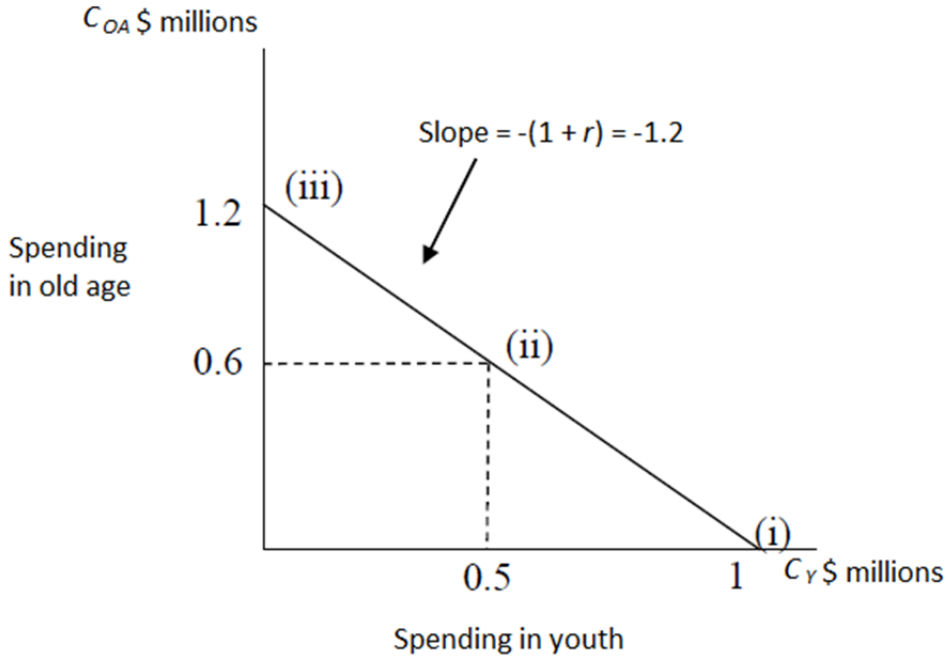
The Net Present Value Rule

In the Appendix to Chapter 1, we outlined the reasoning behind the net present value rule with the aid of the ant and grasshopper example. Here we provide more flesh on this reasoning.

Bill Ross has inherited $1 million. He has developed a real aversion to work; he completely detests it. He therefore plans to use his inheritance to finance himself for the rest of his life. For simplicity we'll divide his life into two periods, youth and old age. Also, we assume that there is only one financial institution, a bank, which lends and borrows at a rate of r = 20%, so that for every dollar deposited in youth, $1.20 is received in old age.

Bank Alone

Assuming this bank is the only opportunity open to Bill, what can he do?



(i) He could go on a fantastic trip around the world, spend the whole $1 million, and then live in poverty with nothing for his old age. He is at ($1 million, 0) on the diagram.

(ii) He could spend $0.5 million in his youth, have a moderate lifestyle, put $0.5 million in the bank, and still have $0.6 million for his old age. He is at ($0.5 million, $0.6 million).

(iii) He could put all his money in the bank for his old age and spend nothing in his youth, so that he can take an even better trip around the world in his old age. In this case he gets (0, $1.2 million).

(iv) If we consider all the other possibilities, we get a straight line between (i) and (iii).

The basic point is that the bank allows him to transform $1 today into $1.20 next period and vice-versa. He can move up or down the line by putting more or less money in the bank. As a result, he can consume anywhere on the straight line between $1 million in youth and $1.2 million in old age. This line represents his budget constraint. He does not have the resources to move north-east of the line and spend more. Analytically we have:

$$C_{OA} = 1.2 - 1.2C_Y$$

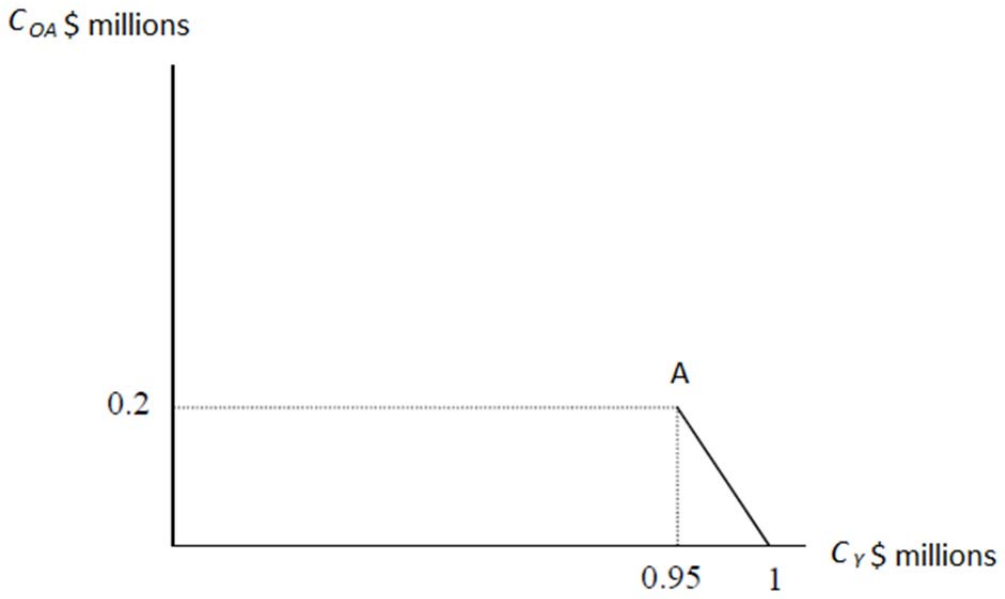
Projects Alone

We will now look at the case where there is no bank and there are only productive opportunities or projects that allow him to transfer wealth from his youth to his old age.

Bill fancies himself as an entrepreneur and sits down to work out what investments he can make. He ranks them in terms of profitability with the most profitable being first and the least profitable last.

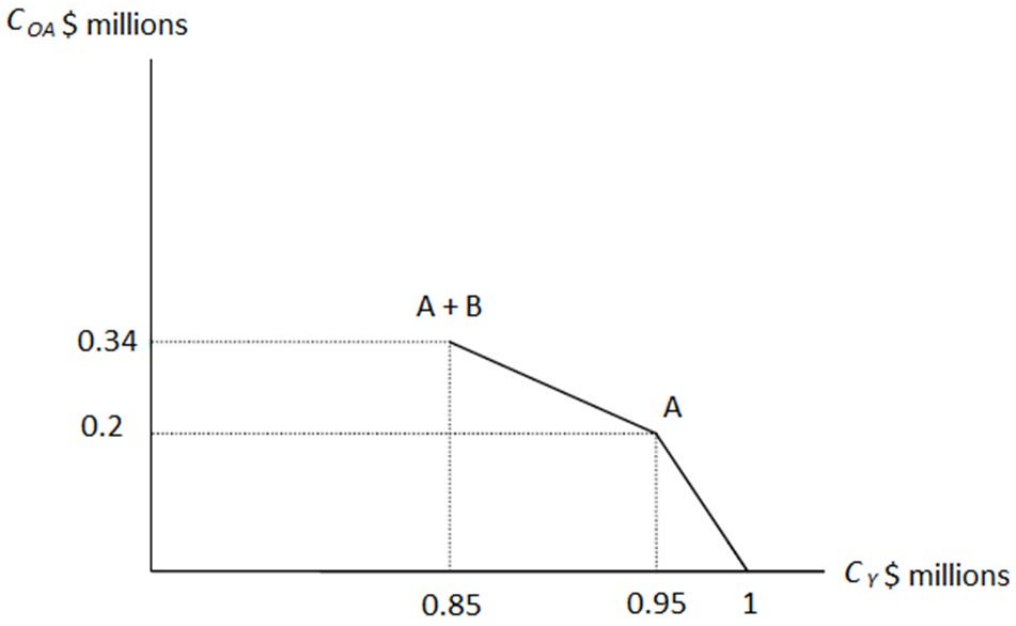
Project A

Bill is a wine lover. He estimates that a small vineyard that has recently come on the market will cost him $50,000 now and will yield him $200,000 for his old age. This is the best project he can think of. Hence, if he invests just $50,000 in this project, he can consume $1 million - $50,000 = $950,000 now and he can consume the $200,000 project A generates in his old age. He is at point A in the diagram and consumes ($0.95 million, $0.2 million).



