

Deriving AD from the aggregate expenditures model

In earlier math notes, we derived the formula for equilibrium GDP as the solution to the following equation: $Y = C + I_g + G + X_n$ where $C = a(W, E, B, i) + b(Y - T)$, $I_g = f(i, r(A, B, C, K, E)) + \Delta V$ and $X_n = X_n(Y_f, t, P_s)$. In words, consumption is assumed to be a linear function of disposable income, but the intercept is a function of household wealth, consumer expectations, household borrowing, and real interest rates. Planned investment is a function of the real rate of interest, the rate of return on investments, plus planned changes in inventories. The rate of return is itself a function of acquisition, maintenance, and operating cost, business taxes, technological change, the stock of capital, and expectations. Net exports are a function of foreign income, tariff rates, and the exchange rate. At equilibrium GDP, production equals planned expenditures.

Here we can note that many of these spending determinants are themselves dependent on the domestic price level. Specifically, wealth, the real rate of interest, and the exchange rate are all functions of the domestic price level: $W = W(P)$; $i = i(P)$; $P_s = P_s(P)$. A higher price level reduces autonomous consumption by reducing wealth. Likewise, it reduces planned investment by increasing the real rate of interest. Finally, it reduces net exports by causing the dollar to appreciate.

If we hold all non-price-level determinants of autonomous spending constant, we can simplify our expression of GDP to the following: $Y = a(P) + b(Y - T) + I_g(P) + G + X_n(P)$, or even more compactly, $Y = A(P) + bY$ where $A(P) = a(P) - bT + I_g(P) + G + X_n(P)$ includes all autonomous spending (that is, spending that is independent of the level of GDP). Since a higher price level reduces autonomous consumption, planned investment, and net exports, we know that dA/dP is negative: $A'(P) = a'(P) + I_g'(P) + X_n'(P) < 0$. The first term is the “real balances effect,” the second is the “interest-rate effect,” and the final term is the “foreign purchases effect.”

Solving our equation for Y we obtain $Y = \left(\frac{1}{1-b}\right) \cdot A(P)$. This relationship between equilibrium GDP and the price level is the aggregate demand curve, or AD. We see that it is downward sloping: $dY/dP = \left(\frac{1}{1-b}\right) \cdot A'(P) < 0$. Further, we note that the AD curve is an “equilibrium” curve of sorts. Total production equals planned purchases at every combination of Y and P along the AD curve.