CHAPTER

4

Factoring

INTRODUCTION

Developing secret codes is big business because of the widespread use of computers and the Internet. Corporations all over the world sell encryption systems that are supposed to keep data secure and safe.

In 1977, three professors from the Massachusetts Institute of Technology developed an encryption system they called RSA, a name derived from the first letters of their last names. Their security code was based on a number that has *129 digits.* They called the code RSA-129. For the code to be broken, the 129-digit number must be factored into two prime numbers.

A data security company says that people who are using their system are safe because as yet no truly efficient algorithm for finding prime factors of massive numbers has been found, although one may someday exist. This company, hoping to test its encrypting system, now sponsors contests challenging people to factor more very large numbers into two prime numbers. RSA-576 up to RSA-2048 are being worked on now.

The U.S. government does not allow any codes to be used for which it does not have the key. The software firms claim that this prohibition is costing them about $60 billion in lost sales because many companies will not buy an encryption system knowing they can be monitored by the U.S. government.

CHAPTER 4 OUTLINE

The Streeter/Hutchison Series in Mathematics Beginning Algebra

4 pretest

Name ____________

Section ________ Date ________

Answers

1. ____________
2. ____________
3. ____________
4. ____________
5. ____________
6. ____________
7. ____________
8. ____________
9. ____________
10. ____________
11. ____________
12. ____________
13. ____________
14. ____________
15. ____________
16. ____________
17. ____________
18. ____________
19. ____________
20. ____________

This pretest provides a preview of the types of exercises you will encounter in each section of this chapter. The answers for these exercises can be found in the back of the text. If you are working on your own, or ahead of the class, this pretest can help you identify the sections in which you should focus more of your time.

Factor each of the following polynomials.

4.1

1. $15c + 35$ **2.** $8q^4 - 20q^3$

3. $6x^2 - 12x + 24$ **4.** $7c^3d^2 - 21cd + 14cd^3$

Factor each of the following trinomials.

4.2

5. $b^2 + 2b - 15$ **6.** $x^2 + 10x + 24$

7. $x^2 - 14x + 45$ **8.** $a^2 + 7ab + 12b^2$

Factor each of the following trinomials completely.

4.3

9. $3y^2 + 5y - 12$ **10.** $5w^2 + 23w + 12$

11. $6x^2 + 5xy - 21y^2$ **12.** $2x^3 - 7x^2 - 15x$

Factor each of the following polynomials completely.

4.4–4.5

13. $b^2 - 49$ **14.** $36p^2 - q^2$

15. $9x^2 - 12xy + 4y^2$ **16.** $27xy^2 - 48x^3$

Solve each of the following equations.

4.6

17. $x^2 - 11x + 28 = 0$ **18.** $x^2 - 5x = 14$

19. $5x^2 + 7x - 6 = 0$ **20.** $9p^2 - 18p = 0$

4.1 An Introduction to Factoring

< 4.1 Objectives >

1 > Factor out the greatest common factor (GCF)

2 > Factor out a binomial GCF

3 > Factor a polynomial by grouping terms

Overcoming Math Anxiety

Hint #5

Working Together

How many of your classmates do you know? Whether you are by nature gregarious or shy, you have much to gain by getting to know your classmates.

1. It is important to have someone to call when you miss class or are unclear on an assignment.
2. Working with another person is almost always beneficial to both people. If you don't understand something, it helps to have someone to ask about it. If you do understand something, nothing cements that understanding more than explaining the idea to another person.
3. Sometimes we need to commiserate. If an assignment is particularly frustrating, it is reassuring to find that it is also frustrating for other students.
4. Have you ever thought you had the right answer, but it doesn't match the answer in the text? Frequently the answers are equivalent, but that's not always easy to see. A different perspective can help you see that. Occasionally there is an error in a textbook (here we are talking about *other* textbooks). In such cases it is wonderfully reassuring to find that someone else has the same answer you do.

In Chapter 3 you were given factors and asked to find a product. We are now going to reverse the process. You will be given a polynomial and asked to find its factors. This is called **factoring.**

We start with an example from arithmetic. To *multiply* $5 \cdot 7$, you write

$$5 \cdot 7 = 35$$

To *factor* 35, you would write

$$35 = 5 \cdot 7$$

Factoring is the *reverse* of multiplication.

