A Certainty-Equivalent Version of the CAPM

When calculating present value you can take account of risk in either of two ways. You can discount the expected cash flow C_1 by the risk-adjusted discount rate r:

$$PV = C_1/(1 + r)$$

In practice this is the formulation that most managers use.

The alternative is to discount the certainty-equivalent cash flow CEQ_1 by the risk-free rate of interest r_f:

$$PV = CEQ_1/(1 + r_f)$$

The certainty-equivalent cash flow, CEQ_1, is lower than the expected cash flow C_1 because it contains a mark-down to reflect the fact that the cash flow is risky. We now show how you can use the CAPM to calculate this mark-down.

We know from our present value formula that $1 + r$ equals the expected dollar payoff on the asset divided by its present value:

$$1 + r = C_1/PV$$

The capital asset pricing model also tells us that $1 + r$ equals

$$1 + r = 1 + r_f + \beta(r_m - r_f)$$

Therefore

$$C_1/PV = 1 + r_f + \beta(r_m - r_f)$$

In order to find beta, we calculate the covariance between the asset return and the market return and divide by the market variance:

$$\beta = cov(r,r_m)/\sigma_m^2 = cov[(C_1/PV - 1,r_m)]/\ \sigma_m^2$$

The quantity, C_1 is the future cash flow and is therefore uncertain. But PV is the asset's present value: It is not unknown and so does not "covary" with r_m. Therefore, we can rewrite the expression for beta as

$$\beta = cov(C_1,r_m)/PV\sigma_m^2$$

Substituting this expression back into our equation for C_1/PV gives

$$C_1/PV = \frac{1+r_f+cov(C_1,r_m)}{PV} \times \frac{r_m-r_f}{\sigma_m{}^2}$$

The expression $(r_m - r_f)/\sigma_m^2$ is the expected risk premium on the market per unit of variance. It is often known as the market price of risk and is written as λ (lambda).

Thus

$$C_1/\text{PV} = \frac{1+r_f+\lambda\text{cov}(C_1,r_m)}{\text{PV}}$$

Multiplying through by PV and rearranging, gives

$$\text{PV} = \frac{C_1-\lambda\text{cov}(C_1,r_m)}{1+r_f}$$

This is the certainty-equivalent form of the capital asset pricing model. It tells us that, if the asset is risk-free, $\text{cov}(C_1,r_m)$ is zero and we simply discount C_1 by the risk-free rate. But, if the asset is risky, we must discount the certainty-equivalent of C_1. The deduction that we make from C_1 depends on the market price of risk, λ , and on the covariance between the cash flows on the project and the return on the market.