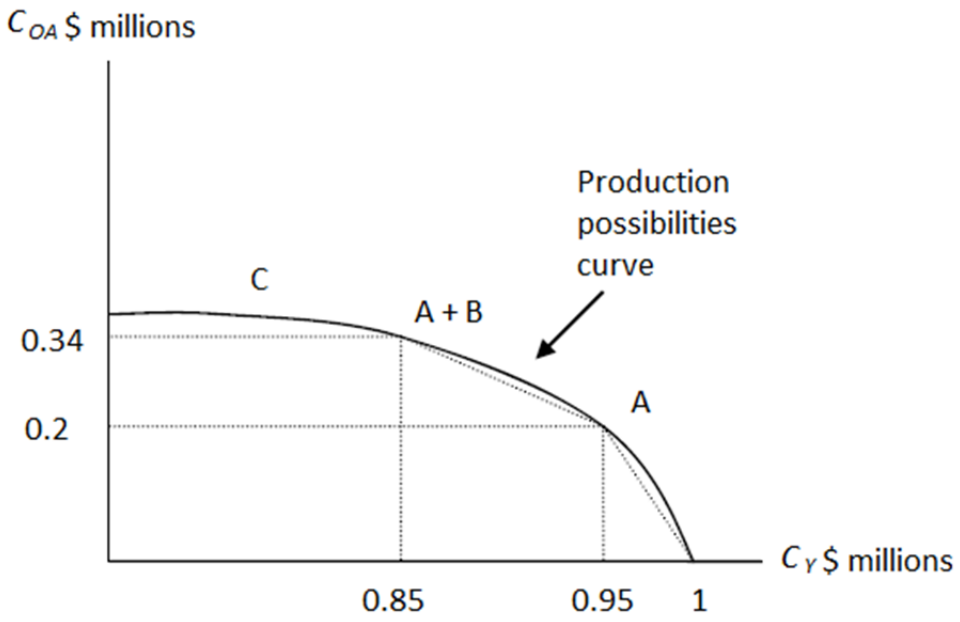
Project B

Bill is also a gourmet. The next best project he can think of is to run a restaurant in the town he lives in. He reckons that for a $100,000 outlay now he can get $140,000 in his old age. If he just undertakes this project and project A, then he can consume $950,000 − $100,000 = $850,000 now and $200,000 + $140,000 = $340,000 that the two projects will generate in his old age. He is at the point A + B in the diagram below and consumes ($0.85 million, $0.34 million).



Project C and so on

There are a number of other projects he thinks of. We can trace out a curve, called the production possibilities curve, to approximate these series of short straight lines as shown below. With projects alone, he can consume anywhere along the curve.



Projects and the Bank

We now consider what Bill can do if we take account of both his projects and the possibility of borrowing and lending at the bank.

Project A

Suppose he just undertakes the first project, A. He has $950,000 now and $200,000 later on—what can he do?

(i) He could simply consume $950,000 now and $200,000 later on without going to the bank. He consumes at ($0.95 million, $0.2 million).

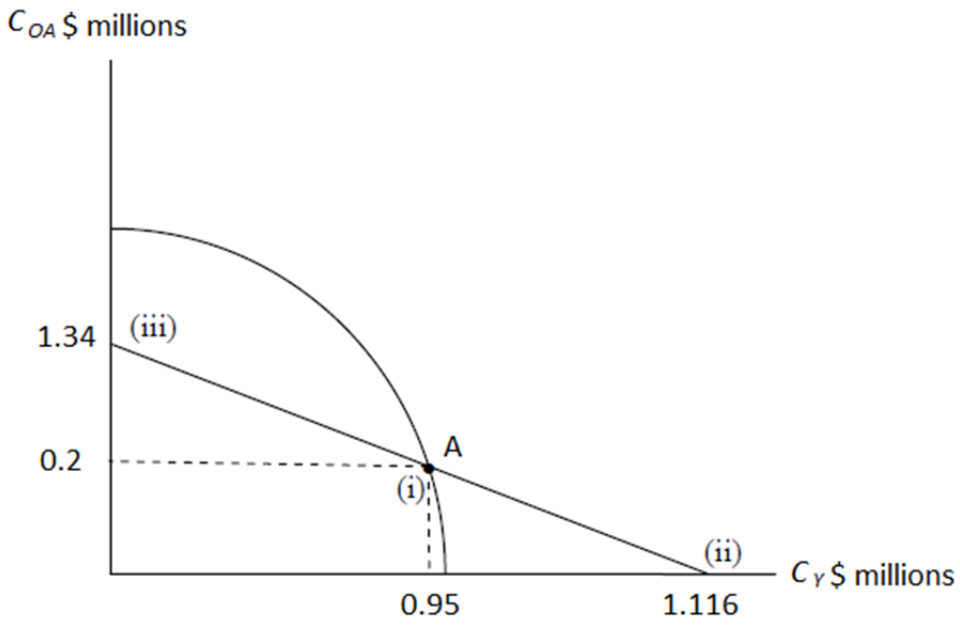
(ii) Alternatively, he can use all the money he receives to have a marvelous time. He has $950,000 now and $200,000 later on, which he can use to repay a loan that he spends now. How much can he borrow to repay with the $200,000 later on; i.e., what is the

present value of $200,000 at 20%?

$$\text{PV}(200{,}000_{OA} \text{ at } 20\%) = \frac{200{,}000}{1.20} = 166{,}667$$

Hence,

Total possible in consumption youth = 950,000 + 166,667 = $1.1167 million



(iii) Alternatively he can plan to spend it all next period.

Total in old age = 1.2 x 950,000 + 200,000 = $1.34 million

As before, we can go on doing this until we trace out a straight-line budget constraint as before. The equation for this line is

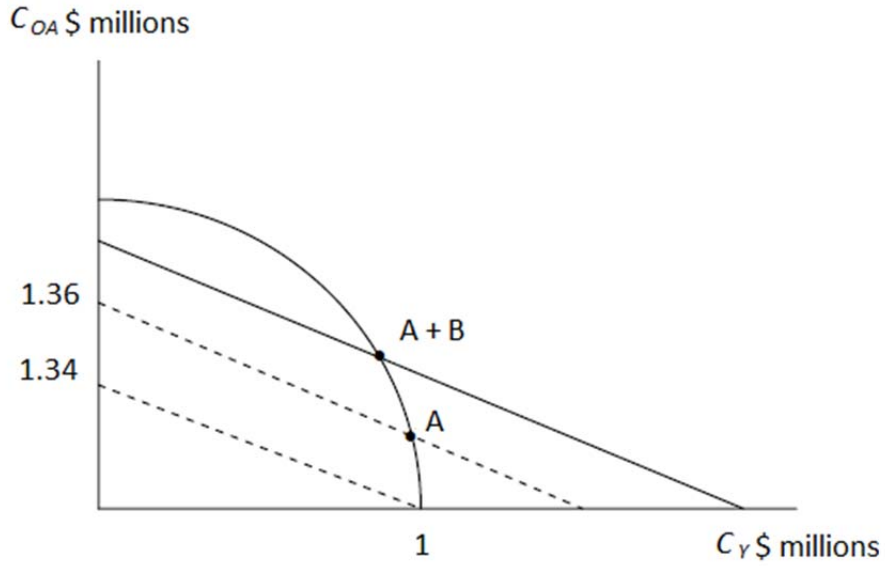
$$C_{OA} = 1.34 \text{ million} - 1.2C_Y$$

The slope is again −1.2 since the interest rate is 20%. This follows from the structure of the problem since all values in old age are multiplied by 1.20 compared with their values in youth. The bank always allows Bill to move up and down a straight line with slope −1.2.

