How Dilution Affects Option Value

If you buy a call option on an options exchange and then exercise it, you have no effect on the number of outstanding shares. The investor who sold the call simply hands over to you his or her shares. However, sometimes the company itself may issue options to buy its own shares. For example, we saw in Section 21-4 that in 2008 Oracle Corporation issued a total of 69 million executive stock options.

Companies also issue convertible bonds, which give investors the option to exchange their bonds in the future for common stock. Therefore, a convertible bond resembles a package of a straight bond and an option to buy the stock. Alternatively, the company may sell a package of bonds and warrants. These warrants are long-term call options to buy the company's stock. Presumably the company hopes that the warrants will serve as a "sweetener," so that by includ-ing options in the package investors will be induced to pay a much higher price. If the holders of the convertible bonds or the warrants decide to exercise their option, the company must issue the additional shares to them.

Options that are issued by the company are somewhat trickier to value than exchange-traded options. When these options are exercised, the firm's assets and profits are spread over a larger number of shares. Sometimes this dilution is negligible and can safely be ignored. But, if the number of shares can increase substantially, you need to take it into account when valuing the options. To illustrate how you can do so, we will work through the example of United Glue's warrants.

Example: Valuing United Glue's Warrants United Glue has just issued a $2 million package of debt and warrants. Here are some basic data that we can use to value the warrants:

Number of shares outstanding (N)	1 million
Current stock price (P)	$12
Number of warrants issued per share outstanding (q)	.10
Total number of warrants issued (Nq)	100,000
Exercise price of warrants (EX)	$10
Time to expiration of warrants (t)	4 years
Annual standard deviation of stock price changes (σ)	.40
Rate of interest (r)	10%
United stock pays no dividends	

Suppose that without the warrants the debt is worth $1.5 million. Then investors must be paying $.5 million for the warrants:

$$\text{Cost of warrants} = \text{total amount of financing} - \text{value of loan without warrants}$$

$$\$500{,}000 = \$2{,}000{,}000 - \$1{,}500{,}000$$

$$\text{Each warrant costs investors } 500{,}000/100{,}000 = \$5$$

Table 21A.1 shows the market value of United's assets and liabilities both before and after the issue.

Now let us take a stab at checking whether the warrants are really worth the $500,000 that investors are paying for them. Since the warrant is a call option to buy the United stock, we can use the Black–Scholes formula to value the warrant. It turns out that a four-year call to buy United stock at $10 is worth $6.15.[1] Thus the warrant issue looks like a good deal for investors and a bad deal for United. Investors are paying $5 a share for warrants that are worth $6.15.

Table 21A.1 United Glue's market for value balance sheet (in $ millions)

Before the Issue			
Existing assets	$16	$ 4	Existing loans
		12	Common stock (1 million shares at $12 a share)
Total	$16	$16	Total
After the Issue			
Existing assets	$16	$ 4	Existing loans
New assets financed		1.5	New loan without warrants
by debt and warrants	2	5.5	Total debt
		.5	Warrants
		12	Common stock
		12.5	Total equity
Total	$18	$18.0	Total

[1] Plugging the data for United into the Black–Scholes formula gives

$$d_1 = \log[12/(10/1.1^4)]/(.40 \times \sqrt{4}) + (.40 \times \sqrt{4}/2 = 1.104 \text{ and } d_2 = 1.104 - .40 \times \sqrt{4} = .304$$

The Excel NORMSDIST function tells us that $N(d_1) = .865$, and $N(d_2) = .620$. Therefore, estimated warrant value = $.865 \times 12 - .620 \times (10/1.1^4) = \$6.15$.

How the Value of United Warrants Is Affected by Dilution Unfortunately, our calculations for United warrants do not tell the whole story. Remember that when investors exercise a traded call or put option, there is no change in either the company's assets or the number of shares outstanding. But, if United's warrants are exercised, the number of shares outstanding will increase by Nq = 100,000. Also the assets will increase by the amount of the exercise money (Nq × EX = 100,000 × \$10 = \$1 million). In other words, there will be dilution. We need to allow for this dilution when we value the warrants.

