

11

Differential costing and linear programming

Objectives

By the end of this chapter you will be able to:	Page
1. Explain what is meant by differential costing	323
2. Differentiate between relevant and irrelevant data for decisions	323
3. Define some terms used in differential costing	323
4. Distinguish between quantitative and qualitative data and information for decisions	327
5. Analyse the benefits and costs for some decisions: advertising; accept or reject special order prices or offers; make or buy a product or component; add, modify or drop a service, product or department; sell now or process further; shut down or seasonal closure	327
6. Explain when and how linear programming is used for decisions	345
7. Solve some simple constraint problems using graphical analysis	345

Introduction

Differential costing is not an alternative costing method. It involves analysing data and providing information that will assist in making decisions. Despite its name, the data used in the analysis is not restricted to the possible costs involved in a decision but also to revenues. Decisions also need to take into account some factors that are difficult to put a dollar value on, and these qualitative factors must be included in any report to decision makers.

Any decision requires examining the difference between the costs and the benefits of alternate proposals. This is where the term ‘differential costing’ comes from—an analysis of the difference in costs and/or benefits between two or more competing proposals. This chapter looks at a number of situations in which quantitative and qualitative information is required. The chapter finishes with an introduction to linear programming, a technique that is used when there are constraints on the available resources which will affect decisions.

Relevant and irrelevant data

Relevant costs (and revenues) are those that will affect, or be affected by, the decision. For example, a business has to make a decision on whether it should produce an extra 100 items or not. The total cost of raw material for these 100 items, where each item requires \$100 of raw material, is relevant to the decision. If the items are made, the raw material cost will go up by \$10 000—that is, the difference in raw material costs between the two alternatives (to make or not make the items) is \$10 000. If the extra 100 units might affect the current selling price, this is also a relevant factor but much more difficult to quantify (put a dollar value on).

Irrelevant costs (and revenues) are those that will not be different regardless of the decision. If, in the example above, the factory rates of \$15 000 per annum will not change regardless of whether the 100 units are made or not, the cost of these rates is not relevant. The rates will be \$15 000 per annum whether or not the extra items are made. Irrelevant data should be ignored when providing information for decisions.

Relevant information is, therefore, the information that has a bearing on the future and is different for the alternatives that are part of the decision.

Which costs are relevant depends on the decision being made, and a number of different types of decisions will be examined in this chapter. Deciding which costs, revenues and qualitative data are relevant is the first and a difficult step in any analysis. With some categories of costs the degree of relevance is fairly easy to determine. Some of the categories of costs and their degree of relevance are outlined below.

Historical costs

Decisions are made about the future—you cannot decide on something that has already occurred. You can learn from past decisions but you cannot change them once they have been implemented. Accountants may use **historical cost** data as a basis for

predicting the future. The historical cost data produced from a normal costing system is not of itself relevant data to be used in the decision, but the predictions based on those historical costs may be relevant to the decision.



Example 11.1

Benjamin's Plastics rents industrial factory space and has plastic extruding machinery that is working at about 75% capacity. This computer controlled machinery cost \$118 800 two years ago. A toy manufacturer and assembler has asked for a quotation for the supply of doll parts, which would require some additional casual labour and overtime.

The historical cost of the extruding machinery is not relevant to this decision as there is spare capacity. Benjamin's Plastics cannot decide not to purchase the machinery! The predicted additional cost of casual labour and overtime is relevant to this decision as these costs will be incurred in the future if the quotation is accepted and the doll parts are made.



Self-test problem 11.1

Indicate which of the following costs are historical costs:

- Cost of machinery purchased one year ago.
- Expected increase in casual labour costs due to accepting a new order.
- Amount paid for setting up and incorporating the business.
- Budgeted sales revenue from additional sales.
- Special packing that will be required if an overseas order is accepted.
- Pallets purchased last year for stacking inventory of raw materials.
- The cost of clothes purchased by a retailer that are on sale at a reduced price because they are out of fashion.
- The cost of new machinery that is being considered as a replacement in the next financial year.

Sunk costs

Sunk costs are historical costs. They are costs that have already been incurred. Sunk costs are irrelevant to any decision as they cannot be changed by a decision now or in the future. The historical costs in Example 11.1 and Self-test problem 11.1 were sunk costs.



Self-test problem 11.2

Sydney Coach Builders has some obsolete (but new) seats that are in the books at the original cost of \$58 000. These seats are no longer used in new coaches being built as they do not comply with NSW seat belt regulations. These seats could be modified to comply with regulations at an expected cost of \$24 000, which would give a saving of \$52 000 on buying new seats. South-West Treks Pty Ltd of Tasmania still operates coaches that were built with the old seats and have offered to purchase the entire stock for \$34 000.

- Which cost is a sunk cost?
- Which of the figures is not relevant?
- What should Sydney Coach Builders do—sell the old seats and buy new ones or modify the old seats? Provide figures to support your answer.
- If South-West Treks only offered \$27 000, would your advice to Sydney Coach Builders be different? Why?

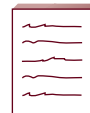
Imputed costs

Imputed costs do not actually involve a cash outlay and they are not recorded in the ledger. These costs are hypothetical costs that represent the value of a resource used. Examples of imputed costs include the interest cost of capital invested, salary cost of a sole proprietor and rental value of buildings owned. While these imputed costs are not recorded in the books of account, they are relevant and important in arriving at valid comparisons. They bridge the gap between accounting for costs and economic reality.

Imputed costs may also be classified as opportunity costs, which are discussed below.

Example 11.2

Ebis MM Pty Ltd has a multimedia product it would like to develop. It is looking to buy an established business that has the expertise and equipment to carry out the development. Two businesses are on offer, one owned by a sole trader and the other a limited liability company. The sole owner acts as the manager for the sole-trader business. If the sole-trader's business is purchased, Ebis will need to employ a manager. Summary profit statements and return on capital for the current year show:



	Sole-trader business	Company business
	\$	\$
Sales	489 900	740 000
<i>less</i> Expenses		
Materials	189 760	285 940
Salaries and wages—general	87 600	140 700
Manager's salary		105 000
Other overheads	54 000	81 550
	<u>331 360</u>	<u>613 190</u>
Net profit (before income tax)	<u>\$158 540</u>	<u>\$126 810</u>
Capital of the business	<u>\$3 000 000</u>	<u>\$3 000 000</u>
Rate of net profit (before tax) on capital	5.3%	4.2%

To make a valid comparison between these two businesses, the figures for the sole trader might be 'adjusted' by imputing (adding in) a manager's salary (if this business is purchased a salary will need to be paid to replace the sole owner). Other possible imputed figures will depend on the methods of financing the purchase of either business. If a manager's salary is an imputed cost and estimated at \$105 000 per annum, the net profit of the sole trader's business will be \$53 540 and the rate of return on capital is 1.8%. Given these comparative figures, a purchase price for each business can now be established.

Self-test problem 11.3

Indicate which of the following would give rise to an imputed cost when a decision about alternatives is to be made. Discuss how the imputed cost would be calculated.

- (a) A loan of \$100 000 to a supplier as part of a purchase contract at a rate of interest that is 4% below similar risk loans.

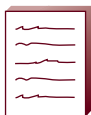


- (b) Labour has been paid for time taken to install new machinery.
- (c) An investment in a capital asset.
- (d) A sole trader provides his/her personal car to the business.
- (e) Electricity cost incurred and paid.
- (f) Partners in a trading partnership do not draw salaries.

Opportunity costs

In general, **opportunity costs** exist whenever a scarce resource has multiple uses. An opportunity cost is the value of one alternative forgone by deciding on another use of the resource. For example, using funds (a resource) to invest in one particular asset means that the alternative earnings from using those funds is forgone (passed up). Opportunity costs are taken into consideration when making decisions because the earnings from alternative uses must be compared.

Opportunity costs measure sacrifices associated with alternative opportunities. If there is no sacrifice, no opportunity cost exists. Resources such as factories, equipment, money, time and employee talent may all have opportunity costs. Opportunity costs may be relevant to a particular decision but may be overlooked in practice because it may be difficult to put a value on them.



Example 11.3

K. & N. Canteens Pty Ltd operates a number of canteens in colleges throughout Victoria. One canteen needs remodelling to cater for an increased number of students. This remodelling will take two weeks. What opportunity cost is involved and how would K. & N. Canteens minimise this cost?

Solution

While the canteen is closed, no sales will be made. The opportunity cost is the lost contribution margin on those sales. This cost would be minimised by closing down and remodelling during a holiday break.



Self-test problem 11.4

Hills Warehouse & Transport Pty Ltd leases a warehouse at \$25 000 per month. Management is considering alternatives for using available space. Two alternatives are:

- (a) sublease the space for \$5000 per month;
- (b) use the space as a turn-around area for a new route—expected net revenues amount to \$52 800 per annum.

For *each* of these alternatives, indicate what the opportunity cost is. Which alternative would you recommend?

Future costs/benefits

As the name implies, **future costs** or **benefits** are those that are expected or budgeted. Decisions are made about the future so these are the costs or benefits that are used in making a decision. Future costs or benefits are not necessarily all relevant. When a future cost or benefit is the same under two proposals it is not relevant.

You should now be able to do Question 11.1.



Quantitative and qualitative data and information

Accountants usually provide data or information expressed in numerical terms. Data or information that is stated in numbers is known as **quantitative** data. Typically, these numbers will show which alternative gives the higher profit or the least cost. Decisions are rarely made based on quantitative information alone. Other factors cannot be expressed effectively in numerical terms but must be considered when making a rational decision. These are called **qualitative** factors. For example, when considering a long-term supply contract, management would need to take into consideration the reputation of the supplier, the supplier's quality control, whether the supplier will deliver on time, the financial position and future stability of the supplier, and alternative suppliers if the one selected were to cease business. If the supplier is in an overseas country, the stability and political situation in that country will also need to be considered.

The different decisions in the remainder of this chapter consider both the quantitative and the qualitative factors involved.

Self-test problem 11.5

For each of the following factors, indicate whether it is a qualitative or a quantitative factor in a decision:



- (a) The cost of materials expected to be consumed in manufacturing a product.
- (b) The cost of importing raw materials.
- (c) The political stability in an overseas country from which supplies may be purchased.
- (d) The possible loss of customer goodwill from closing down a product line.
- (e) The possible loss of current customers by selling some product at a special short-term price to only one new customer.
- (f) Future government policy on the GST.
- (g) A product sold in Australia requires 10% added for the GST whereas export sales of this product are currently GST-free.
- (h) A soft drink manufacturer has obtained a long-range weather forecast for the forthcoming summer and this indicates very hot and dry conditions.
- (i) The budgeted sales of soft drinks for the predicted very hot and dry summer.
- (j) The perceived environmental impact of building an oil refining plant on the Derwent River in Tasmania.

Differential cost analysis

There are two main ways in which the data can be analysed and presented as information:

1. present total figures for the alternatives in columns and show a final column for differences; or

2. present only the differences, or increments, in revenues and costs (the reason why this topic is also called 'incremental costing').