Project B

Now suppose he undertakes the first and second projects, A and B, so that he produces at A +

B. We can go through the same calculations again and get another straight line representing his consumption possibilities.



In this case,

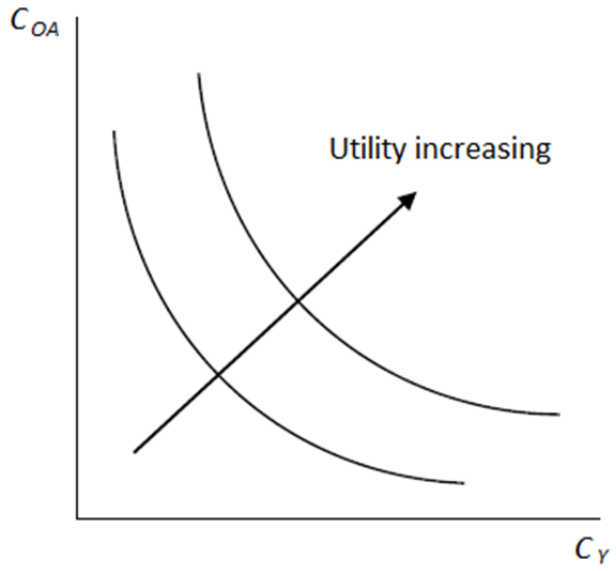
$$\text{Intercept on } C_{OA} \text{ axis} = 1.2 \times 850{,}000 + 340{,}000 = \$1.36 \text{ million}$$

Hence, analytically the budget constraint is given by

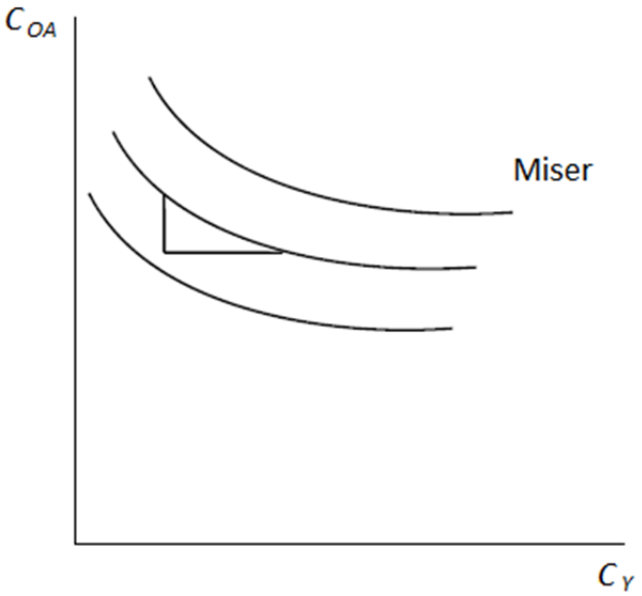
$$C_{OA} = 1.36 \text{ million} - 1.2C_Y$$

We can see that by undertaking the first project, Bill can push out the line representing his possible consumption. Similarly, he can push it out when he undertakes the second project, and so on. If he prefers more money to less, he is better off if his budget constraint is pushed out further since this allows him to consume more in both periods. Hence no matter what Bill's preferences are, he is better off with a budget constraint that is farther out.

To see this, we can represent Bill Ross's preferences for consumption in old age and youth by a set of indifference curves. Each indifference curve shows the mixture of combinations of consumption in old age and youth that he would be equally happy to have. Since (we assume) he would like *more* rather than less consumption, his utility is increased by moving in a northeasterly direction.

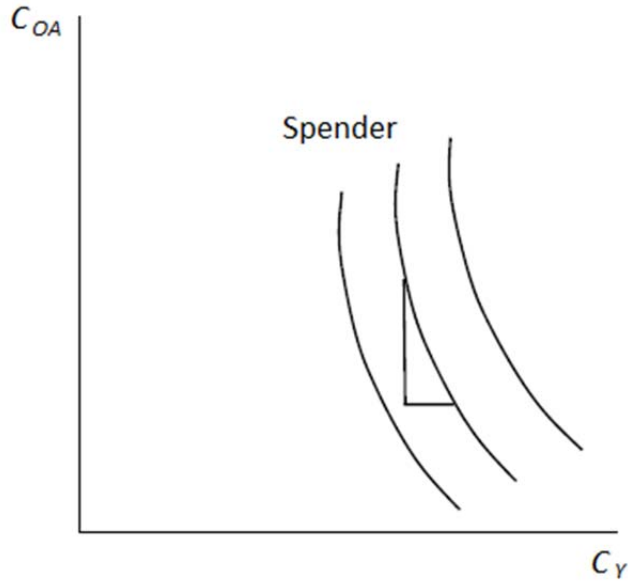


Suppose, for example, that Bill has a strong preference for consumption in old age. In other words, he is a miser. Then his indifference curves will look something like this:

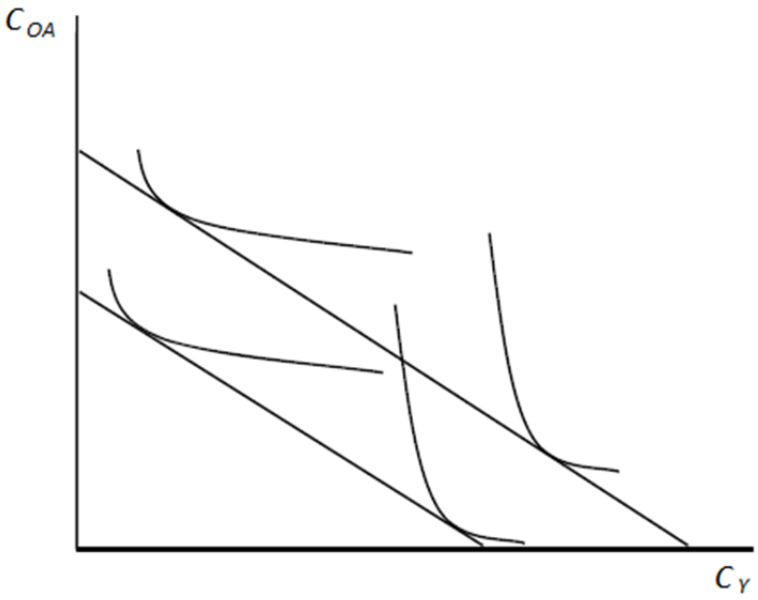


Since he is a miser, a small reduction in consumption in old age would need to be compensated by a large increase in consumption in youth. Therefore, the indifference curves are relatively flat. Conversely, if Bill strongly prefers consumption in his youth (a spender), he will have the type of indifference curves shown below. They are steep because a small reduction in consumption in youth would need to be compensated by a large increase in consumption in old

age.