Now let's look at factoring in algebra. You have used the distributive property as

$$a(b + c) = ab + ac$$

NOTE

3 and $x + 5$ are the factors of $3x + 15$.

For instance,

$3(x + 5) = 3x + 15$

To use the distributive property in factoring, we reverse that property as

$ab + ac = a(b + c)$

The property lets us factor out the common factor a from the terms of $ab + ac$. To use this in factoring, the first step is to see whether each term of the polynomial has a common monomial factor. In our earlier example,

$3x + 15 = 3 \cdot x + 3 \cdot 5$

Common factor

So, by the distributive property,

$3x + 15 = 3(x + 5)$ The original terms are each divided by the greatest common factor to determine the terms in parentheses.

NOTES

Again, factoring is the reverse of multiplication.

Factoring out the GCF is the *first* method to try in any of the factoring problems we will discuss.

To check this, multiply $3(x + 5)$.

Multiplying →

$3(x + 5) = 3x + 15$

← Factoring

The first step in factoring is to identify the *greatest common factor* (GCF) of a set of terms. This factor is the product of the largest common numerical coefficient and the largest common factor of each variable.

Definition

Greatest Common Factor

The **greatest common factor (GCF)** of a polynomial is the factor that is the product of the largest common numerical coefficient factor of the polynomial and each variable with the smallest exponent in any term.

Example 1 Finding the GCF

< Objective 1 >

Find the GCF for each set of terms.

(a) 9 and 12 The largest number that is a factor of both is 3.

(b) 10, 25, 150 The GCF is 5.

(c) x^4 and x^7

$x^4 = (x) \cdot (x) \cdot (x) \cdot (x)$

$x^7 = (x) \cdot (x) \cdot (x) \cdot (x) \cdot x \cdot x \cdot x$

The largest power that divides both terms is x^4.

(d) $12a^3$ and $18a^2$

$12a^3 = 2 \cdot (2) \cdot (3) \cdot (a) \cdot (a) \cdot a$

$18a^2 = (2) \cdot (3) \cdot 3 \cdot (a) \cdot (a)$

The GCF is $6a^2$.

Check Yourself 1

Find the GCF for each set of terms.

(a) 14, 24 **(b)** 9, 27, 81
(c) a^9, a^5 **(d)** $10x^5$, $35x^4$

Step by Step

To Factor a Monomial from a Polynomial

Step 1 Find the GCF for all the terms.

Step 2 Use the GCF to factor each term and then apply the distributive property.

Step 3 Mentally check your factoring by multiplication. Checking your answer is always important and perhaps is never easier than after you have factored.

Example 2 Finding the GCF of a Binomial

(a) Factor $8x^2 + 12x$.

The largest common numerical factor of 8 and 12 is 4, and x is the common variable factor with the largest power. So $4x$ is the GCF. Write

$$8x^2 + 12x = 4x \cdot 2x + 4x \cdot 3$$

GCF

Now, by the distributive property, we have

$$8x^2 + 12x = 4x(2x + 3)$$

It is always a good idea to check your answer by multiplying to make sure that you get the original polynomial. Try it here. Multiply $4x$ by $2x + 3$.

(b) Factor $6a^4 - 18a^2$.

The GCF in this case is $6a^2$. Write

$$6a^4 + (-18a^2) = 6a^2 \cdot a^2 + 6a^2 \cdot (-3)$$

GCF

Again, using the distributive property yields

$$6a^4 - 18a^2 = 6a^2(a^2 - 3)$$

You should check this by multiplying.

NOTE

It is also true that

$6a^4 + (-18a^2)$
$= 3a(2a^3 + (-6a))$.

However, this is *not completely factored.* Do you see why? You want to find the common monomial factor with the *largest possible* coefficient and the *largest* exponent, in this case $6a^2$.

Check Yourself 2

Factor each of the following polynomials.

(a) $5x + 20$ **(b)** $6x^2 - 24x$ **(c)** $10a^3 - 15a^2$

The process is exactly the same for polynomials with more than two terms. Consider Example 3.

Example 3 Finding the GCF of a Polynomial

NOTES

The GCF is 5.

(a) Factor $5x^2 - 10x + 15$.

$$5x^2 - 10x + 15 = 5 \cdot x^2 - 5 \cdot 2x + 5 \cdot 3$$

GCF

$$= 5(x^2 - 2x + 3)$$

The GCF is $3a$.