In the decisions that follow, you should try and present the information in ways that will be easy to follow by managers who may not necessarily be accountants. Include both qualitative and quantitative information in answers, and only include information that would be relevant in arriving at a rational decision.

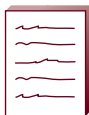
Most of the decisions that follow are short-term, non-routine decisions. This means that they are one-off decisions that are not major strategic decisions. Strategic decisions involving large amounts of finance usually affect the business over a number of years and may change the way it does business or finances operations. Short-term decisions, on the other hand, often ignore the time value of money as they are assumed to be for a short period of time.

Advertising

Many businesses will set an advertising budget for the year and that amount is spent. Often there is no analysis on the effectiveness of such advertising and advertising is seen as a means of keeping the product or the business name in the public's 'eye'. There are occasions when a particular advertising expenditure is decided upon for a particular project (e.g. the launch of a new product) or in an effort to increase the sales of a current product. This advertising expenditure must be justified and is subject to management decision.

A lot of judgment is required in determining how much advertising will increase sales. It is one of the more difficult problems that business managers have to deal with. You would probably be aware of some companies that ask you where you heard about them (e.g. newspapers, television, radio, telephone). They do this so that they can get some idea of how effective their advertising strategies are.

If management decides to spend more on advertising, it needs to know what type of advertising to spend it on (e.g. television, radio, newspaper, mail drops). Advertising costs in each sector of the media would need to be compared with projected benefits from each type of advertising. Remember that the extra cost of advertising needs to be compared with the extra 'margin' from sales generated by the advertising.



Example 11.4

If a publisher has an overall margin of $33\frac{1}{3}\%$ from its sales, by how much would sales need to increase to pay for an advertising expenditure of \$30 000?

The answer is not \$30 000. The answer is $\$30\,000/0.333 = \$90\,000$.

This is because the 'cost of sales' also increases when sales increase. In our example, sales of \$90 000 mean that there would be costs incurred in publishing the books. As the margin is $33\frac{1}{3}\%$, the cost would be $66\frac{2}{3}\%$ of \$90 000, namely \$60 000. Therefore, sales would have to increase by \$90 000 to pay for the extra advertising expense of \$30 000. Naturally, management would prefer sales greater than \$90 000 so that additional profit can be earned.

Self-test problem 11.6

If a retailer has an overall margin of 20% from its sales, by how much would sales need to increase to pay for an advertising expenditure of \$36 000?

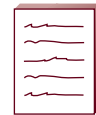


You should now be able to do Question 11.2.



An analysis of advertising is often carried out on an incremental basis. The objective is to determine the point at which extra money spent on advertising no longer produces an increase in the total contribution margin.

Example 11.5



Epsanal Pty Ltd makes ice and ice-cream products sold under its own brand name. The manager of the ice products division has proposed an increase in the advertising budget in November and December as new flavours and packaging are being introduced. A market analysis for the forthcoming six months 1 October to 31 March shows the following budgeted sales of cartons with varying increases in advertising expenditure:

	Cartons
No increase in advertising budget	17 800
An increase of \$25 000 in advertising	20 300
An increase of \$50 000 in advertising	22 300

The sales price of \$38 per carton and the unit variable costs of \$27 per carton are not expected to change over the six months. Fixed selling and administrative costs allocated to the ice division, excluding the additional advertising, are expected to be \$125 000 for the six months.

Should the advertising budget be increased? Explain.

Solution

The contribution margin per carton is \$11 (sales price of \$38 less unit variable cost \$27).

	Total analysis		
Advertising level	Present	Plus \$25 000	Plus \$50 000
Cartons	<u>17 800</u>	<u>20 300</u>	<u>22 300</u>
Contribution margin	\$195 800	\$223 300	\$245 300
Selling and administrative	<u>\$125 000</u>	<u>\$150 000</u>	<u>\$175 000</u>
Net profit	<u>\$70 800</u>	<u>\$73 300</u>	<u>\$70 300</u>

	Incremental analysis	
Advertising level	Plus \$25 000	Plus \$50 000
Increase in cartons sold	<u>2 500</u>	<u>2 000</u>
Increase in contribution margin	27 500	\$22 000
Additional advertising cost	<u>\$25 000</u>	<u>\$25 000</u>
Increase (decrease) in net profit	<u>\$2 500</u>	<u>(\$3 000)</u>

It can be seen from this relatively simple decision that profit will probably increase if advertising is increased by \$25 000. Spending a further \$25 000 (i.e. \$50 000 in total) will actually reduce profit overall even though sales will increase.

Notice that the contribution margin was used in the answer. This is a usual technique in short-term decisions, and where the total contribution margin exceeds any increases in fixed costs, the proposal would be accepted.



Self-test problem 11.7

A. D. Manufacturing Company has spent \$289 780 on the research and development of a digital board that improves the graphics on inexpensive computers. The manager of Research and Development estimates that it will take another \$86 000 to finish development and to make the board a marketable product. The sales team has surveyed the market and has provided the following estimates of advertising launch costs and the resulting sales levels for the first six months of sales:

Advertising \$	Sales Units
40 000	13 000
80 000	19 000
100 000	24 000
120 000	26 000
140 000	27 000

The projected selling price is \$38.00 per board and the cost accountant has estimated unit variable manufacturing and distribution costs at \$30.00 per board. The current fixed manufacturing and administration costs are \$381 520 per annum and this amount is not expected to change if this product is made. The advertising launch costs are not included in the cost accountant's figures.

Management must make a decision on whether or not to complete the development and the amount to spend on launching the product. Advise management and provide a report on the quantitative factors.



You should now be able to do Question 11.3.

Accept or reject a special order

Setting the price for a one-off order is a short-term decision that depends not only on the economic analysis (e.g. special price, costs involved and resulting profit on the order) but several qualitative factors. This decision is not about setting the normal selling prices of products, which is a substantial topic deserving a chapter of its own and requiring a thorough economic and marketing analysis.

A special order may be received with an offer price that may be below the normal selling price. In fact, the price may be below the apparent full cost. At first sight it may seem sensible to reject an offer that is below normal selling price, especially if the price is below our calculated total unit cost.

You should recall from earlier chapters that a unit cost that includes allocated fixed costs might not be the appropriate cost for making decisions. The factors that you need to consider in most decisions on accepting or rejecting a special offer are:

Quantitative factors:

- Unit special selling price.
- Unit variable cost of making, selling and distribution.
- The contribution margin per unit on this order (this is, of course, the result of the above two factors—selling price less variable cost. If there is no contribution margin, then the special price would be unacceptable in economic terms.
- Any additional special costs that may be incurred on this order (e.g. special packing for an export order). These are deducted from the contribution margin to establish if there is still a positive profit on the order. If there is a profit on the order, the qualitative factors will need to be considered (remember that qualitative factors are those that would be difficult to value in dollar terms).

Qualitative factors in special order decisions:

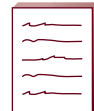
- The effect on current customers of accepting this order. Customers who found out about a special price given to another customer would not be happy and there is, therefore, a possibility of losing future sales (with the consequent loss of the contribution margin on those sales) and the loss of customer goodwill.
- Spare capacity must be available and, therefore, current fixed costs will not be affected. If there is no spare capacity available, the opportunity cost of the facilities used on the special order will need to be added in to the relevant costs.
- Even if the present special order price is not profitable, the prospect of increasing market share might lead to a decision to accept the special price.

Example 11.6

Poly-Flooring Manufacturing Pty Ltd makes and sells vinyl flooring in sheets and tiles. The present factory facilities are operating at about 80% capacity, producing 78 000 square metres of varying thickness.

One of the products produced is an antistatic vinyl sheet that can be hot-air welded together to form seamless flooring and wall covering. This is used in hospitals and laboratories as it is easier to keep germ-free. The normal selling price throughout Australia is \$8.00 per square metre, excluding freight. A summary of production costs for making 11 000 square metres of this product in the last six months shows:

	Total	Per metre
	\$	\$
Direct raw materials	9 350	0.85
Packing materials	2 750	0.25
Direct labour	15 400	1.40
Variable manufacturing overhead	3 300	0.30
Fixed manufacturing overhead	20 900	1.90
Variable selling and administrative overhead	2 750	0.25
Fixed selling and administrative overhead	11 550	<u>1.05</u>
		<u><u>\$6.00</u></u>



An agent in the Philippines states that he can obtain a contract for the supply of 6000 square metres of antistatic sheeting at A\$4.20 per square metre. The agent would be entitled to 10% commission on the sale price. Special export packaging would cost an additional \$2400 on this order. Freight and insurance on this order, not included in the selling price, would cost \$1500.

Advise management on whether the order should be accepted or rejected.

Solution

As the capacity using the current facilities is 97 500 square metres, there is available capacity. Fixed manufacturing costs will, therefore, not change and are not relevant to this decision. It is assumed that both variable and fixed selling and administrative costs will also not be affected by this decision and are, therefore, not relevant.

	\$	\$
Sales price		4.20
<i>less</i> Variable manufacturing costs		
Direct raw materials	0.85	
Packing (present)	0.25	
Direct labour	1.40	
Variable factory overhead	<u>0.30</u>	<u>2.80</u>
Contribution margin per square metre		<u>1.40</u>
Total contribution from sale 6000 square metres × \$1.40		8400
<i>less</i> Special order costs		
Freight	1500	
Export packing	2400	
Agent's commission (6000 × \$0.42)	2520	<u>6420</u>
Profit from this order		<u>1980</u>

The order should be accepted as it contributes towards fixed costs and profits. Capacity is available and even after relaxing the assumption about current variable selling expense there would still be a profit.

A special price on an export sale is unlikely to affect current customers in Australia. There is also the possibility of expanding the export sales business if this contract goes ahead.

It would be more profitable if the company could increase local sales volume at prices above \$4.20 per square metres but the effect on local customers would need to be considered.

Self-test problem 11.8

A company at present operates at 80% of capacity, producing 50 000 units per annum. A budget for the year is:

		Total \$	Per unit \$
Sales		500 000	10.00
<i>less</i> Manufacturing expense			
Variable	100 000		
Fixed	<u>200 000</u>	<u>300 000</u>	<u>6.00</u>

(Continued)

		Total	Per unit
		\$	\$
Gross profit		200 000	4.00
less Selling and administration expense			
Variable	50 000		
Fixed	<u>50 000</u>	<u>100 000</u>	<u>2.00</u>
Net profit		<u>100 000</u>	<u>2.00</u>

An order is received from Tanzania for 5000 units at \$5 FOB. Special export packing will increase manufacturing costs by \$0.50 per unit on this order. The present variable selling and administrative costs will be reduced by \$0.20 per unit on this order. A commission of 10% on sales price will be payable to the Tanzanian agent.

Required

- Calculate the increase or decrease in profit if this order is accepted.
- Discuss factors other than profit that should be taken into account when considering short-term special order price decisions.