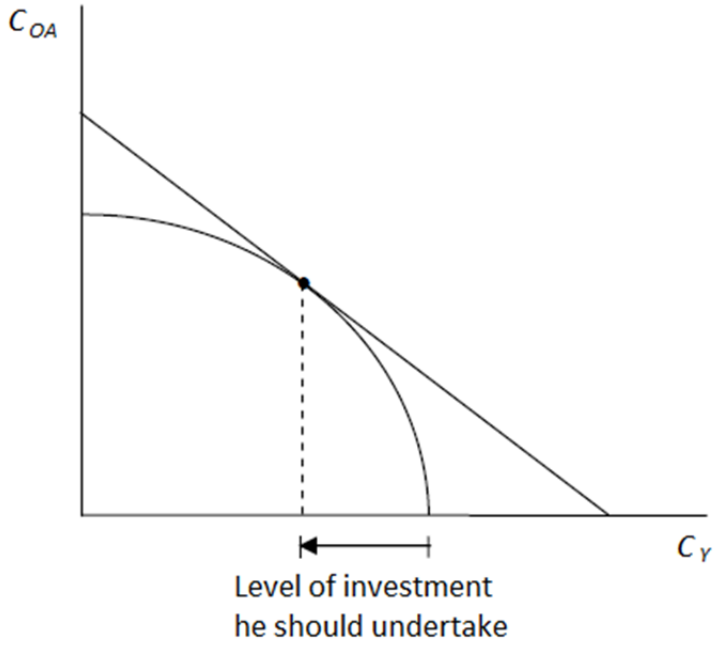


Now if we put budget constraints on these diagrams, we can see that pushing a budget constraint out will always get Bill onto a higher indifference curve and increase his utility.



Hence no matter what Bill Ross's preferences are, he must be made better off if the budget constraint is pushed out. Clearly the best that he can do is to go on investing in projects until he can push out his budget constraint no farther, as in the following diagram. This is true no

matter what his preferences are.

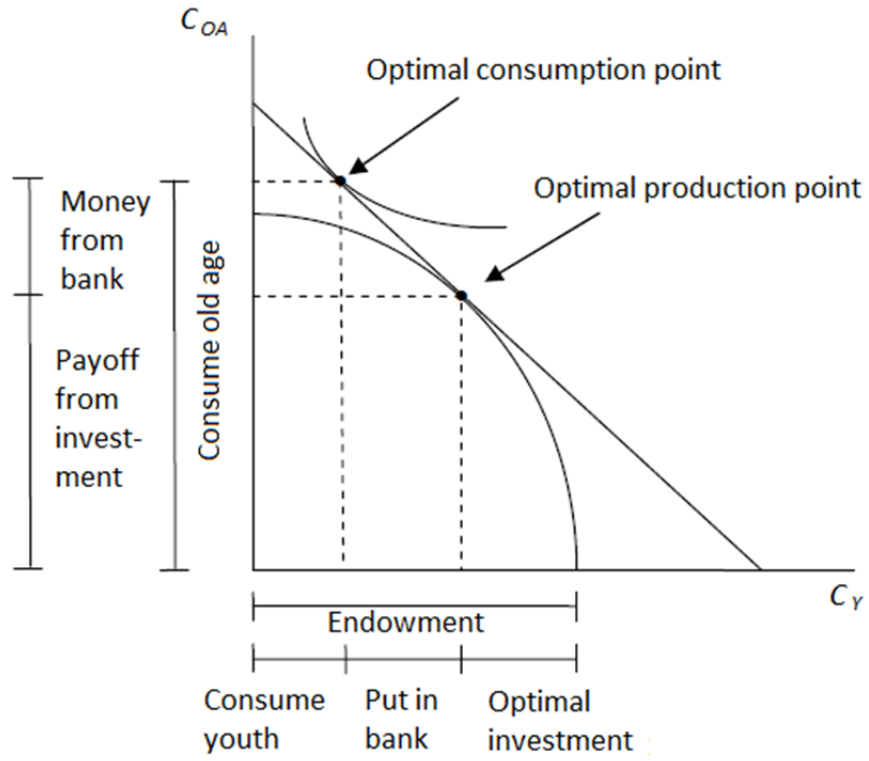


The crucial point here is that the level of investment that Bill should undertake does not depend on his preferences. In other words, his consumption and production decisions are separate. Let's look at a couple of examples and diagrams to illustrate how somebody actually maximizes his utility given this separation.

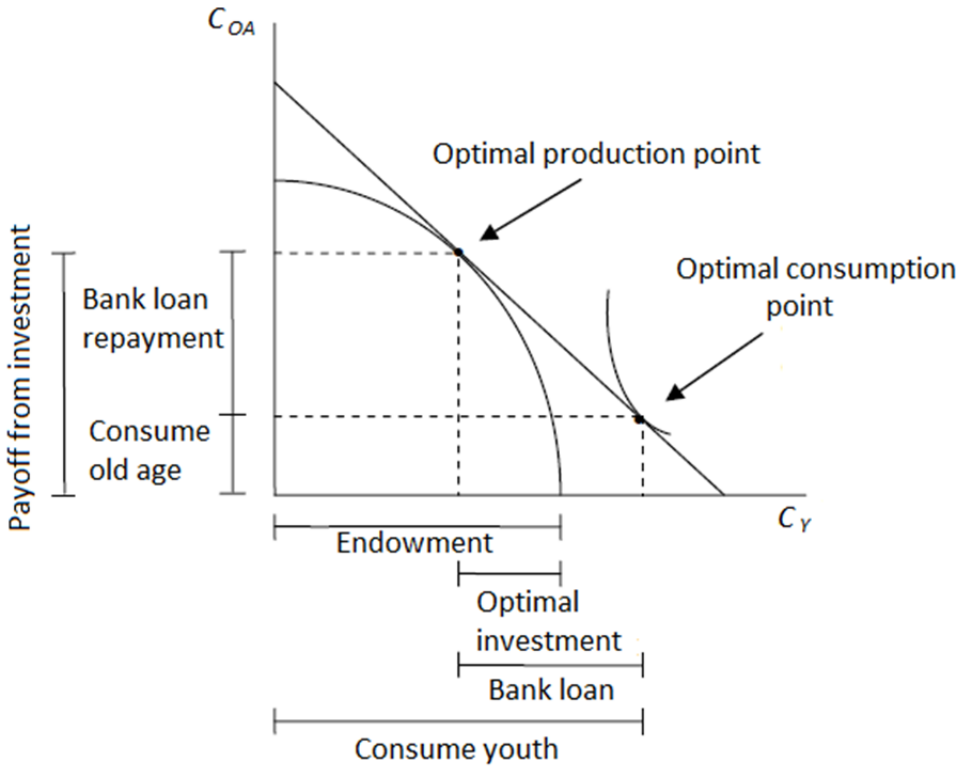
Miser – Lends

If Bill is a miser, he would use his funds in three ways. He would invest in projects up to the optimal investment point, consume modestly, and put the balance in the bank. Then in old age he would spend both the payoff from the projects and the money in the bank.

If he is a spender, he would invest the same amount in projects, but he would borrow against their future payoff to finance his current consumption. Then in old age he would spend the payoff from the projects less the amount that he must repay the bank.