(b) Factor $6ab + 9ab^2 - 15a^2$.

$$6ab + 9ab^2 - 15a^2 = 3a \cdot 2b + 3a \cdot 3b^2 - 3a \cdot 5a$$

GCF

$$= 3a(2b + 3b^2 - 5a)$$

The GCF is $4a^2$.

(c) Factor $4a^4 + 12a^3 - 20a^2$.

$$4a^4 + 12a^3 - 20a^2 = 4a^2 \cdot a^2 + 4a^2 \cdot 3a - 4a^2 \cdot 5$$

GCF

$$= 4a^2(a^2 + 3a - 5)$$

In each of these examples, you should check the result by multiplying the factors.

(d) Factor $6a^2b + 9ab^2 + 3ab$.

Mentally note that 3, a, and b are factors of each term, so

$$6a^2b + 9ab^2 + 3ab = 3ab(2a + 3b + 1)$$

Check Yourself 3

Factor each of the following polynomials.

(a) $8b^2 + 16b - 32$ **(b)** $4xy - 8x^2y + 12x^3$

(c) $7x^4 - 14x^3 + 21x^2$ **(d)** $5x^2y^2 - 10xy^2 + 15x^2y$

We can have two or more terms that have a binomial factor in common, as is the case in Example 4.

Example 4 Finding a Common Factor

< Objective 2 >

NOTE

Because of the commutative property, the factors can be written in either order.

(a) Factor $3x(x + y) + 2(x + y)$.

We see that *the binomial* $x + y$ is a common factor and can be removed.

$$3x(x + y) + 2(x + y)$$
$$= (x + y) \cdot 3x + (x + y) \cdot 2$$
$$= (x + y)(3x + 2)$$

(b) Factor $3x^2(x - y) + 6x(x - y) + 9(x - y)$.

We note that here the GCF is $3(x - y)$. Factoring as before, we have

$3(x - y)(x^2 + 2x + 3)$.

Check Yourself 4

Completely factor each of the polynomials.

(a) $7a(a - 2b) + 3(a - 2b)$ **(b)** $4x^2(x + y) - 8x(x + y) - 16(x + y)$

Some polynomials can be factored by grouping the terms and finding common factors within each group. Such a process is called factoring by grouping and will be explored now.

In Example 4, we looked at the expression

$$3x(x + y) + 2(x + y)$$

and found that we could factor out the common binomial, $(x + y)$, giving us

$$(x + y)(3x + 2)$$

That technique will be used in Example 5.

Example 5 Factoring by Grouping Terms

< Objective 3 >

NOTE

Note that our example has *four* terms. That is a clue for trying the factoring by grouping method.

Suppose we want to factor the polynomial

$$ax - ay + bx - by$$

As you can see, the polynomial has no common factors. However, look at what happens if we separate the polynomial into *two groups of two terms.*

$$ax - ay + bx - by$$

$$= \underbrace{ax - ay}_{(1)} + \underbrace{bx - by}_{(2)}$$

Now *each* group has a common factor, and we can write the polynomial as

$$a(x - y) + b(x - y)$$

In this form, we can see that $x - y$ is the GCF. Factoring out $x - y$, we get

$$a(x - y) + b(x - y) = (x - y)(a + b)$$

Check Yourself 5

Use the factoring by grouping method.

$x^2 - 2xy + 3x - 6y$

Be particularly careful of your treatment of algebraic signs when applying the factoring by grouping method. Consider Example 6.

Example 6 Factoring by Grouping Terms

Factor $2x^3 - 3x^2 - 6x + 9$.

We group the polynomial as follows.

$$\underbrace{2x^3 - 3x^2}_{(1)} \underbrace{- 6x + 9}_{(2)}$$

Factor out the common factor of -3 from the second two terms.

$$= x^2(2x - 3) - 3(2x - 3)$$

$$= (2x - 3)(x^2 - 3)$$

NOTE

$9 = (-3)(-3)$

Check Yourself 6

Factor by grouping.

$3y^3 + 2y^2 - 6y - 4$

It may also be necessary to change the order of the terms as they are grouped. Look at Example 7.

Example 7 Factoring by Grouping Terms

Factor $x^2 - 6yz + 2xy - 3xz$.