You should now be able to do Questions 11.4 and 11.5.

Make or buy a product or component

Many manufacturers use component parts in the assembly of finished products. For example, General Motors (Holden) purchases and installs transmissions from Borg-Warner and tyres from several tyre sellers.

Manufacturers may produce component parts themselves or purchase them from an outside source. For example, a refrigerator manufacturer may produce the electric motor itself or may purchase the electric motor from a specialist and install it in the refrigerator.

The factors that you need to consider in most decisions on making or buying a component part are:

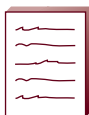
Quantitative factors:

- Variable costs of producing the part.
- Unit variable cost of purchasing from external supplier.
- Costs that would be avoided if the change is from producing the part to purchasing it.
- Contribution margin available from alternative uses of the space made available by purchasing rather than making.

Qualitative factors in special order decisions:

- Control over quality of the part—usually if the part is made, the business has control over the quality.
- The long-term stability of the supplier, including the political stability of the country of the supplier if the part is purchased from overseas.
- Adverse relationship with the supplier if the decision is made to make a part instead of purchasing.

- Future prices to be paid if the decision is made to purchase.
- Spare capacity must be available and therefore current fixed costs will not be affected.



Example 11.7

K. & N. Canteens Pty Ltd operates a canteen in a major metropolitan college. At present, sandwiches are made at the counter by canteen staff. Part-time staff are employed in this job during busy periods of the day, lunch and evening.

A catering business, Chestertons Pty Ltd, has proposed the supply of ready-made sandwiches packed in plastic containers. The canteen manager has provided you with the following estimated figures:

Present monthly costs of sandwich operation during term time	\$
Staff costs involved with making sandwiches	7 220
Staff costs involved with serving the customer	4 500
Cost of space used by sandwich bar (e.g. rent, electricity)	3 000
Portion of overheads (e.g. depreciation, cleaning, insurance)	4 000
Materials (e.g. bread, fillings)	<u>9 780</u>
	<u><u>\$28 500</u></u>

Average number of sandwiches sold in a month is 30 000.

Average price of producing and selling a sandwich is, therefore, \$0.95 each.

The offer from Chestertons is for the supply of 30 000 sandwiches per month at a cost per sandwich of \$0.55 each. The supply will be limited to a maximum of eight varieties per day and varieties will be subject to available supplies at the time.

Recommend to K. & N. Canteens Pty Ltd whether they should continue making the sandwiches or purchase them from Chestertons. Include in your recommendation both quantitative and qualitative factors that need to be considered.

Solution

The first step is to determine the relevant costs in the decision. Certain assumptions may need to be made and these should be clearly stated in any report or recommendation. In this case, the costs included in the manager's estimates that are probably not relevant are the 'occupancy' costs of space used, the share of the other overhead costs and the staff costs of serving customers. These costs will probably not be affected either way, as sandwiches will still be sold in the space allowed.

The relevant costs of making sandwiches that will be saved if the sandwiches are purchased from Chestertons will be:

	\$
Staff costs involved with making sandwiches	7 220
Materials (e.g. bread, fillings)	<u>9 780</u>
	<u>17 000</u>

The average variable cost of producing a sandwich is, therefore, \$0.57 each.

The quote from Chestertons is slightly cheaper than this variable cost and might be accepted but other factors need to be considered:

- The quality of the supply from Chestertons and possible complaints from college students if the quality is not high. This may result in jeopardising the canteen lease.
- The financial stability of Chestertons.
- The continuity of supply and supply when required.
- The lack of variety and reliance on the supplier for variety.
- Staff morale by dropping some casual staff if sandwiches are purchased.

Self-test problem 11.9

Elecwiz Ltd manufactures a number of kitchen gadgets each requiring a small electric motor. It has a separate motor production section making all the various sizes of motors required. The most popular motor made by this section has a unit cost of:

	\$
Direct material	7.50
Direct labour	6.40
Variable factory overhead	2.00
Fixed factory overhead (2 hours @ \$2.00 per hour)	<u>4.00</u>
	<u>19.90</u>

This unit cost is based on an annual requirement of 30 000 of these motors.

The local agent of a Korean manufacturer has stated that his principal could supply an identical motor at a price of \$17 into store. The production manager at Elecwiz is interested in this proposal, especially as the cost department has indicated that fixed factory costs would be reduced by \$20 000 per annum if production of this particular motor stopped.

Required

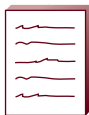
- Advise management on whether or not it should pursue investigation of the agent's proposal. Give figures supporting your advice and state any assumptions that you have made.
- Using the figures above, advise management of the price below which it would be economically beneficial to purchase motors rather than make them. State other qualitative factors that management would need to consider if the quote was below this price.

You should now be able to do Questions 11.6 and 11.7.

Add, modify or drop a service, product or department

Another common management decision is the mix of products (or departments) that will maximise returns to the business. Typical departmental or product line statements will not necessarily provide the best figures for this type of decision because they usually include joint costs that have been allocated to the departments or products using an arbitrary method.

Once again, the problem revolves around relevant costs and revenue and decisions are based on the contribution provided by the product or department compared with any change in fixed costs arising from the decision.



Example 11.8

Manacc Ltd makes three hardware products, A, B and C, in its factory. The products are sold to hardware distributors throughout Australia and no stocks are kept of finished products.

A summary profit statement by product for the last year shows:

	A	Products B	C	Total \$
Units	80 000	90 000	25 000	
Sales	<u>800 000</u>	<u>450 000</u>	<u>500 000</u>	<u>1 750 000</u>
<i>less Costs</i>				
Variable	500 000	270 000	300 000	1 070 000
Fixed	<u>250 000</u>	<u>150 000</u>	<u>220 000</u>	<u>620 000</u>
	<u>750 000</u>	<u>420 000</u>	<u>520 000</u>	<u>1 690 000</u>
Profit/(loss)	<u>50 000</u>	<u>30 000</u>	<u>(20 000)</u>	<u>60 000</u>

Management is concerned with the loss of \$20 000 shown on product C. It is considering dropping this product from its operation. Advice to management will depend on a number of assumptions. Each of the following possible assumptions should be treated independently.

- (a) Assume that management has no immediate plans to utilise the capacity made available by dropping product C, and that none of the fixed costs will be reduced should product C be dropped.

If the profit statement above is recast after dropping product C, the total column will show:

	\$
Sales	<u>1 250 000</u>
<i>less Costs</i>	
Variable	770 000
Fixed	<u>620 000</u>
	<u>1 390 000</u>
Profit/(loss)	<u>(140 000)</u>

Obviously, the company will be worse off by \$200 000. A quicker way of answering this type of question is to consider the relevant figures only. The answer may then be restated as:

	\$
Sales that will be lost	500 000
Variable costs avoided	<u>300 000</u>
Lost contribution margin from dropping C	200 000
less Savings in fixed costs	<u>Nil</u>
Increase (decrease) in company profit	<u>(200 000)</u>

- (b) Now assume that \$150 000 of the fixed costs that are allocated to product C represent supervision and other costs that will be avoided (not incurred) in the short term if product C is dropped. Assume also that management has no immediate plans to utilise the capacity made available by dropping product C.

	\$
Lost contribution margin from dropping C	200 000
less Savings in fixed costs	<u>150 000</u>
Increase (decrease) in company profit	<u>(50 000)</u>

The answer shows that product C should not be dropped because the company will still be \$50 000 worse off if it is dropped.

- (c) Assume that the facilities made available by dropping product C could be used to produce another 50 000 units of product B. As there is a market for 50 000 additional units of product B, these could all be sold at the current selling price of product B. To answer this question only the relevant contribution margins from each alternative need be compared.

	\$
Lost contribution margin from dropping C	200 000
Additional contribution margin from B (50 000 units @ \$2.00 per unit ^(a))	<u>100 000</u>
Increase (decrease) in company profit	<u>(100 000)</u>

(a) The contribution margin per unit for B is calculated as (\$450 000 – \$270 000) divided by 90 000 units = \$2.00 per unit.

The company would be worse off by \$100 000 by swapping from producing product C to producing 50 000 additional units of product B.

- (d) Suppose that management has for some time considered producing another item, product D. Market estimates indicate that 60 000 units could be sold in a year at a selling price of \$7.00 per unit. The variable cost per unit to make and sell product D will be \$3.50. The new product could be made using the facilities at present allocated to either product B or product C (i.e. one of either product B or product C would have to be deleted from the range). Advise management, with figures, if either product B or product C should be deleted in favour of producing product D.

Once again, the only figures that need to be considered are the relative contribution margins from each of the three products.

	\$
Contribution margin provided by C	200 000
Contribution margin provided by B	180 000
Contribution margin provided by D (60 000 units @ \$3.50 per unit)	210 000

Based on the figures alone, product D might replace product B. Other qualitative factors that need to be considered and that are not stated in the case are:

- Are any of the products complimentary—that is, will dropping one affect the sales from any of the others?
- While the analysis is in the short term, product changes are often part of a longer term strategic plan. Will the market for product D, for example, continue in the longer term?
- Adding a new product usually requires a greater cost in advertising and additional working capital to build up raw material inventory. None of these costs are included above.

In general, decisions about changes in a product mix are made by examining the contribution margin of each product and the possible changes in avoidable fixed costs.



Self-test problem 11.10

Doolight Manufacturing Pty Ltd manufactures three types of flooring products, cork, vinyl and parquet. A budgeted profit statement for the forthcoming year shows:

	Vinyl \$	Cork \$	Parquet \$	Total \$
Sales	500 000	400 000	600 000	1 500 000
less Variable costs	<u>300 000</u>	<u>250 000</u>	<u>350 000</u>	<u>900 000</u>
Contribution margin	200 000	150 000	250 000	600 000
less Fixed costs	<u>170 000</u>	<u>180 000</u>	<u>200 000</u>	<u>550 000</u>
Profit (loss)	<u>30 000</u>	<u>(30 000)</u>	<u>50 000</u>	<u>50 000</u>

The fixed costs of the business have been classified into those that are directly related to the individual departments making each product (mainly supervision costs) and those that have been allocated on the bases of relative floor space. This analysis shows:

	Vinyl \$	Cork \$	Parquet \$	Total \$
Direct (avoidable)	75 000	50 000	80 000	205 000
Allocated on relative space	<u>95 000</u>	<u>130 000</u>	<u>120 000</u>	<u>345 000</u>
Total fixed costs	<u>170 000</u>	<u>180 000</u>	<u>200 000</u>	<u>550 000</u>

Required

- (a) Management is considering whether or not to cease the manufacture and sale of cork flooring because it cannot raise the selling price without affecting volume. If no other use can be made of the available facilities, advise management of the consequences of stopping the production of cork.

- (b) Assume that the facilities made available by stopping production of cork can be used to produce a new product, artificial grass. Estimates for this product are for \$500 000 sales with variable costs of \$300 000 and additional annual fixed costs of \$60 000. Advise management on whether or not artificial grass should replace the cork product.