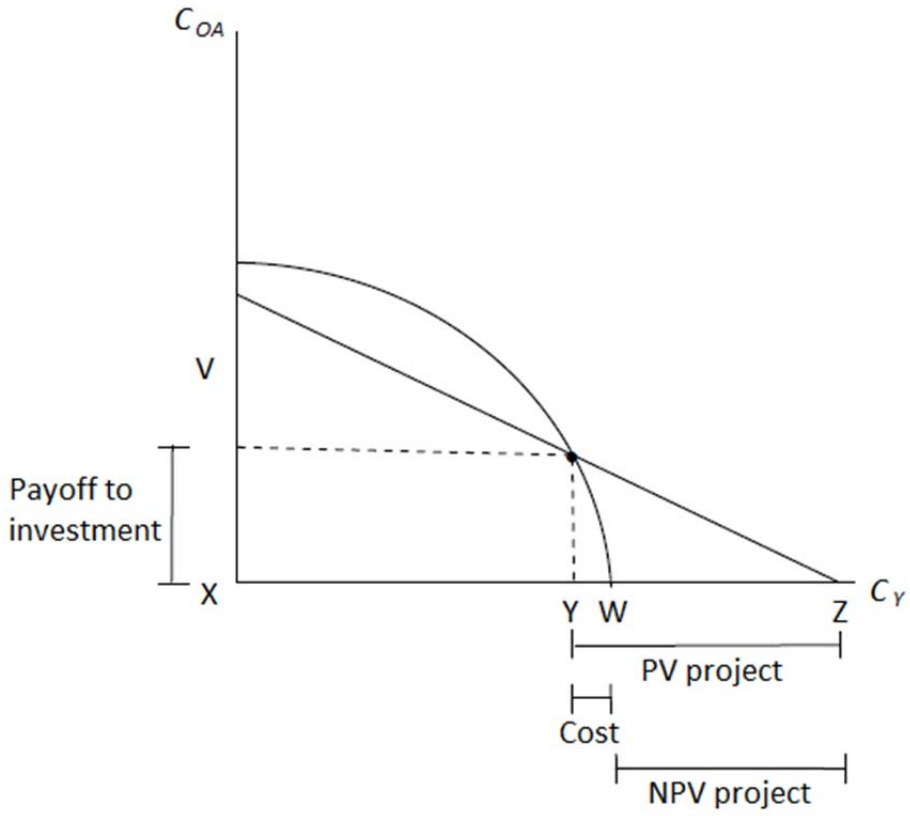


Spender – Borrows



Equivalence of Pushing Out Line to Maximization of NPV

What we now need to show is that pushing out the budget line is equivalent to the maximization of NPV.



The thing to remember is that vertical distances are future values; horizontal distances are PVs. What is XV? It is the payoff to the investment. What, therefore, is YZ? It's the PV of the project. What's YW? It's the cost. Thus, what is WZ? It's the NPV of the project since NPV = PV − Cost. Hence the distance between the intercept for the production possibilities curve and the intercept on the budget constraint is NPV. Making this as large as possible allows a person to consume more in both periods, and she should do it.

Pushing the line out as far as possible is thus equivalent to making the distance WZ as large as possible and hence is equivalent to maximizing NPV.

One important thing to notice from the diagram is that NPV is a monetary measure of how

much better off Bill Ross is from undertaking the investment. It measures the increase in his wealth from the project. NPV is a measure of wealth creation.

Another important thing here is that the discount rate used to find the NPV is the opportunity cost of capital. If Bill Ross doesn't do the project, the best alternative use of his money is to put it in the bank. We use this best alternative rate to discount so we can find out which is the best thing to do.

The bank (or, more generally, the financial market) enables people to choose their desired consumption. The firm's role is to push out the line as far as possible.

Rate of Return

Another way of looking at the problem is in terms of the rate of return.

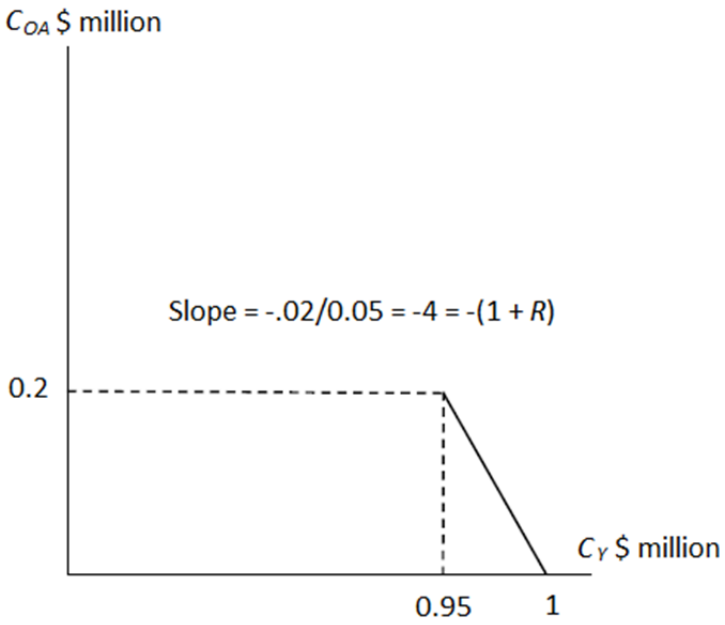
Consider Project A in the following diagram. The rate of return, R, that this project earns is given by

$$1 + R = \frac{200{,}000}{50{,}000} = 4$$

Hence,

R = 3 or 300%

We can represent 1 + R by the slope of the line between A and 1 million on the CY axis:

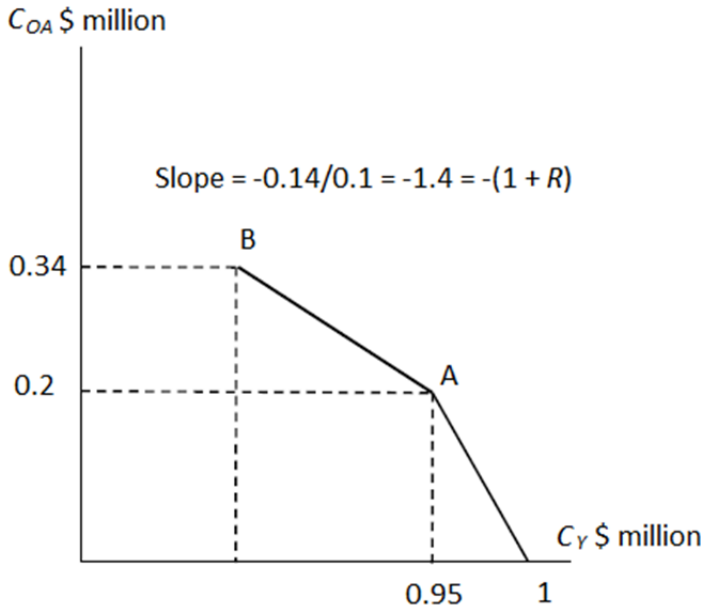


With project B below, the rate of return is given by

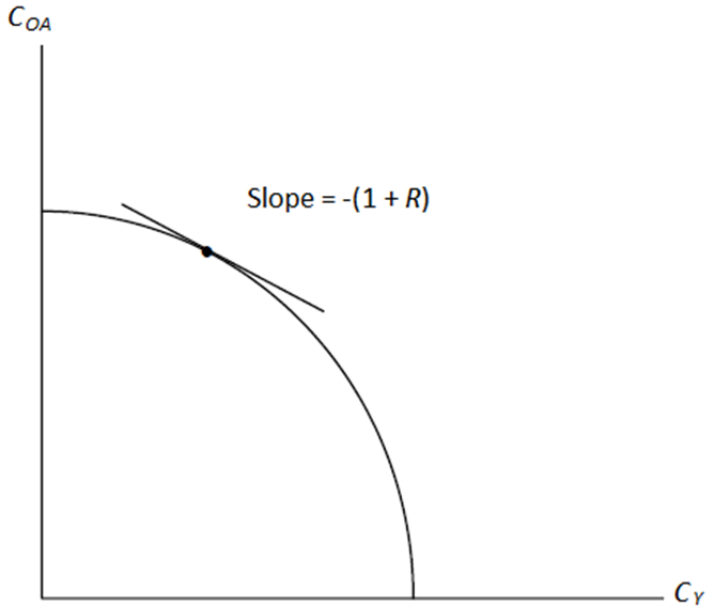
$$1 + R = \frac{140{,}000}{100{,}000} = 1.4$$

R = 0.4 or 40%

Graphically, 1 + R is again the slope of the line between B and A.

