Arbitrage Models of the Term Structure

If we know how the returns on bonds with different maturities move together, then we can make some useful statements about the *relative* prices of these bonds.

Suppose, for example, that you can invest in three possible government loans: a three-month Treasury bill, a medium-term bond, and a long-term bond. The return on the Treasury bill over the next three months is certain; we will assume that it yields a 2% quarterly rate. But the return on each of the other bonds depends on what happens to interest rates. Suppose that you foresee only two possible outcomes – a sharp rise in interest rates or a sharp fall. The following table summarizes how the prices of the three investments would be affected:

Illustrative payoffs from three government securities. Note the wider range of outcomes from the longer-term bonds. We don't know what the medium bond sells for, but we can figure it out from the way that its value changes when interest rates change.

		Change in Value		
	Beginning Price	Interest Rates Rise	Interest Rates Fall	Ending Value
Treasury bill	98	+2	+2	100
Medium-term bond	?	-6.5	+10	?
Long-term bond	105	-15	+18	90 or 123

Notice that the long-term bond has a longer duration and therefore a wider range of possible outcomes.

Now suppose that you start with $100. You invest half of this money in the Treasury bill and half in the long-term bond. If interest rates rise, the change in the value of your portfolio will be (.5 x 2) + [.5 x (-15)] = -$6.5. If rates fall, the change in value will be (.5 x 2) + (.5 x 18) = +$10. Thus, regardless of whether rates rise or fall, your portfolio will provide exactly the same payoffs as an investment in the medium-term bond. Since the two investments provide identical payoffs, they must sell for the same price or there will be an opportunity for arbitrage. So, the value of the medium-term bond must be halfway between the value of a three-month bill and that of a long-term bond, that is, (98 + 105)/2 = 101.5. Knowing this, you can calculate the yield on the medium-term bond. You can also calculate its value next year, either 101.5 – 6.5 = 95 or 101.5 + 10 = 111.5.

Our example is grossly oversimplified, but you probably get the basic idea. If you know how bond prices move together, you may be able to construct a package of two bonds that will provide identical payoffs to a third bond. That allows you to value one bond given the prices of the other two bonds.

In practice, bond traders use extremely intricate and complex models, but the underlying idea is the same as in our example. First identify how changes in the prices of bonds with different maturities are related, then calculate the proper relationship between prices.