You should now be able to do Questions 11.8 and 11.9.



Sell now or process further

Chapter 16 deals more fully with the manufacturing situation where more than one product emerges from a single process. Techniques for allocating the joint cost between the products are shown. A product emerging from a joint process may have a market value as is or it may be processed further to create another product. A decision may be required on whether to sell the product at the end of the joint process (at 'split-off' point) or to process it further and sell it as another product.

The important part of this decision is that the costs of processing up to the point where it becomes a separately identifiable product may not be relevant. The decision involves further processing costs and the change in sales price to be gained from that further processing.

Example 11.9

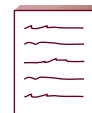
Thomas Blakely Ltd obtains cylinder stock from the Shell refinery and after further refining it produces two joint products, bright stock and petrolatum.

The cost of cylinder stock and the joint process amounts to \$300 000 per month and the output is 100 000 litres of bright stock and 50 000 litres of petrolatum. No adjustments can be made to the process to vary these relative amounts. The selling prices of these products at split-off point (the end of the joint process) are:

Bright stock	\$2.80 per litre
Petrolatum	\$1.05 per litre

Management is considering a proposal to further process the petrolatum into superfine oil to be sold to a sewing machine manufacturer. This further process also requires packaging the oil in small plastic bottles with the sewing machine manufacturer's brand name. The sewing machine manufacturer currently imports this oil from Korea at an in-store cost of A\$3.80 per litre.

The factory manager has produced the following figures for a senior management meeting at which a decision will be made on providing the sewing machine manufacturer with a price for the superfine oil:



	Cost per litre
	\$
Cost from joint process (\$300 000/150 000 L)	2.00
Further variable processing cost of petrolatum	1.95
Additional fixed cost (\$25 000 per month/50 000 L)	<u>0.50</u>
	4.45
Plus mark-up 25%	<u>1.10</u>
Price	<u><u>5.55</u></u>

Comment on the factory manager’s figures and establish the cost per litre that is relevant to this decision. If the sales price cannot exceed the current import price, should Thomas Blakely Ltd sell at split off-point or process and sell the superfine oil to the sewing machine manufacturer?

Solution

The cost from the joint process is not relevant to the decision because this is incurred regardless of the decision to process *after* the split-off point. The further processing costs and the selling price at split-off point is the minimum price below which it would be better to sell at split-off.

	\$
Further processing cost per litre	2.45
Selling price at split-off point	<u>1.05</u>
Minimum selling price	<u><u>3.50</u></u>

If the sales price after further processing is \$3.80 per litre, the company will be better off by \$15 000 per month as follows:

	\$	\$
Sales of superfine oil (50 000 L @ \$3.80)		190 000
<i>less</i> sale value of petrolatum (50 000 L @ \$1.05)		<u>52 500</u>
Incremental revenue from processing further		137 500
<i>less</i> Further processing costs		
Variable (50 000 L @ \$1.95)	\$97 500	
Fixed facility cost	<u>\$25 000</u>	<u>122 500</u>
Increase in contribution margin from processing further		<u><u>\$15 000</u></u>

Any sale price over \$3.50 will result in an increased contribution margin from further processing.



Self-test problem 11.11

Tantara Co-Op Ltd has a joint process from which two joint products emerge. Monthly average production and sales prices are:

Product X—20 000 kg. Market price is \$10.00 per kilogram.

Product Y—50 000 kg. Market price is \$1.00 per kilogram.

The joint process cost to produce these quantities is \$140 000.

The relative quantities of each product cannot be changed in the joint process.

Management is considering processing product Y further into a new product called Z. The additional processing cost would be \$1.50 per kilogram of product Y input into the process. The market price of product Z would be \$4.00 per kilogram. The yield in this additional process will be 80%—that is, for each input of 10 kilograms of product Y, only 8 kilograms of product Z will be derived.

The factory clerk has prepared the following schedule for management:

	\$	\$
Allocation of \$140 000 joint costs		
Product X (20 000/70 000 × \$140 000)		40 000
Product Y (50 000/70 000 × \$140 000)		<u>100 000</u>
		<u>140 000</u>
Net income from new product Z		
Revenue from Z (40 000 kg @ \$4.00 per kilogram)		160 000
less Cost of producing Z		
Allocated joint cost (product Y)	\$100 000	
Additional processing (50 000 × \$1.50)	<u>\$75 000</u>	175 000
Loss from further processing Y into Z		15 000

Required

Management has asked you to examine these figures and to make a recommendation.

You should now be able to do Questions 11.10 and 11.11.

Shut down or seasonal closure

A business that is operating at a loss may consider shutting down. A profitable business that operates on a seasonal basis (e.g. a ski lodge) may also consider shutting down in the 'off-season'. A decision to shut down needs to take into consideration the following:

- The business may be better off remaining open if the products or services cover their variable costs and contribute towards the recovery of fixed costs.
- Certain costs will continue even if the business closes down. For example, depreciation, council rates, insurance and security of premises will continue after shutting down.
- The costs of retrenchment of current staff and other costs of shutting down.
- The costs of reopening that will be incurred after shutting down.
- The cost of recruitment and training of new staff when reopening after shutting down.



Example 11.10

Laksa Catering Company operates a cafeteria and coffee shop at a college. The area is leased for \$192 000 per annum. A profit statement for a month during term and a month during vacation shows:

	During term time	During vacation
	\$	\$
Revenue	<u>287 650</u>	<u>128 900</u>
<i>less</i> Direct costs		
Wages	148 000	82 000
Supplies	88 290	30 325
Cutlery and crockery	6 900	1 500
Lease	8 000	8 000
Cleaning	5 000	5 000
Sundry fixed overheads	2 800	2 800
Depreciation equipment	<u>2 000</u>	<u>2 000</u>
Total direct costs	<u>260 990</u>	<u>131 625</u>
Net profit (loss)	26 660	(2 725)

Additional information

The lease, sundry fixed overheads and a yearly cleaning contract for \$10 000 are allocated equally each month. Cutlery and crockery replacement costs are expensed when purchased.

Management is considering closing the operation during holidays because it shows a loss. Advise management, including any assumptions that you make.

Solution

Contribution during holidays	\$
Revenue	<u>128 900</u>
<i>less</i> Relevant direct costs	
Wages	82 000
Supplies	30 325
Cutlery and crockery	<u>1 500</u>
	<u>113 825</u>
Net contribution	\$15 075

In the short term, only those costs that would be avoided are relevant to this decision. The operation contributes at least \$15 075 towards the lease and other fixed overhead during the holidays and the business would be worse off by this amount if it closed down during this period. There would also be additional staff costs for retraining each time the operation started up and staff morale might be affected.



Self-test problem 11.12

Mt Wilhelmina Lodge operates in the Victorian snowfields. Last year's revenue statement by season and by the two main revenue areas showed:

	Winter		Summer	
	Accommodation	Restaurant	Accommodation	Restaurant
	\$	\$	\$	\$
Revenue	<u>2 190 670</u>	<u>498 000</u>	<u>487 000</u>	<u>98 000</u>
less Costs				
Staff—operations	772 100	232 600	153 650	33 300
Staff—administration	160 000	55 000	95 000	55 000
Supplies	289 590	158 855	58 775	27 325
Crockery, cutlery, linen	88 420	19 900	18 430	3 100
Rates and insurance	12 550	4 420	5 490	600
Electricity, gas, wood	38 980	15 880	12 889	1 760
Cleaning	25 890	10 400	5 980	2 885
Repairs and maintenance				
Normal	8 995	6 880	2 400	1 980
Major			29 470	9 525
Depreciation				
Lodge	150 040	8 090	150 040	8 090
Restaurant equipment		2 600		2 600
Other overheads (fixed)	<u>12 000</u>	<u>6 000</u>	<u>4 000</u>	<u>2 000</u>
Total costs	<u>1 558 565</u>	<u>520 625</u>	<u>536 124</u>	<u>148 165</u>
Net profit (loss)	632 105	(22 625)	(49 124)	(50 165)

Additional information

1. Staff costs include all salary oncosts. Assume that all staff costs for the restaurant would cease if that operation were closed. Also assume that total accommodation administrative costs would continue, even if the accommodation were closed during summer.
2. Rates and insurance, depreciation and other fixed overheads are allocated to periods and operations using various methods.
3. Major repairs and maintenance are carried out in the off-season. These repairs would still be necessary even if the lodge were closed in summer.
4. Cutlery and crockery replacement costs are expensed when purchased.

Required

Management is considering whether or not to close restaurant operations and whether or not to close down in the summer months. You are required to advise management, taking into considering the additional information. You will have to make some assumptions and state other matters that would need to be considered.

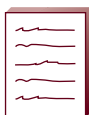
You should now be able to do Question 11.12. 

Decisions with constraints on resources

In many problems there may be limits to the availability of resources. A limitation on the availability of a resource is called a *constraint*. A problem with only one constraint does not require any special techniques, but linear (or in some cases integer) programming techniques are used if there are several constraints.

Contribution margin per unit of constraint

Where a resource is limited, the contribution margin per unit of that constraint must be determined.



Example 11.11

A company makes two products, A and B. The costs and contribution margins per unit for these products have been calculated as follows:

	A	B
	\$	\$
Sales price	<u>26.00</u>	<u>16.00</u>
<i>less</i> Variable costs		
Direct material	8.00	6.00
Direct labour	6.00	3.00
Overhead	<u>6.00</u>	<u>3.00</u>
	<u>20.00</u>	<u>12.00</u>
Contribution margin per unit	6.00	4.00

It takes 2 hours to make one unit of A and 1 hour to make one unit of B.

If there are only 30 000 direct labour hours available (i.e. direct labour is a constraint), which product should be produced?

If product A is selected, 15 000 units could be produced with a total contribution of \$90 000 (15 000 units × \$6 per unit). If product B is selected, 30 000 units could be produced with a total contribution margin of \$120 000 (30 000 units × \$4 per unit).

Therefore, product B is more profitable, and if there are no other constraints it should be produced in preference to product A.

The contribution margin should be expressed as an amount per unit of constraint rather than per unit of output. That is:

Product A contribution margin = \$3 per direct labour hour

Product B contribution margin = \$4 per direct labour hour



Self-test problem 11.13

Constraint Company has plant available to make two products, X and Y. The unit profits for these two products have been estimated as:

	X	Y
	\$	\$
Sales price	<u>45.00</u>	<u>30.00</u>
<i>less</i> Variable costs		
Direct material	10.00	12.00
Direct labour	12.00	6.00
Variable overhead	8.00	4.00
Fixed overhead	<u>4.00</u>	<u>2.00</u>
	<u>34.00</u>	<u>24.00</u>
Profit per unit	11.00	6.00

Factory overheads are applied at a rate of \$6.00 per direct labour hour. The maximum annual practical capacity is 60 000 hours with a fixed factory overhead of \$120 000.

