Portfolio Selection in Practice

Table 8.1 illustrates an efficient portfolio for 10 stocks. But you can build a portfolio from thousands of stocks. Say you work with a list of 5,000 stocks. Calculating the efficient portfolio from this list would require 5,000 standard deviations, and $(5{,}000^2 – 5{,}000)/2 = 12{,}497{,}500$ correlation coefficients or covariances. *Impossible*: You would not just get GIGO (garbage in garbage out), but GIGOG (garbage in, garbage out guaranteed).

Practical applications of portfolio theory find a way to simplify. One approach groups stocks by type into portfolios, for example large-cap growth, large-cap income, medium-cap growth, medium-cap income, small-cap growth and small-cap income. ("Cap" refers to market capitalization, defined as stock price times number of shares outstanding.) This gives six portfolios, which are treated just like individual securities, and a manageable number of inputs to estimate.

An international investor could work with portfolios by country or region and estimate a manageable number of inputs.

Another approach: Define the first stock as a benchmark portfolio, for example the S&P 500 index. Then consider adding stocks from a short list for which you think you have special information.

Suppose you think that you have special information about GE. Then you have only four boxes to fill in.

	GE	S&P
GE		
S&P		

Assume the following inputs:

	GE		S&P
Expected Return	13.49%		8.5%
Standard Deviation	39.99%		19.85%
Correlation		0.826	
Covariance		655.9	

The risk-free interest rate is r_f = 1.0% and the expected market risk premium is 7.5%.

The following table shows portfolio weights, the portfolio expected return (r_P) and standard deviation (σ_P). The goal is to maximize the Sharpe ratio, defined as the ratio of expected portfolio risk premium to portfolio standard deviation.

S&P	GE	r_p	σ_p	$(r_p - r_f)/\sigma_p$	
1.2	− .2	7.5	17.79	0.3655	
1.1	− .1	8.0	18.67	0.3750	
1.0	0	8.5	19.85	0.3778	← Highest Sharpe ratio
.9	+ .1	9.0	21.29	0.3757	
.8	+ .2	9.5	22.94	0.3705	
.7	+ .3	10.0	24.75	0.3635	

Notice that the Sharpe ratio is maximized by putting 100% in the S&P index. With these inputs, the optimal portfolio is 100% in the index and 0% in GE.

But GE is already included in the S&P 500 index. Therefore a zero portfolio weight on GE means investing in GE at its percentage weight in the market index. A positive portfolio weight here means *over-weighting* GE, that is, investing more in GE than its weight in the market index. A negative portfolio weight means *under-weighting* GE. A large underweight would require a short position in GE.

Why does our calculation recommend investing in GE at exactly its market weight? You can see the trick by calculating GE's beta (the ratio of its covariance with the market to the market variance) and then calculating GE's expected rate of return from the CAPM. The expected return of 13.49% is exactly what the CAPM predicts.

The CAPM assumes that investors hold a well-diversified market portfolio. Put differently, the CAPM says that investors who share the same information will be willing to hold all stocks at their market weights.

Figure 1 shows that a portfolio that does not over- or under-weight GE is efficient.

Now assume you are strongly bullish on GE and believe that the expected rate of return is 15%, about 1.5% higher than the CAPM predicts. The new Sharpe ratios are:

S&P	GE	r_p	σ_p	$(r_p - r_f)/\sigma_p$	
1.0	0	8.5	19.85	0.3778	
.9	+ .1	9.15	21.29	0.3828	
.875	+ .125	9.31	21.68	0.3834	
.85	+ .15	9.48	22.09	0.3837	
.825	+ .175	9.64	22.51	0.3838	← Highest Sharpe ratio
.80	+ .20	9.8	22.94	0.3837	

With these inputs, the optimal portfolio would over-weight GE by about 17.5%.

You may wish to refer to these calculations when you tackle the mini-case at the end of this chapter.

Figure 1