Rule of 70

As stated in the text, the "Rule of 70" says that real GDP will double in 70/*r* years, where *r* is the annual rate of growth expressed as a percentage. For example, real annual GDP growth of 5% will cause real GDP to double in (70/5) = 14 years. More generally, whether it be real GDP, the price level, or the value of your Individual Retirement Account (IRA), any initial value growing at *r*% per year will double in 70/*r* years. Why does this formula work?

For the remainder of this note, it will be convenient to work with a growth rate of g = *r*/100. For example, if r = 5%, then $g = .05$. Suppose we start with some initial amount, say V_0. Growing at a rate of g, an additional amount of gV_0 will be added to its value at the end of the first year. If V_1 is the value at the end of the first year, then $V_1 = V_0 + gV_0 = V_0(1 + g)$. Using similar reasoning, its value at the end of the second year will be $V_2 = V_1(1 + g)$. But since $V_1 = V_0(1 + g)$, we can substitute for V_1 to obtain $V_2 = V_0(1 + g)^2$. By induction, we can see that after T years, the initial value will have grown to $V_T = V_0(1 + g)^T$.

The question at hand is this: at what value of T will $V_T = 2V_0$? Substituting $2V_0$ for V_T, $2V_0 = V_0(1 + g)^T$, or upon dividing by V_0, $2 = (1 + g)^T$. Our task now is to solve this for T in terms of g. If we take the natural logarithm of both sides, we get $\ln(2) = T\ln(1 + g)$. Next we make use of an important result: for very small values of g, as we might find for reasonable rates of real GDP growth, inflation or returns on investments, $\ln(1 + g)$ is approximately equal to g. (You may recall that $\ln(1) = 0$.) Making this substitution, and noting that $\ln(2) = 0.693147\ldots \approx .70$, we have our result: $T \approx .70/g$. If growth is expressed as a percentage, simply multiply both top and bottom by 100: $T \approx 70/(g\cdot 100\%)$ or $T \approx 70/r$.

Incidentally, the rough approximation inherent in $\ln(1 + g) \approx g$ disappears when there is continuous exponential compounding of the growth, rather than annual compounding. With continuous compounding, it can be shown that after T years, an initial value of V_0 will grow to $V_T = V_0e^{gT}$ where e is the base of natural logarithms. As before, we wish to know the value of T for which $V_T = 2V_0$, or equivalently, solve for T in the equation $2 = e^{gT}$. Taking the natural logarithm of both sides, we obtain $\ln(2) = gT$, or $T = 0.693147\ldots/g \approx 70/r$ as before.