Required

- If direct labour hours is the only constraint, which product or combination of products should be produced?
- If the estimated maximum demand for product X is 25 000 units and for product Y is 45 000 units, which product, or combination of products, should be produced? (The labour constraint still applies.)

Note: This problem is used to demonstrate the use of graphical analysis where constraints occur. You should make sure that you understand this question before looking at the section in the chapter on linear programming.

You should now be able to do Question 11.13.

Linear programming

A problem with more than one constraint and with more than two products can be solved using a technique called **linear programming**. Where there are only two or three constraints and a maximum of two products, the problem can be shown in a graph. Larger problems can now be solved easily with computers and even in spreadsheet programs (e.g. using Solver in Microsoft Excel).

The objective in problems is to either maximise profit or to minimise costs. The problem is usually set out in equation form with an objective function (maximise or minimise) and a set of constraints. The objective function and the constraints can be plotted on graph paper and the solution obtained by reading off values on the graph.

Example 11.12

Self-test problem 11.13 will be solved using graphical analysis. A summary of this problem is:

Contribution margins per unit are \$15.00 for product X and \$8.00 for product Y.

A maximum 60 000 direct labour hours are available and it takes 2 hours to make one unit of product X and 1 hour to make one unit of product Y.

Maximum demand is 25 000 units of product X and 45 000 units of product Y.

The objective is to maximise the total contribution margin subject to the constraints. This is set out as a 'model':

Let X = number of units of X to produce and sell

Let Y = number of units of Y to produce and sell

Maximise:

$Z = 15X + 8Y$ (maximise the number of units of X \times \$15 plus the number of units of Y \times \$8)

Subject to:

$2X + 1Y \leq 60\,000$ (2 hours \times the number of units of X plus 1 hour \times number of units of Y must be less than or equal to 60 000 hours)

$X \leq 25\,000$ (the number of units of X must be less than the maximum demand of 25 000)



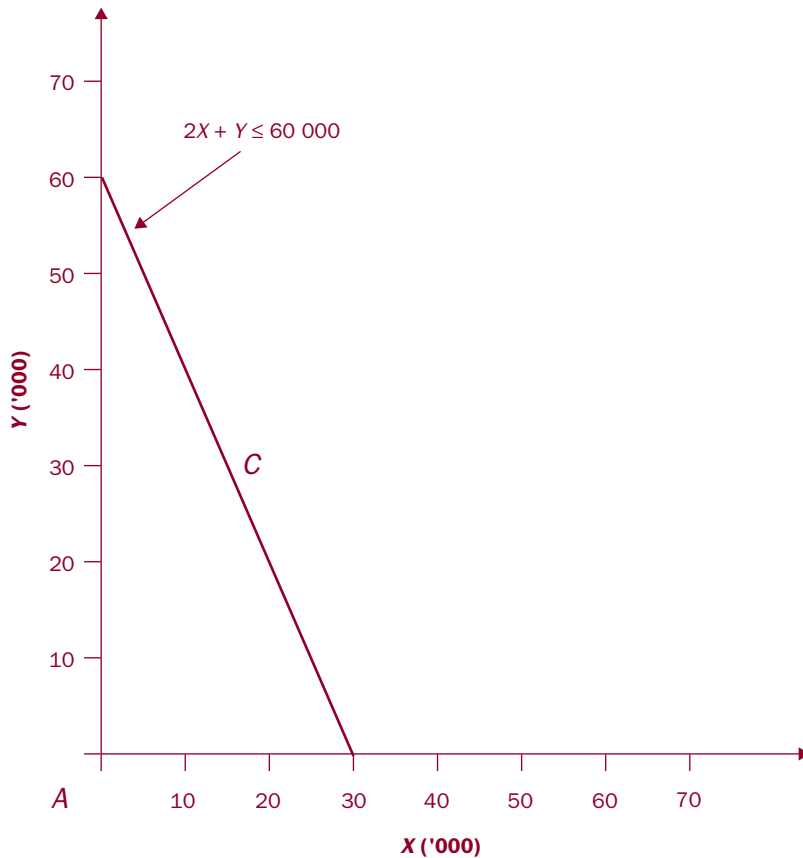
$Y \leq 45\ 000$ (the number of units of Y must be less than the maximum demand of 45 000)
 $X, Y \geq 0$ (both X and Y must not be negative, i.e. greater than or equal to zero)

The mathematical symbol \leq is used to represent 'less than or equal to' and \geq is used to represent 'greater than or equal to'.

To plot this 'model' on a graph, use the horizontal axis as the number of units of one product (in this example, X will be represented on the horizontal axis) and the vertical axis for the other product (product Y in this example).

With the constraint $2X + 1Y \leq 60\ 000$, if X is zero, then the maximum $Y = 60\ 000$. That is, if no units of X are produced, the whole of the 60 000 hours will produce 60 000 units of Y. Therefore, this point can be plotted on the graph. In Figure 11.1 this point is the 60 000 units mark on the Y-axis and zero on the X-axis. Similarly, if no units of Y are produced, the whole 60 000 hours can be used to produce 30 000 units of X. This point is the 30 000 units mark on the X-axis. A line joining all of the points between 60 000 on the Y-axis and 30 000 on the X-axis will contain all of the combinations of products X and Y that will satisfy the constraint. This line is shown in Figure 11.1 and is labelled C. The origin of the graph is marked A and at this point no units of either product X or Y are produced.

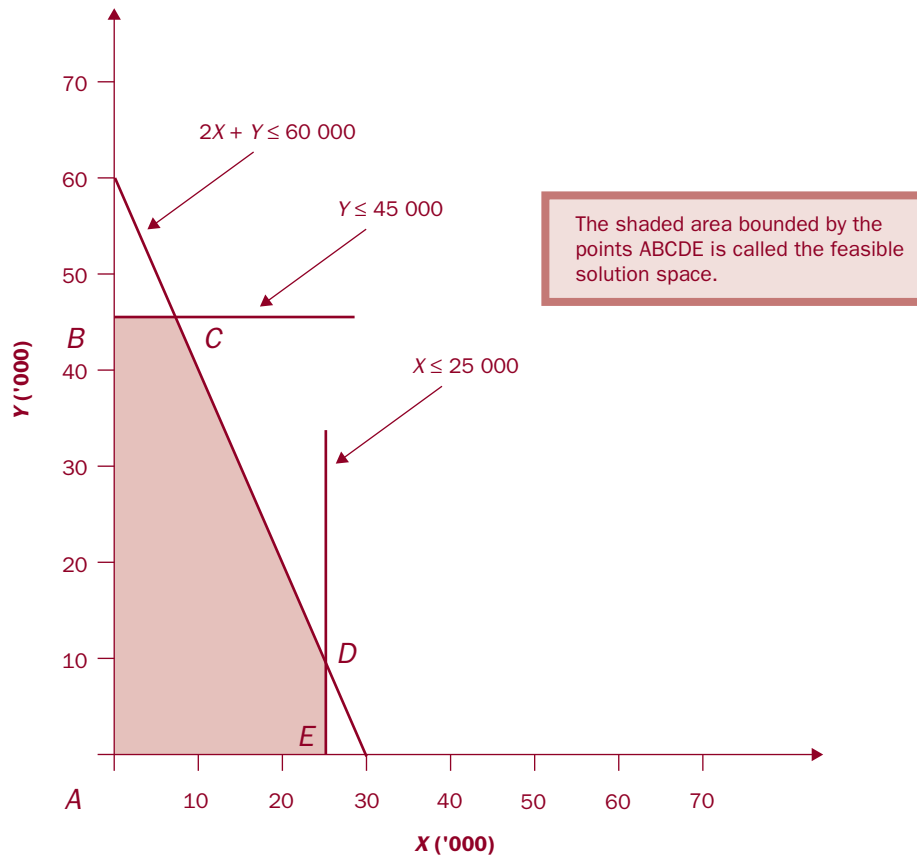
Figure 11.1 • First constraint $2X + 1Y \leq 60\ 000$



Now the other two constraints need to be added. It would not make sense to produce more than 45 000 units of Y as that is the maximum market for that product. So a constraint line is added to restrict product Y to a maximum of 45 000 units. Similarly, a constraint line is needed restricting the quantity of product X to a maximum market demand of 25 000 units. Figure 11.2 shows these new constraint lines added.

Any point in the shaded area bounded by the corner points (vertices) marked as A, B, C, D and E in Figure 11.2 will provide a possible combination of products X and Y. This area is called the *feasible solution space*. The optimum solution always lies on an outside corner point (vertex) of the feasible solution space.

Figure 11.2 • All constraints added

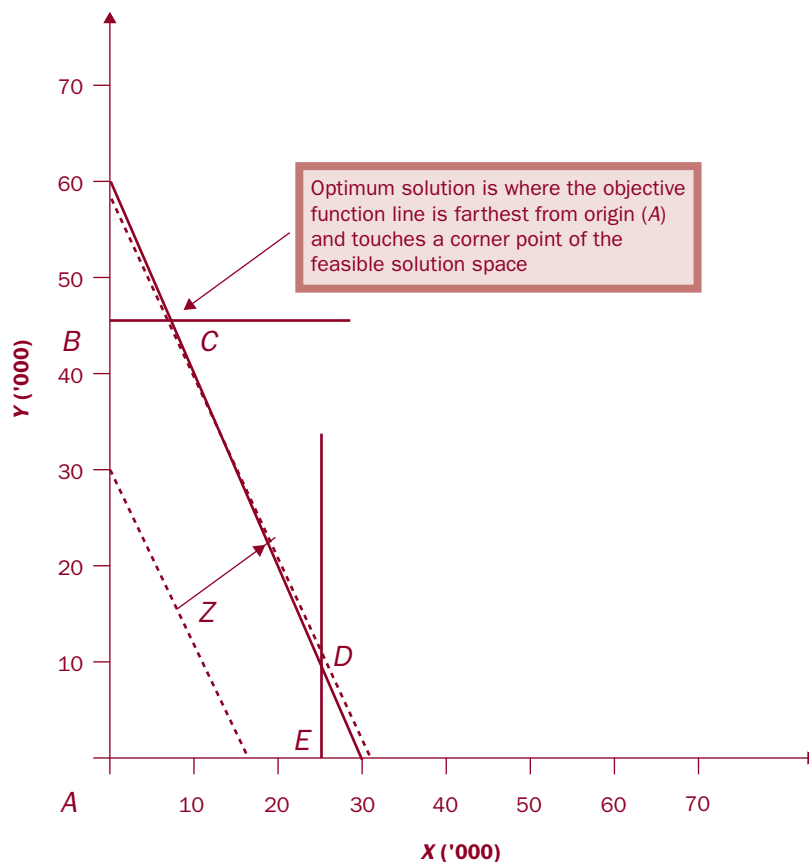


To establish which corner point provides the optimum solution, the objective function that is being maximised ($Z = 15X + 8Y$) can be drawn on the graph. The problem is that the maximum contribution margin (represented by Z) is not known, and there are many lines that can be drawn that would satisfy this equation. However, the objective function line will be the line that is farthest from the origin (point A on the graph) and just touches a corner point.

