2. ✦ A single electron orbits a nucleus with charge $+Ze$ at constant speed in a circle of radius r. (a) What is the electron's kinetic energy in terms of k, Z, e, and r? Use the expression for the electron's speed found in Problem 1. (b) What is the electron's electric potential energy? (Assume $U = 0$ when $r = \infty$.) (c) Show that the electron's mechanical energy $(K + U)$ is $E = -kZe^2/(2r)$. (d) Show that the energy of the n^{th} Bohr orbit is

$$E_n = -\frac{m_e k^2 Z^2 e^4}{2n^2\hbar^2}$$

3. According to the Bohr model, the speed of the electron in the ground state of singly ionized helium (He^+, with $Z = 2$) is 4.4×10^6 m/s. Use this information to find the speed of an electron in the first excited state of triply ionized beryllium (Be^{3+} with $Z = 4$).

Answers to Problems

27.S1 (a) $F = kZe^2/r^2$

27.S3 4.4×10^6 m/s