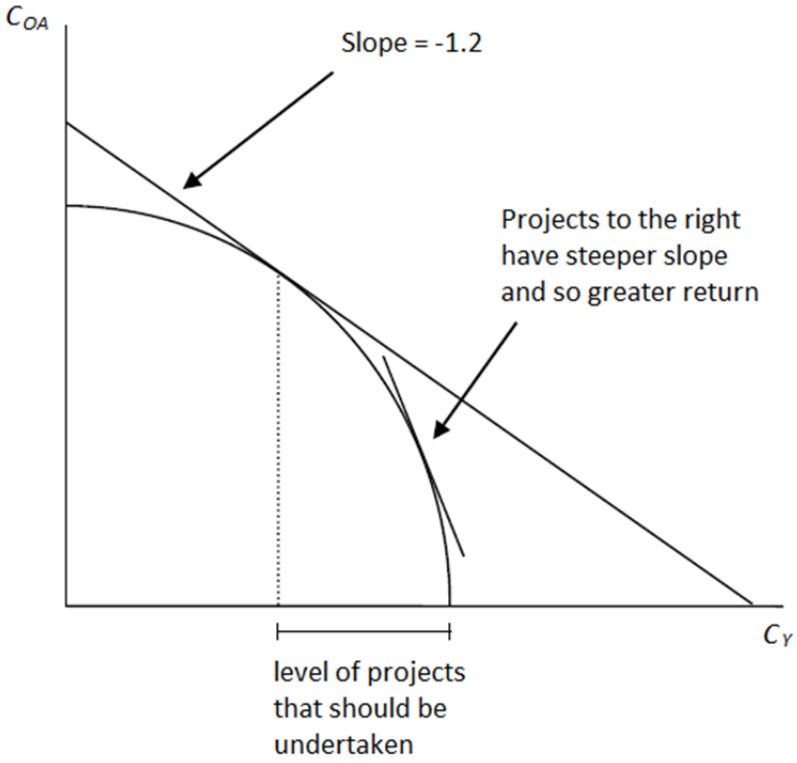


In the more general case where we have the curve representing the production possibilities, the rate of return is represented by the slope of the curve.



Intuitively, it is worthwhile undertaking a project if the rate of return on it is above the rate of interest at which you can borrow or lend at the bank.

We can then see that this is equivalent to the rule we have just derived.



We therefore have two ways of choosing the optimal amount of investment and we have seen these are equivalent.

1. Net Present Value

   Invest so as to maximize the NPV of the investment. This is the difference between the discounted present value of the future income and the amount of the initial investment.

2. Rate of Return Rule

   Invest up to the point at which the rate of return on the investment is equal to the rate of return on alternative investments (in our case, this was the bank).

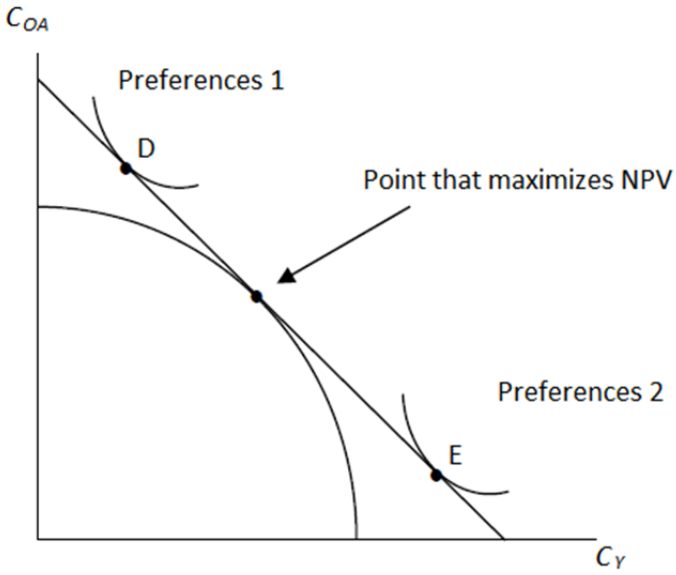
Rules for Managers of Corporations

We have derived the rules for an individual to follow when choosing his investments in real assets. But are they relevant for the type of firm we are interested in where (i) ownership and control are separated and (ii) there are many shareholders who may be very different?

To answer this question, we go back to our example. Bill Ross has been thinking a bit about the implementation of these projects and decides they involve more work than he first thought. He therefore decides to hire managers to implement the investment in real assets. In order to ensure that they pursue his interests all he has to do is to tell them to *maximize NPV*, and this will guarantee that he will be as well off as possible.

The important point to note here is:

Bill's decision on how to allocate his consumption between youth and old age is independent of the need to maximize the NPV of the firm.

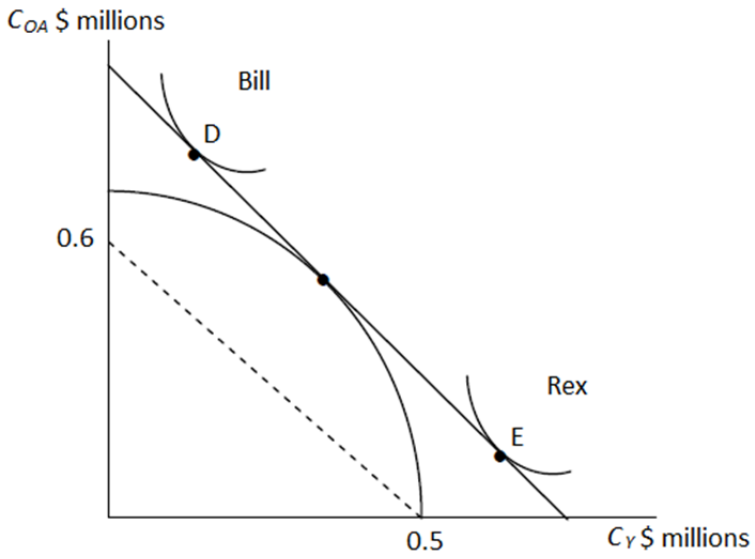


No matter whether he has Preferences 1 (a miser) or 2 (a spender), he will choose to produce at the point that maximizes NPV. With Preferences 1 he consumes at D. With Preferences 2 he consumes at E. Thus ownership and control can be separated. It is only necessary for the owner to tell the manager to maximize NPV; it is not necessary to give a detailed description of how to run the firm.

The fact that it doesn't matter what Bill Ross' preferences are as far as the production decision is concerned has very important implications if there is more than one shareholder. Suppose that in his father's will, the terms under which Bill Ross inherits actually say that the inheritance "is to be divided equally among the children." The $1 million figure was based on the assumption that Bill Ross was the only child. In actual fact, it turns out that his father had another son, Rex, by a former liaison. Bill's inheritance therefore drops to 0.5 million.

Should the managers change the operation of the firm that Bill Ross has set up? No. Since the NPV rule we derived was independent of the owner's preferences, it follows that both Bill and Rex are content to leave the firm the way it is.

Why is this? What is the situation they face? The production possibilities curve is now half what it was before.



1/2 production possibilities curve in the previous case

Thus, even with separate ownership and control and diverse ownership, it is the case that it is in the interest of the owners to maximize NPV (or equivalently to use the rate of return rule). The argument is similar for more than two shareholders.

Significance of Separation of Ownership and Control

The fact that it is possible for many very diverse shareholders to simply tell managers to maximize NPV is crucial for the type of capitalist economy in the U.S. It means that shareholders don't have to be involved in the day-to-day affairs of the companies they own. What they need to know is that managers have incentives to maximize NPV. As long as this is the case they can own many different firms and can diversify their wealth among hundreds or even thousands of companies. If shareholders had to follow the day-to-day affairs of all the companies that they owned shares in, they would only be able to own shares in a few firms.