The objective function line can be drawn anywhere on the graph so long as the total contribution margin is the same amount along the line and is determined by the equation $Z = 15X + 8Y$. All of these contribution margin lines will be parallel to each other (i.e. the slope of the line will be the same). The method of obtaining the objective function (contribution margin) line is:

1. Make the contribution margin (Z) any figure that will make the arithmetic easy. The figure of \$240 000 has been selected in this example because each contribution margin per unit (\$15 and \$8) divides evenly into \$240 000. If the equation is restated as $15X + 8Y = \$240\,000$, then a line can be plotted in the same manner as the constraints. This line will be where $X = 16\,000$ and $Y = 30\,000$ and this is shown as a dashed line in Figure 11.3 and labelled Z .
2. Place a ruler along this contribution margin line and move the ruler away from the origin (A) in the direction of the arrow shown in Figure 11.3 so that it remains parallel to the drawn objective function line labelled Z . When the ruler is farthest away from the origin and just touches a corner point (i.e. is still feasible), the contribution margin is maximised. The dashed line touching corner point C in Figure 11.3 is the objective function line that is farthest from the origin and just touching the feasible solution space.

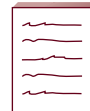
Figure 11.3 • Objective function lines plotted



The maximum contribution margin is obtained at corner point C in Figure 11.3 where 45 000 units of product Y are produced and 7500 units of X are produced. This can be proved by looking at all of the possible contribution margins at the corner points in the feasible solution space:

Corner points	Units of products		Contribution margin
	X	Y	\$
A	0	0	0
B	0	45 000	360 000
C	7 500	45 000	472 500
D	25 000	10 000	455 000
E	25 000	0	375 000

Example 11.13



Flat-track Products Pty Ltd has estimated the contribution margin of its two products to be:

- Product A—\$14 per unit
- Product B—\$21 per unit

Both products require machining and assembly. Machine hours and assembly hours per product, and maximum hours available per month are:

	Machining	Assembly
Product A—hours per unit	0.5	3.0
Product B—hours per unit	1.0	2.0
Maximum hours available	4000	12 000

Management would like to know which combination of products to produce.

- (a) Formulate the problem as a linear programming model.
- (b) Solve the problem by preparing a graph to determine the optimal product mix. On the graph, show all constraints, the objective function line and the feasible solution space. Advise management.

Solution

- (a) The linear programming model is represented by:
 Let A = the number of units of product A to produce
 Let B = the number of units of product B to produce

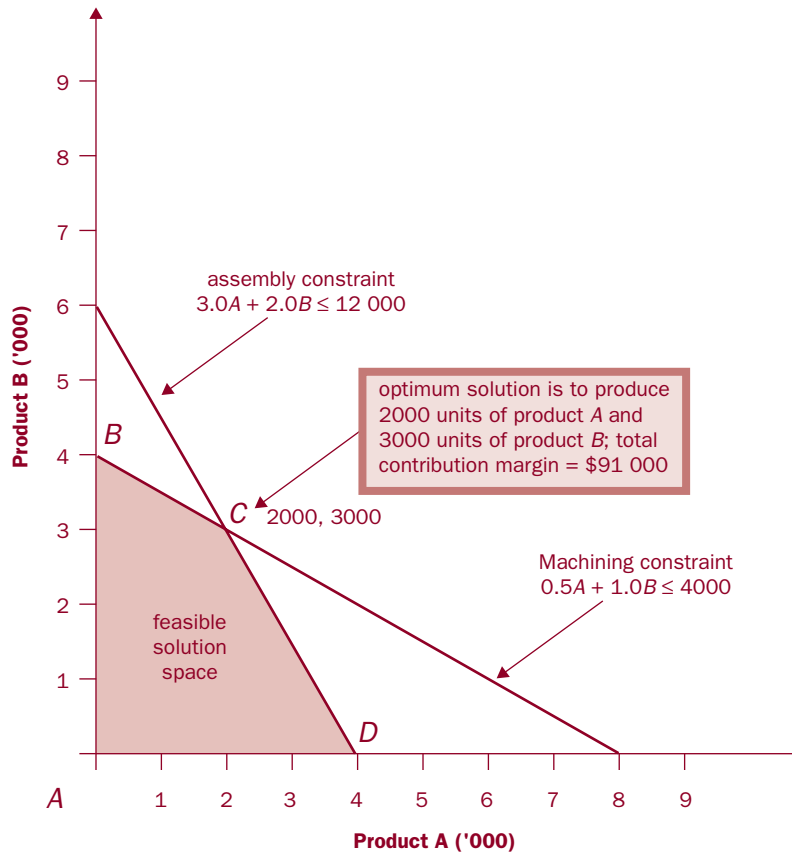
Maximise:
 $Z = 14A + 21B$ (maximise the total contribution margin)

Subject to:
 $0.5A + 1.0B \leq 4\ 000$ (machine hours constraint)
 $3.0A + 2.0B \leq 12\ 000$ (assembly hours constraint)
 $A, B \geq 0$ (A and B cannot be negative)

- (b) The machining constraint $0.5A + 1.0B \leq 4000$ means that the maximum number of product A that could be produced would be 8000 (i.e. if no machining time was allocated to product B, 8000 units of A could be produced, each unit requiring 0.5 hours). Similarly, the maximum number of product B that could be produced would be 4000. This is then plotted as shown in Figure 11.4.

The assembly constraint $3.0A + 2.0B \leq 12\ 000$ means that the maximum number of product A that could be produced would be 4000 (i.e. if no assembly time was allocated to product B, 4000 units of A could be produced, each unit requiring 3 hours). Similarly, the maximum number of product B that could be produced would be 6000. This is then plotted as shown in Figure 11.4.

Figure 11.4 • Constraints plotted and feasible solution space shaded



The optimum solution where the contribution is maximised is at corner point C shown in Figure 11.4. The contribution at each corner point can be calculated as:

Corner points	Units of products		Contribution margin
	A	B	\$
A	0	0	0
B	0	4 000	84 000
C	2 000	3 000	91 000
D	4 000	0	56 000



Self-test problem 11.14

MicroMoth Ltd assembles and sells two models of computers, A and B. Demand for these computers is high and there is a two-month backlog of orders. Model A gives a unit contribution margin of \$120, while model B gives a unit contribution margin of \$90.

Due to a shipping strike by waterfront workers, three of the imported components are in short supply for the coming month. The number of these components available and the number required in each model are:

Component	Total available	No. of parts required for:	
		Model A	Model B
IC301 chips	3000	6	3
PIA	1600	2	2
T40 controller	1200	3	0

All other components are readily available and the above supply shortages are the only constraints that need to be considered.

Required

Using a graph, determine the quantity of each model to be produced to maximise the total contribution margin. Your graph should indicate the constraint lines, the feasible solution space and an estimated objective function line.

You should now be able to do Questions 11.14 to 11.16. 

Problems of minimising costs can also be solved using a graph. Minimisation problems are usually concerned with minimising costs but with constraints requiring a minimum level of production (otherwise costs would be minimised by producing nothing!). The constraints are plotted in the same way as in maximisation problems but are usually expressed as ‘greater than or equal to’ and the symbol \geq is used.

Example 11.14



Agrifeed Ltd operates two production lines and each line produces a mixed output of animal feed in the form of cake, pellets and meal. The two production lines are run intermittently as required and the hourly output of the two lines is as follows:

	Cake	Pellets	Meal
Production line A	25 tonnes	15 tonnes	10 tonnes
Production line B	50 tonnes	5 tonnes	75 tonnes

It costs \$10 an hour to run production line A and \$25 an hour to run production line B. The orders on hand require 2000 tonnes of cake, 525 tonnes of pellets and 1350 tonnes of meal, and the factories will be run simultaneously until these quantities are available. How long should each line operate in order to provide sufficient output to meet the orders on hand?

This problem is set out as:

Let X = the hours that production line A will operate

Let Y = the hours that production line B will operate

The objective is to minimise the total cost from running the production lines. The total cost of running production line A will be $\$10 \times X$ hours and for running production line B it will be $\$25 \times Y$ hours. The objective function is:

Minimise:

$$C \text{ (cost)} = 10X + 25Y$$

Subject to:

$$25X + 50Y \geq 2000 \quad \text{(the output of cake from both production lines must be at least equal to 2000 tonnes)}$$

$$15X + 5Y \geq 525 \quad \text{(the output of pellets from both production lines must be at least equal to 525 tonnes)}$$

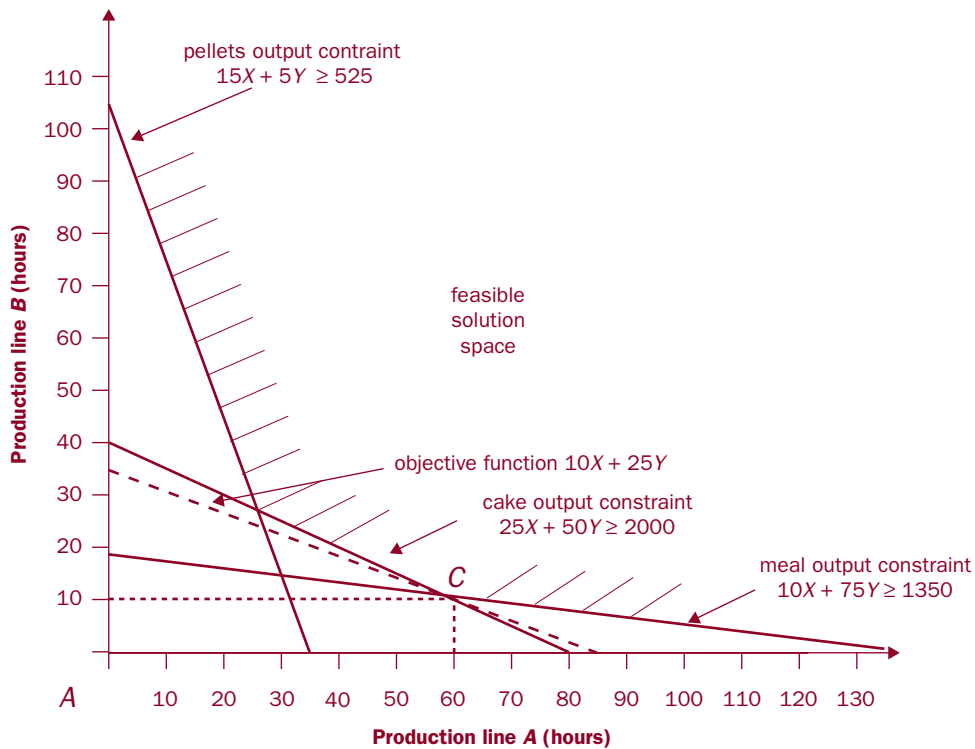
$$10X + 75Y \geq 1350 \quad \text{(the output of meal from both production lines must be at least equal to 1350 tonnes)}$$

$$X, Y \geq 0 \quad \text{(the hours } X \text{ and } Y \text{ must not be less than zero)}$$

The graphical solution is set up in the same way as a maximisation problem. This example uses the X -axis to show the number of hours to run production line A and the Y -axis for the hours to run production line B. Each constraint is plotted on the graph. For the first constraint, if production line A is used to produce the whole 2000 tonnes, then it will need to run for 80 hours. If production line B is used to produce the whole 2000 tonnes, then it will need to run for 40 hours. A line is drawn on the graph from 40 hours on the Y -axis to 80 hours on the X -axis. And any point on this line will represent the combination of hours for both lines in which the total output will be 2000 tonnes. This line, together with lines drawn for the other constraints, is shown in Figure 11.5.

