Capital Rationing Models

In Section 5-4 we looked at a capital rationing project where the firm had a capital investment limit of $10 million in each of years 1 and 2. The firm needs to choose from the following four projects.

	Cash Flows Millions of Dollars				
Project	C_0	C_1	C_2	NPV at 10% $ millions	Profitability Index
A	-10	+30	+5	21	3.1
B	-5	+5	+20	16	4.2
C	-5	+5	+15	12	3.4
D	0	-40	+60	13	1.4

Ranking by the profitability index suggests that the firm should take projects B and C. However, if instead we invested in project A in period 0, we would generate enough cash to undertake D in period 1. The combination of A and D has a higher NPV than B and C.

In this example ranking projects on the basis of the profitability index fails because resources are constrained in each of years 1 and 2. In fact this method is inadequate whenever there is *any* other constraint on the choice of projects. This means that it cannot cope with cases in which two projects are mutually exclusive or in which one project is dependent on another.

The simplicity of the profitability-index method may outweigh its limitations. For example, it may not pay to worry about expenditures if you have only a hazy notion of future capital availability or investment opportunities. But there are occasions when we need a more general method of choosing between a set of projects.

Consider the above set of projects. Suppose that we were to accept proportion x_A of project A. Then the net present value of this investment would be $21x_A$. Similarly, the net present value of our investment in project B would be $16x_B$ and so on. Our objective is to select the set of projects with the highest total net present value. In other words, we wish to find the values of x that maximize

$$\text{NPV} = 21x_A + 16x_B + 12x_C + 13x_D$$

Our choice of projects is subject to several constraints. First, total cash outflow in period 0 must not be greater than $10 million. In other words,

$$10x_A + 5x_B + 5x_C + 0x_D <= 10$$

Similarly, total outflow in period 1 must also not be greater than \$10 million:

$$-30x_A - 5x_B - 5x_C + 40x_D <= 10$$

Finally, we cannot invest a negative amount in a project, and we cannot purchase more than one of each. Thus

$$0 <= x_A <= 1, \quad 0 <= x_B <= 1\ldots\ldots$$

Collecting all these conditions, we can summarize the problem as follows:

Maximize $21x_A + 16x_B + 12x_C + 13x_D$

Subject to

$10x_A + 5x_B + 5x_C + 0x_D <= 10$

$-30x_A - 5x_B - 5x_C + 40x_D <= 10$

$0 <= x_A <= 1, \quad 0 <= x_B <= 1\ldots\ldots$

One way to tackle such a problem is to keep selecting different values for the *x*s, noting which combination both satisfies the constraints and gives the highest net present value. But it is smarter to recognize that the equations above constitute a linear programming (LP) problem, which can be handed over to a computer. For example, you could use the Solver tool in Excel.

So far we have assumed that it is possible to invest in a fraction of a project. In some case this is reasonable. For example, if project A represents an investment in 1000 tons of steel plate, it might be feasible instead to buy 500 tons. If, however, project A is a single crane or oil well, such fractional investments make little sense. In such case you need to use a variant of linear programming which limits all the *x*s to integers. For example, if you are using Excel's solver, you can specify that the *x*s must be binary.

Some Embellishments

When choosing between our four projects, you may also need to add some additional constraints.

Cash carry-forward A plant manager who is forced to return the unspent part of an annual capital allocation may be goaded into a substantial year-end investment in pink carpeting for the foundry flow or other equally silly assets. (How can you argue for a high budget next year if there is money left over this year?) Headquarters can alleviate this problem by permitting the manager to carry forward any unspent balance. For example, to allow the possibility of cash carry-forward, we simply need to add another term to the spending constraint. Let s denote the funds transferred from year 0 to year 1 and let them earn interest at the rate r. Then we can rewrite the constraint for year 0 as

$$10x_A + 5x_B + 5x_C + 0x_D + s = 10$$

Similarly, the constraint for year 1 becomes

$$-30x_A - 5x_B - 5x_C + 40x_D <= 10$$

Since carrying forward a negative amount is equivalent to borrowing we will probably wish to add the constraint $s => 0$.

Mutually exclusive projects Suppose now that projects B and C are mutually exclusive. We can take care of this in an integer program by specifying that our total investment in the two projects cannot be greater than 1

$$x_B + x_C < = 1, \qquad x_B, x_C = 0 \text{ or } 1$$

In other words, if x_B is 1, x_C must be zero; if x_C is 1, x_B must be 0.

Contingent projects Suppose next that project D is an attachment to project A, and we cannot accept D, *unless* we also accept A. In this case we need to add

$$x_D - x_A < = 0 \qquad x_D, x_A = 0 \text{ or } 1$$

In other words, if x_A is 1, x_D can be 0 or 1; but if x_A is 0, x_D must likewise be zero.

Constraints on nonfinancial resources Money may not be the only scarce resource. Each of our projects may require the services from a 12-person design department. If project would employ three designers, project B two, and so on, we would need to add a constraint like

$$3x_A + 2x_B + 8x_C + 3x_D <= 12$$

Constraints on nonfinancial output Sometimes it makes sense to place constraints on the total increase in physical capacity. Suppose that projects A and C produce four and three units respectively of the same product. If the company is unable to sell more than five units, it is necessary to add

$$4x_A + 3x_C <= 5$$

We could go on – but you get the idea.