Assumptions Used in the Derivation of NPV Rule

Is it the case that the NPV rule is always a good rule for corporations to use? The answer is no. In our theory we have made a number of assumptions. If these assumptions do not hold, then the result may not hold. It is important to know when it holds and when it doesn't.

There were a number of simplifying assumptions. The bank we talked about represented the whole range of financial possibilities in capital markets. We also assumed certainty. The rate of interest at which you discount is essentially the opportunity cost of capital—this is the rate of return you can get on a similar asset. We also had only two periods. We can clearly extend to the case where there are more than two periods, but we would have to use more sophisticated mathematical techniques.

The crucial assumption in deriving the NPV rule is that "capital markets" are "perfect."

Perfect Capital Markets

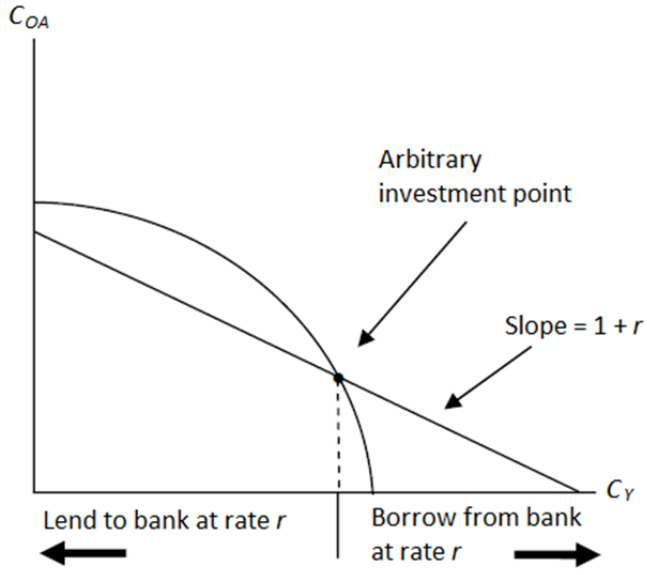
What we mean by this is the following:

1. There are no frictions. Examples of frictions are brokerage costs, fixed fees, and any type of transaction cost such as when lending and borrowing rates differ. To see what happens in this latter case, let's start by considering the case where the two are equal (i.e. from the point of view of consumers):

lending rate = borrowing rate

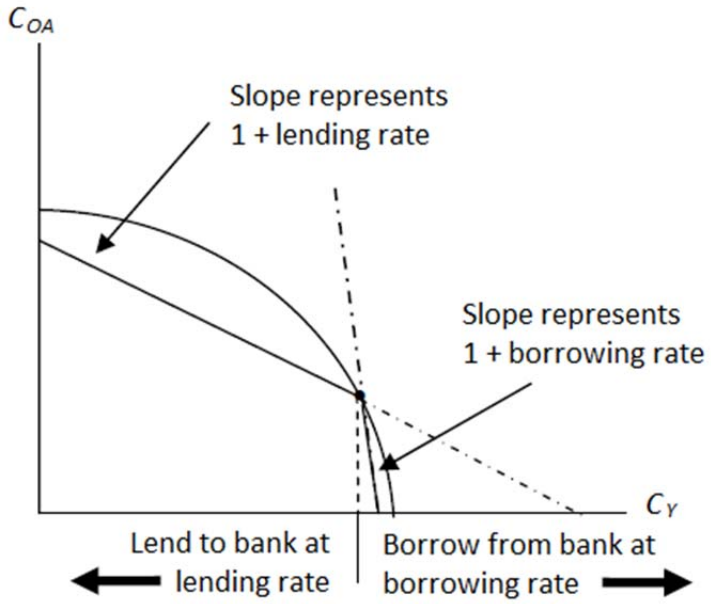
(i.e., put money in bank) (i.e., loan from bank)

First of all consider what happens at an arbitrary point on the investment curve:

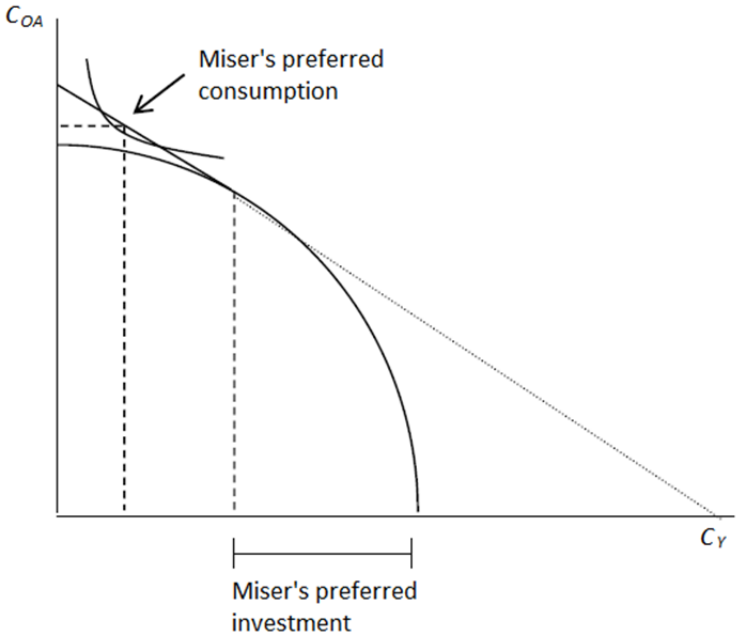


To the left of the point the person is lending to the bank; to the right, the person is borrowing from the bank.

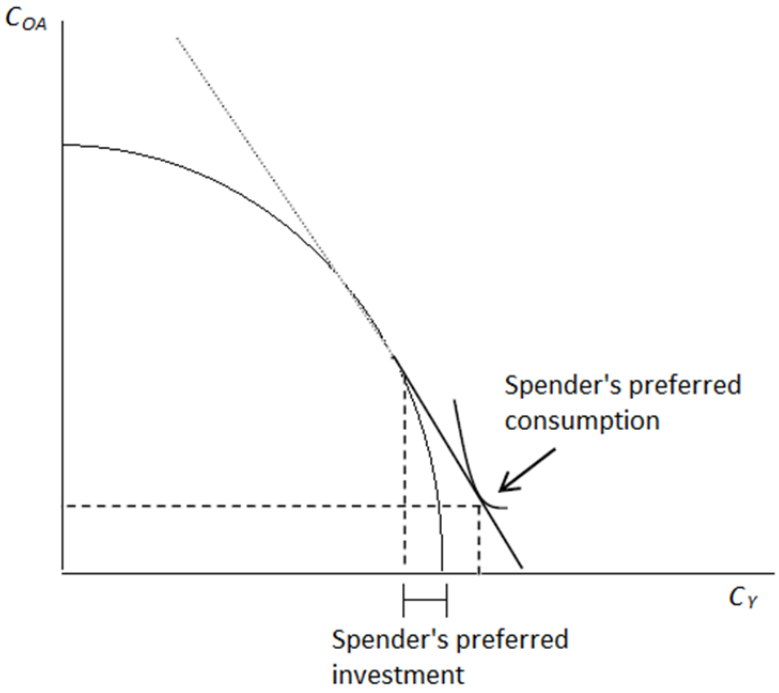
Next consider what happens if the lending rate is less than the borrowing rate so that the bank costs are covered.