The hours required to minimise cost is shown on the graph in Figure 11.5 as point C. This shows that running production line A for 60 hours and production line B for 10 hours will produce a minimum cost. Putting these hours into the objective function, the minimum cost is \$850 (60 hours \times \$10 plus 10 hours \times \$25). You may have noticed that while this solution

Figure 11.5 • Graph for minimisation example



minimises the cost, it produces 950 tonnes of pellets (15 tonnes per hour in line A \times 60 hours, plus 5 tonnes per hour in line B \times 10 hours) and this is 450 tonnes more than the orders. This would need to be considered in the final decision on running the lines.

The feasible solution space in this example is the area to the right of all of the constraint lines. The minimum cost line is drawn as before but it is moved *towards* the origin (point A) until it reaches the corner point of the feasible solution space that is nearest the origin.

Self-test problem 11.15

An animal feed must provide at least 2000 calories and 120 protein units per day to provide required growth rates. It is possible to use two ingredients, the particulars of which are:

	Calories per kilogram	Protein units per kilogram	Cost per kilogram
Ingredient 1	500	10	\$0.75
Ingredient 2	200	20	\$0.50

Required

Using a graph, determine the optimal mix of ingredients to minimise cost.

You should now be able to do Question 11.17.

Summary

1. *Differential costing is the process of providing relevant information to management that will assist it in making decisions.*
2. *Relevant quantitative information consists of those costs and revenues that will be affected by any decision. Relevant information, therefore, has a bearing on the future.*
3. *Historical costs are not affected by a decision but may be used to predict relevant future costs.*
4. *Sunk costs are historical costs that cannot be altered by any decision. They are not relevant and can be ignored in any quantitative analysis.*
5. *Imputed costs are relevant costs that are not usually recorded in the accounts of a business. They are costs that do not involve an outlay of cash but are included in order to make valid comparisons.*
6. *An opportunity cost is the value of one alternative forgone (given up) by a decision made on another use of the resource.*
7. *Quantitative data or information refers to the numbers (units or dollars) used in an analysis and recommendation. Qualitative data or information refers to factors that must be considered but which cannot be expressed in numerical terms.*

8. *The differential cost analysis may be presented in tables showing totals for each alternative, or as differences between two alternatives.*
9. *There are many possible differential cost decisions for which both quantitative and qualitative data or information will be required. The main areas covered in this chapter are advertising, accept or reject a special order price, make or buy a product or component, change in product lines or departments, sell now or process further, and temporary shutdowns.*
10. *Decisions may involve constraints on resources and the relevant figure to use in these decisions is the contribution margin per unit of that constraint.*
11. *Decisions involving more than one constraint and a number of products can be formulated and solved using linear programming. Computerised linear programming packages may be used. In simple cases, a graph can be drawn that will provide answers.*
12. *Linear programming provides answers to the mix of products that should be made or sold so as to maximise the total contribution margin or to minimise the total relevant costs.*

Questions

- 11.1 A summary of cost types that are relevant and not relevant for decision making is:

Relevant costs

Future costs
Incremental costs
Avoidable costs
Imputed costs
Opportunity costs

Irrelevant costs

Historical (sunk) costs
Non-incremental
Unavoidable
No-cash expenses

For each of the following, indicate whether the costs involved are relevant or irrelevant, and classify them into one of the cost types listed above:

- (a) In a decision about replacing a vehicle, the cost of the vehicle to be replaced is in the ledger at \$35 000.
- (b) A business intends to purchase machinery for \$50 000. The cost of the capital to be invested in this business is 10%.
- (c) The cost of indirect labour paid in a period was \$12 000.
- (d) The fixed supervision salary costs for a division will no longer be incurred if the division is closed down.
- (e) The expected cost of labour will be reduced by 10% if machinery is purchased.
- (f) If machinery is purchased, the salary for the factory supervisor will remain the same.

- (g) Garments that had been purchased for \$85 000 need alterations costing \$12 000 before they can be sold for \$40 000. If they are not altered they can be sold for \$15 000.
- (h) Fixed occupancy costs (e.g. lease, security, property insurance) will not change with a proposed increase in production of 5%.
- 11.2 If a retailer has an overall margin of 30% from its sales, by how much would sales need to increase to pay for an advertising expenditure of \$45 000?
- 11.3 R. V. B. Enterprises Pty Ltd operates a number of computer stores. The company is considering setting up a website to increase its market. It also considers that at the end of two years the customer base could be sold for a substantial gain.

Two options are being considered: to develop its own website or to use a well-known website, such as, Yahoo! to host the electronic store. The set-up, advertising, potential contribution margin, and infrastructure costs for the two options have been reported by a marketing consultant and show:

	Own website \$	Professional hosting \$
Initial set-up	84 000	36 000
Advertising costs		
First six month period	186 000	170 000
Second six month period	240 000	210 000
Third six month period	180 000	160 000
Fourth six month period	150 000	150 000
Sales contribution margin		
First six month period	100 000	90 000
Second six month period	190 000	200 000
Third six month period	220 000	230 000
Fourth six month period	250 000	240 000
Administration and other fixed costs		
Each six months	180 000	

The administration and other fixed costs are an allocation of the costs of current information technology facilities in the company. The professional website service fee (in addition to the advertising costs) is \$4000 per month plus turnover royalty that averages 5% of the contribution margin. Under each alternative, it is estimated that the business (customer base) will be sold at the end of the two-year period for \$250 000.

Required

- (a) Advise management on which alternative would be best. Set your answer out so that it indicates the profit or loss for each six-month period and cumulatively for each alternative.
- (b) In view of the poor survival rate of this type of business (a 'dot com' company) in recent history, what other matters would you raise with management?

11.4 The sales estimates for Dartford Piping Ltd are based on a forecast that for the next six months the regular market for its product Decla will be at 70% of available normal manufacturing capacity.

Budget estimates for the full year show the following:

	At normal capacity	At 80% of normal capacity
Sales in units	72 000	57 600
Sales price per unit	\$7	\$7
Total costs	\$384 000	\$326 400

In addition to the regular sales budgeted above, two offers to purchase have been received at reduced prices. One is for a local order for 2000 units at \$6 per unit. The other calls for 2400 units delivered to an overseas market at \$5.00 per unit. The overseas order would require an additional \$0.40 per unit for packing, and \$600 for shipping and forwarding costs for the whole consignment.

Required

- (a) Determine the profit at 70% of normal capacity for the next six months without the two special orders.
- (b) Advise management on the desirability of accepting or rejecting either or both of the two special orders, indicating the profit or loss and other factors involved in each case if they are accepted.
- (c) State the minimum unit price below which any further orders should be rejected.

11.5 Dural Manufacturing Company manufactures office furniture for major retailers. One of the items is an executive return desk made from Tasmanian oak. The normal selling price of this desk is \$1760 including 10% GST.

The Sultan of Zarawak has offered to purchase 200 of these desks at a price of A\$1100. Exports are GST-free. Production of these 200 desks would not increase fixed production costs as they can be produced with the current facilities.

A budget schedule showing the costs of producing and selling 500 of these desks shows:

	Unit cost \$	Total costs for 500 desks \$
Timber, leather, brackets	390	195 000
Direct labour	275	137 500
Factory overhead—variable	275	137 500
Factory overhead—fixed	180	90 000
Fixed marketing	30	15 000
Fixed administration	50	25 000
Sales commissions	40	20 000
	<u>1240</u>	<u>620 000</u>

The export order will require special packing that will cost \$20.00 per desk. Freight and insurance on the 200 desks will total \$6000. An agent's commission of 5% of the sales price will replace the local sales commission of \$40.00 per desk.

Required

Management is considering the offer and would like to know the effect on the overall profits if this order were accepted. Provide management with advice.

- 11.6 Gem Electric makes electrical appliances. Electric ovens and microwaves require an electronic timing device that Gem Electric currently makes. The annual budgeted production level is 10 000 of these timing devices.

A & C Controls Pty Ltd has said that it can deliver these into stores at \$10 per timing device. The cost accountant for Gem Electric has provided a unit cost of manufacturing this timing device and this shows:

	\$
Direct materials	3.00
Direct labour	5.00
Manufacturing overhead—variable	1.00
Manufacturing overhead—fixed	<u>2.50</u>
	<u>11.50</u>

The fixed manufacturing overhead of \$2.50 includes \$0.75 of discretionary costs that would cease if the timers were purchased from A & C Controls. The other fixed manufacturing costs are allocated committed costs. However, if the timers were purchased, the space made available would be used for storage and this would save \$10 000 per annum.

Required

- (a) Prepare a report for management that will show the effect on total costs if the timers were purchased rather than manufactured.
 - (b) Advise management on this decision, providing it with any other factors that should be considered.
- 11.7 Campaire Ltd manufactures small air-conditioners suitable for installation in trucks and caravans. The condenser unit in each air-conditioner is formed from copper tubing, and this process is currently carried out in Campaire’s forming area in the factory. Due to a lack of expansion space, the management of Campaire is considering the purchase of condensers from a specialist copper manufacturer, Coptube Ltd.

Allowing for normal damage of 100 units during the assembly of the condensers into the air-conditioners, Campaire has extracted the following information relating to forming an annual requirement of 20 300 good condensers:

	\$
Cost of copper tubing delivered into store	175 440
Direct labour	199 200
Factory overhead (excluding depreciation)	61 200
Cost of carrying average copper tubing inventory	2 924
The depreciated value of equipment used to form the condensers is \$12 000 and its expected remaining useful life is five years. This equipment could readily be used elsewhere in the plant.	
Sale of damaged condensers as scrap	600
Sale of scrap copper tubing	800

The factory overhead is considered as fixed but these costs will be used in expanding other production.

The quotation from Coptube is for an order of 20 400 condensers, delivered in lots of 425 per week. The price quoted is \$20 per condenser delivered into store. Coptube will also collect damaged condensers and pay Campaire \$7 per unit. This supply contract requires renegotiation each year.

Required

- (a) State whether Campaire should make or purchase the condensers. Support your decision with a schedule of costs and income.
- (b) State some of the factors other than costs that would favour a decision to: make the condensers or purchase the condensers.