Let us call the value of United's equity V:

Value of equity = V = value of United's total assets – value of debt

If the warrants are exercised, equity value will increase by the amount of the exercise money to $V + Nq\text{EX}$. At the same time the number of shares will increase to $N + Nq$. So the share price after the warrants are exercised will be:

$$\text{Share price after exercise} = \frac{V+Nq\text{EX}}{N+Nq}$$

At maturity the warrant holder can choose to let the warrants lapse or to exercise them and receive the share price less the exercise price. Thus the value of the warrants will be the share price minus the exercise price or zero, whichever is the higher. Another way to write this is:

$$\text{Warrant value at maturity} = \text{maximum(share price} - \text{exercise price, zero)}$$

$$= \text{maximum}\left(\frac{V+Nq\text{EX}}{N+Nq} - \text{EX}, 0\right)$$

$$= \text{maximum}\left(\frac{V/N-\text{EX}}{1+q}, 0\right)$$

$$= \frac{1}{1+q}\text{maximum}\left(\frac{V}{N} - \text{EX}, 0\right)$$

This tells us the effect of dilution on the value of United's warrants. The warrant value is the value of $1/(1 + q)$ call options written on the stock of an alternative firm with the same total equity value V, *but with no outstanding warrants*. The alternative firm's stock price would be equal to V/N —that is, the total value of United's equity (V) divided by the number of shares outstanding (N). The stock price of this alternative firm is more variable than United's stock price. So when we value the call option on the alternative firm, we must remember to use the standard deviation of the changes in V/N.

Now we can recalculate the value of United's warrants allowing for dilution. First we find the stock price of the alternative firm:

Current equity value of alternative firm = V = value of United's total assets – value of loans

= 18 – 5.5 = $12.5 million

Current share price of alternative firm = $\frac{V}{N} = \frac{12.5 \text{ million}}{1 \text{ million}}$ = $12.50

Also, suppose the standard deviation of the share price changes of this alternative firm is σ* = .41.[2]

The Black–Scholes formula gives a value of $6.64 for a call option on a stock with a price of $12.50 and a standard deviation of .41. The value of United warrants is equal to the value of 1/(1 + q) call options on the stock of this alternative firm. Thus the warrant value is:

$$\frac{1}{1+q} \times \text{value ofcall on alternative firm} = \frac{1}{1.1} \times 6.64 = \$6.04$$

This is a somewhat lower value than the one we computed when we ignored dilution but still a bad deal for United.

It may sound from all this as if you need to know the value of United warrants to compute their value. This is not so. The formula does not call for warrant value but for V, the value of United's equity (that is, the shares *plus* warrants). Given equity value, the formula calculates how the overall value of equity should be split up between stock and warrants. Thus, suppose that United's underwriter advises that $500,000 extra can be raised by issuing a package of bonds and warrants rather than bonds alone. Is this a fair price? You can check using the Black–Scholes formula with the adjustment for dilution.

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[2] How in practice could we compute σ*? It would be easy if we could wait until the warrants had been trading for some time. In that case σ* could be computed from the returns on a package of all the company's shares and warrants. In the present case we need to value the warrants *before* they start trading. We argue as follows: The standard deviation of the *assets* before the issue is equal to the standard deviation of a package of the common stock and the existing loans. For example, suppose that the company's debt is risk-free and that the standard deviation of stock returns *before* the bond-warrant issue is 38%. Then we calculate the standard deviation of the initial assets as follows:

Standard deviation of initial assets = proportion in common stock × standard deviation of common stock

$$= \frac{12}{16} \times 38 = 28.5\%$$

Now suppose that the assets after the issue are equally risky. Then,

Standard deviation of assets after issue = proportion of equity after issue × standard deviation of equity (σ*)

$$28.5 = \frac{12.5}{18} \times \text{standard deviation of equity } (\sigma^*)$$

Standard deviation of equity (σ*) = 41%

Notice that in our example the standard deviation of the stock returns *before* the warrant issue was slightly lower than the standard deviation of the package of stock and warrants. However, the warrant holders bear proportionately more of this risk than do the stockholders; so the bond-warrant package could either increase or reduce the risk of the stock.

Finally, notice that these modifications are necessary to apply the Black–Scholes formula to value a warrant. They are not needed by the warrant holder, who must decide whether to exercise at maturity. If at maturity the price of the stock exceeds the exercise price of the warrant, the warrant holder will of course exercise.

QUESTION

Here is a question about dilution. The Electric Bassoon Company has outstanding 2,000 shares with a total market value of $20,000 *plus* 1,000 warrants with a total market value of $5,000. Each warrant gives its holder the option to buy one share at $20.

a. To value the warrants, you first need to value a call option on an alternative share. How might you calculate its standard deviation?

b. Suppose that the value of a call option on this alternative share was $6. Calculate whether the Electric Bassoon warrants were undervalued or overvalued.