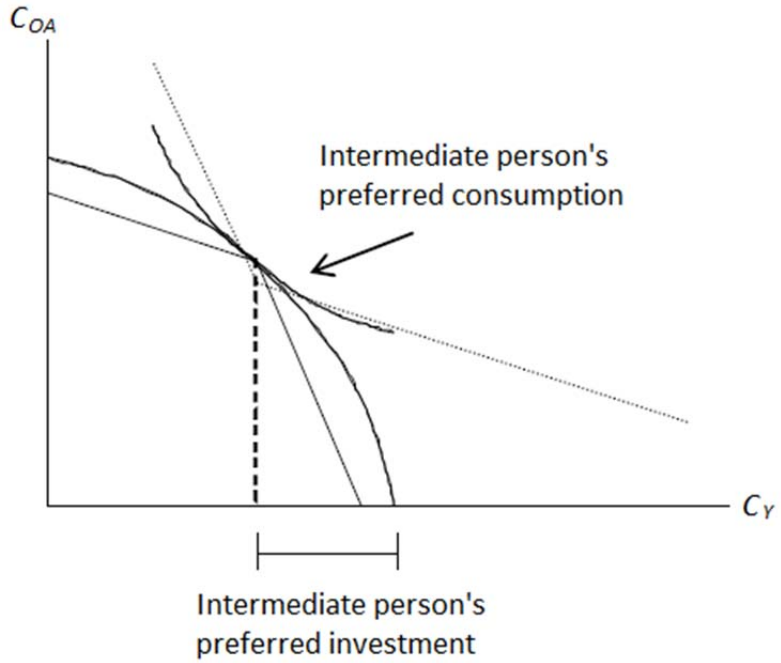
The above represents an arbitrary investment point. What would be the optimal points for a miser and a spender? Consider first the miser. The miser wants to maximize NPV at the lending rate.



Next consider the spender. The spender wants to maximize NPV at the borrowing rate.



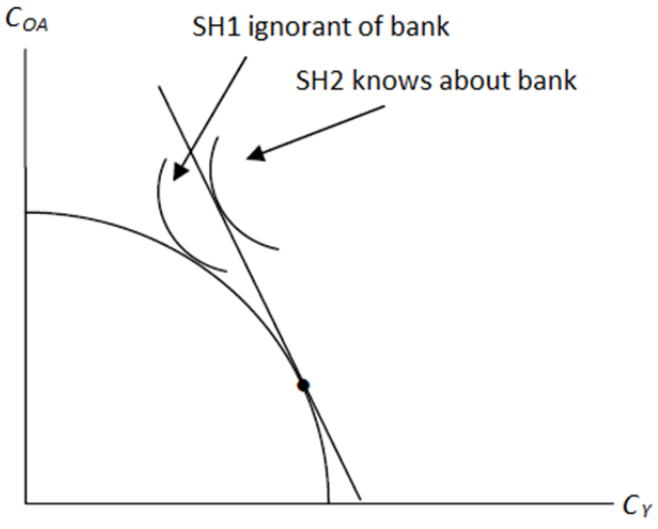
The miser and the spender clearly disagree about the level of investment the firm should undertake. What would somebody with intermediate tastes want?



Again, they no longer agree. Shareholders must vote to determine what the firm should do.

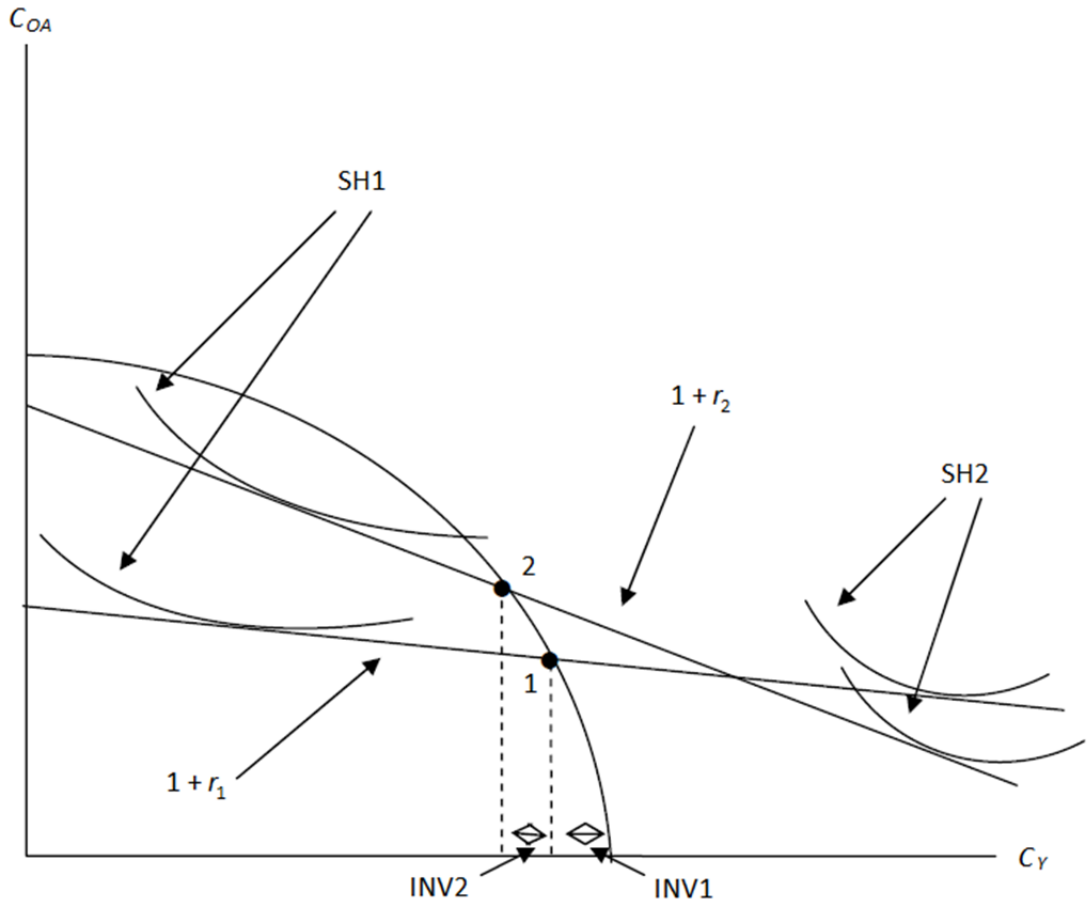
The difference in lending and borrowing rates is just one example of a friction. Whenever there are transaction costs, there will be a friction.

2. Information freely available. If this isn't satisfied, we could, for example, have a case where one shareholder didn't know about the existence of the bank, and so we again have problems.



Again the shareholders must vote.

3. No firm affects interest rates. If a firm (or any person) has a significant effect on interest rates, then stockholders would like the firm to take this into account when making investment decisions. For example, if the firm invests more, so that the demand for funds rises, then the rate of interest, r, may rise from r_1 to r_2, and this may make some shareholders better off and some worse off.



Situation 1 (r_1): One possible level of investment and corresponding interest rate.
Situation 2 (r_2): Higher level of investment and higher corresponding interest rate.

Once again they no longer agree. Shareholder 1 prefers Situation 2 while Shareholder 2 prefers Situation 1. Once again shareholders must vote on what the firm should do.

Assessment of Assumptions

Clearly assumptions 1 to 3 aren't entirely satisfied. However, it is the case that in many economies, particularly in the U.S., they come pretty close to being satisfied. The reason that assumptions 1 and 2 come close to being satisfied is that rich people and institutional holders own most of the stocks in the U.S. For these investors the assumptions are reasonable. As far as the third assumption is concerned, no corporations are large enough to affect interest rates, so this assumption is satisfied. It is for these reasons that we can take the NPV rule as a reasonable guide for managers to follow at least in the U.S.

A number of other countries, particularly emerging economies such as Brazil, India, and Korea, are not as perfect. In these cases it is not clear that maximization of NPV is a sensible goal for corporations with many diverse owners. In such cases shareholders must get involved in the running of the corporation and vote on what it should do. This is why family firms, which avoid these issues, are so much more prevalent in such countries.