11.8 Cavity Sweets manufacturing is divided into three departments: chocolates, toffee and pastilles. The unit manufacturing costs per kilogram of output from these departments were budgeted at the start of the year as follows:

	Chocolates	Toffee	Pastilles
	\$	\$	\$
Direct material	3.05	0.80	1.20
Direct labour	1.00	1.00	1.00
Factory overhead	<u>1.25</u>	<u>1.20</u>	<u>1.25</u>
	<u>5.30</u>	<u>3.00</u>	<u>3.45</u>

The applied factory overhead cost was based on the following summary budgets for the year:

	Chocolates	Toffee	Pastilles
Direct variable (\$)	20 000	10 000	15 000
Direct fixed (\$)	20 000	15 000	15 000
Allocated fixed (\$)	<u>60 000</u>	<u>35 000</u>	<u>45 000</u>
	<u>100 000</u>	<u>60 000</u>	<u>75 000</u>
Budgeted kilograms	80 000	50 000	60 000

The allocated fixed overheads are mainly occupancy costs and relative floor space occupied by each department was used to distribute the total cost between departments.

Actual production and sales for the year were:

	Kilograms	Sales
		\$
Chocolates	70 000	450 000
Toffee	50 000	250 000
Pastilles	55 000	<u>300 000</u>
		<u>1 000 000</u>

Selling and administrative costs (considered as all fixed) for the year amounted to \$160 000. The sales force markets all three products jointly. For

departmental profit statements, the selling and administrative cost are apportioned between departments according to their relative sales value.

Required

- (a) Prepare a profit statement for the year by department and in total showing any under or over applied overhead as an adjustment to cost of goods sold. The statement should show gross profit and net profit.
 - (b) Recast the profit statement in a form that will highlight to management the short-term effect of closing down any one department.
 - (c) Using the results from (a) and (b), prepare advice for management on the effect of a decision to close down chocolate production.
- 11.9 Hills & Frost Pty Ltd makes four different types of fillers for plastics manufacturers. These are designated type A, type B, type C and type D. The management of Hills & Frost is examining possible changes to the product mix so as to improve profitability. Several separate proposals are being considered and you are to provide figures for these proposals.

A summary profit statement and other information is shown in the following table:

	Total	Type A	Type B	Type C	Type D
Sales (\$)	626 000	100 000	180 000	126 000	220 000
Cost of goods sold (\$)	442 740	47 500	70 560	139 680	185 000
Gross profit (\$)	183 260	52 500	109 440	(13 680)	35 000
Selling and administrative (\$)	120 120	19 900	29 760	28 260	42 200
Net profit before tax (\$)	63 140	32 600	79 680	(41 940)	(720)
Kilograms sold		10 000	12 000	18 000	20 000
Variable cost of goods sold per unit (\$)		2.50	3.00	6.50	6.00
Variable selling and administrative per unit (\$)		1.15	1.25	1.00	1.20

Each of the following possibilities is to be considered independently of the others. Only consider the changes proposed for the particular products in each proposal (i.e. assume that the activity of other products remains the same).

Required

- (a) If type C is discontinued, what will be the effect on net profit before tax?
- (b) If type C is discontinued, current customers will cease to buy 2000 kilograms of type B, which is a complimentary product. What will be the combined effect on profit if type C is dropped?
- (c) If the sales price of type C is increased to \$8.00 with a consequent decrease in the number of kilograms sold to 15 000, what will be the effect on profit?

- (d) The department in which type C is produced could be used to produce 16 000 kilograms of a new type E. This new type would sell for \$9.50 per kilogram and its variable costs would total \$8.05 per kilogram. If type E replaces type C, what will be the combined effect on profit?
- (e) Part of the department in which type A is produced can be converted to produce type D. Sales prices will also change. Production and sales of type D can be increased to 25 000 kilograms to be sold at \$10.50 per kilogram. This would require reducing the output of type A to 5000 kilograms at a sales price of \$12 per kilogram. What would be the effect on total income from this change?
- (f) Adding a second shift could double production of type A, but shift allowances and other costs would increase the variable cost of goods sold to \$4.50 per kilogram for each additional kilogram above present sales. If the 10 000 additional kilograms of type A can be sold for \$8.50 each, what will be the total effect on net profit from doubling the output of type A?

11.10 Silverwater Paints Ltd produces anti-rust chemicals that are used by structural engineering businesses. Raw materials are converted through two processes. The output from the second joint process is three different grades of the chemical, A, B and C. Each of these chemicals is further refined in individual processes. The selling prices per litre of these chemicals at the end of all processing are \$14, \$9 and \$8 respectively.

Production and costs estimated for the coming year are:

Product	Litres produced	Costs from		Total cost
		joint processing	Unit costs (\$) Further processing	
Grade A	10 000	5	6	11
Grade B	20 000	5	3	8
Grade C	25 000	5	2	7

The relative quantities of each grade of chemical cannot be changed in the joint process. Further processing costs for each grade of chemical would be eliminated if the product were not processed further.

Required

For each of the following independent situations, advise management:

- (a) A competitor is now importing a product that is similar to grade B chemical but it sells for \$6 per litre. Should Silverwater Paints continue to produce grade B chemical? What price should it charge?
- (b) The grade A chemical can be sold at split-off point (i.e. before further processing) for \$8.50 per litre. Should the product be sold at split-off point or should Silverwater Paints continue to process it further and sell it for \$14 per litre?
- (c) If the sales price of grade C declines to \$5 because of competition, what effect will this have on production and sales?

11.11 Pepi Products makes three types of durable vinyl seat coverings that are stamped with the logo of the purchaser. The products are first produced from a joint process and each is then processed further in separate production departments.

The current month's production schedule and other data are:

Product	Sales price per metre \$	Metres produced	Unit costs (\$)		Total cost	Profit (loss) \$
			Joint cost	Further processing		
Tuff-One	5	10 000	\$3.10	\$2.20	\$6.30	(13 000)
Fortress	11	6 000	3.10	4.10	\$7.20	22 800
Everlast	16	4 000	3.10	5.30	\$8.40	30 400

The relative quantities of each product cannot be changed in the joint process.

Further processing costs for each product would be eliminated if the product were not processed further.

Required

- (a) The managing director would like to know why the biggest selling product is showing a loss. He thinks the figures are wrong. Advise the manager on the figures.
- (b) An importer is marketing a product to government departments that would be a direct substitute for the product Fortress. To compete with this imported product the sales price of Fortress would need to be reduced to \$6.50. Should Pepi Products sell Fortress at this price?
- (c) If the management of Pepi Products has a policy that a gross profit of 20% on sales must be maintained, what would your answer be to part (b) above?

11.12 Pacific Peace Pty Ltd operates several holiday resorts in islands around the South Pacific. Due to political unrest in one island nation, the average occupancy rate (percentage of rooms occupied) in Pacific Peace's resort on that island has reduced from 80% down to 60%. This is expected to continue until the next parliamentary elections in six months time. It is expected that the occupancy rate will return to at least 80% from then onwards.

A summary of last season's profit statement (same six months) for this resort, Peaceful Palms, when the occupancy rate was 80%, is:

	\$
Revenue	
Accommodation	480 000
Restaurant	128 900
Boat, windsurfer, scuba and ski hire	<u>184 300</u>
	<u>793 200</u>
Expenses	
Supplies—accommodation	33 600
Supplies—restaurant	90 230

(Continued)

	\$
Electricity and water	12 180
Salaries	237 960
Insurance	55 520
Lease on island	120 000
Depreciation	138 640
Other overheads (e.g. stationery, travel)	<u>12 690</u>
	<u>700 820</u>
Profit	\$92 380

Management is considering closing Peaceful Palms down for the next six months as it expects a loss. You have been asked to advise. Further information that you have obtained indicates that:

- (a) the revenue from the restaurant and boating equipment hire varies in direct proportion to the accommodation occupancy rate;
- (b) the cost of supplies varies in direct proportion to the revenue to which it relates;
- (c) the cost of electricity and water and other overheads varies in proportion to total revenue;
- (d) both the salaries and insurance would reduce by 5% at an occupancy rate of 60% and would reduce by 50% if the resort closed down for six months—these reductions are on the figures at 80% occupancy rate;
- (e) depreciation would reduce by \$50 000 if the resort closed down—it would remain the same at the 60% occupancy rate.

Required

Prepare a profit statement for the six months at an occupancy rate of 60%. Advise management what the loss would be if the resort were closed for six months.

11.13 Gambole & Donnetti make two types of exercise machines, Gyro and Heavo. Sales prices and unit costs are:

	Gyro	Heavo
	\$	\$
Sales price	<u>200.00</u>	<u>140.00</u>
Costs		
Direct materials	60.00	36.00
Direct labour	20.00	30.00
Variable factory overhead	40.00	20.00
Fixed factory overhead	20.00	10.00
Selling (all variable)	28.00	20.00
Administrative (all fixed)	<u>10.00</u>	<u>10.00</u>
	<u>178.00</u>	<u>126.00</u>
Net margin	22.00	14.00

Required

Factory overhead is based on a rate of \$15 per machine hour.

- (a) If there are 30 000 machine hours available, which product should be produced?
- (b) If there are 30 000 machine hours available but the market for Gyro and Heavo are 10 000 and 4000 respectively, which product or combination of products should be produced?

11.14 Prepare a graph and solve the following linear programming model:

Maximise:

$$Z = 20X + 30Y$$

Subject to:

$$60X + 24Y \leq 600$$

$$20X + 40Y \leq 400$$

$$X, Y \geq 0$$

11.15 Prepare a graph and solve the following linear programming model:

Maximise:

$$Z = 40X + 30Y$$

Subject to:

$$80X + 40Y \leq 3200$$

$$20X + 70Y \leq 1400$$

$$X, Y \geq 0$$

11.16 Woodenway Manufacturing makes table and chairs in four departments. The tables and the chairs both require time in the woodworking and assembly departments, and the products are polished and completed in individual finishing departments.

Output limitations by departments are:

Department	Weekly capacity and hours per product
Woodworking	560 hours (chairs, 4 hours; tables, 8 hours)
Assembly	180 hours (chairs, 2 hours; tables, 2 hours)
Chair finishing	70 chairs (2 hours per chair)
Table finishing	60 tables (2 hours per table)

Chairs and tables contribute \$30 and \$40 respectively towards fixed costs and profits.

Required

Set this problem up as a graph and determine the number of chairs and tables to be made each week so as to maximise profits. Assume that all products can be sold and that the tables and chairs are not sold in sets.

11.17 Fiturn Pty Ltd operates two machines, X and Y, each of which can produce a mixed output of two products, A and B. The possible hourly output from these two machines, and total requirements for the products for the day are shown on page 364.

	Products	
	A	B
Machine X—output per hour	5	2
Machine Y—output per hour	10	1
Total required per day	40	10

The variable cost per hour of running machine X is \$10 per hour and for machine Y is \$25 per hour.

Required

- Set this minimisation problem up as a linear programming model.
- Prepare a graph and determine the number of hours each machine, X and Y, should be run each day to satisfy the constraints and minimise running costs.