Grouping the terms as before, we have

$$\underbrace{x^2 - 6yz}_{(1)} + \underbrace{2xy - 3xz}_{(2)}$$

Do you see that we have accomplished nothing because there are no common factors in the first group?

We can, however, rearrange the terms to write the original polynomial as

$$\underbrace{x^2 + 2xy}_{(1)} \underbrace{- 3xz - 6yz}_{(2)}$$

$$= x(x + 2y) - 3z(x + 2y)$$

We can now factor out the common factor of $x + 2y$ in group (1) and group (2).

$$= (x + 2y)(x - 3z)$$

Note: It is often true that the grouping can be done in more than one way. The factored form will be the same.

Check Yourself 7

We can write the polynomial of Example 7 as

$x^2 - 3xz + 2xy - 6yz$

Factor and verify that the factored form is the same in either case.

Check Yourself ANSWERS

1. **(a)** 2; **(b)** 9; **(c)** a^5; **(d)** $5x^4$ **2.** **(a)** $5(x + 4)$; **(b)** $6x(x - 4)$; **(c)** $5a^2(2a - 3)$ **3.** **(a)** $8(b^2 + 2b - 4)$; **(b)** $4x(y - 2xy + 3x^2)$; **(c)** $7x^2(x^2 - 2x + 3)$; **(d)** $5xy(xy - 2y + 3x)$ **4.** **(a)** $(a - 2b)(7a + 3)$; **(b)** $4(x + y)(x^2 - 2x - 4)$ **5.** $(x - 2y)(x + 3)$ **6.** $(3y + 2)(y^2 - 2)$ **7.** $(x - 3z)(x + 2y)$

Reading Your Text

The following fill-in-the-blank exercises are designed to ensure that you understand some of the key vocabulary used in this section.

SECTION 4.1

(a) The GCF of a polynomial is the factor that is the product of the ______________ numerical coefficient factor of the polynomial and each variable with the smallest exponent in any term.

(b) To factor a monomial from a polynomial, first find the GCF for all of the ______________.

(c) Some polynomials can be factored by grouping the terms and finding common ______________ within each group.

(d) It is often true that grouping can be done in more than one way. The factored form will be the ______________.

4.1 exercises

Basic Skills | Advanced Skills | Vocational-Technical Applications | Calculator/Computer | Above and Beyond | Getting Ready

MathZone

Boost your grade at mathzone.com!
- > Practice Problems
- > NetTutor
- > Self-Tests
- > e-Professors
- > Videos

Name ____________

Section ________ Date ________

Answers

1. ______ 2. ______
3. ______ 4. ______
5. ______ 6. ______
7. ______ 8. ______
9. ______ 10. ______
11. ______ 12. ______
13. ______ 14. ______
15. ______ 16. ______
17. ______ 18. ______
19. ______ 20. ______
21. ______ 22. ______
23. ______
24. ______
25. ______
26. ______
27. ______
28. ______

< Objective 1 >

Find the greatest common factor for each of the following sets of terms.

1. 10, 12

2. 15, 35

3. 16, 32, 88

4. 55, 33, 132

5. x^2, x^5

6. y^7, y^9

7. a^3, a^6, a^9

8. b^4, b^6, b^8

9. $5x^4, 10x^5$

10. $8y^9, 24y^3$

11. $8a^4, 6a^6, 10a^{10}$

12. $9b^3, 6b^5, 12b^4$

13. $9x^2y, 12xy^2, 15x^2y^2$

14. $12a^3b^2, 18a^2b^3, 6a^4b^4$

15. $15ab^3, 10a^2bc, 25b^2c^3$

16. $9x^2, 3xy^3, 6y^3$

17. $15a^2bc^2, 9ab^2c^2, 6a^2b^2c^2$

> Videos

18. $18x^3y^2z^3, 27x^4y^2z^3, 81xy^2z$

19. $(x + y)^2, (x + y)^3$

20. $12(a + b)^4, 4(a + b)^3$

Factor each of the following polynomials.

21. $8a + 4$

22. $5x - 15$

23. $24m - 32n$

24. $7p - 21q$

25. $12m + 8$

26. $24n - 32$

27. $10s^2 + 5s$

28. $12y^2 - 6y$

29. $12x^2 + 12x$

30. $14b^2 + 14b$

31. $15a^3 - 25a^2$

32. $36b^4 + 24b^2$

33. $6pq + 18p^2q$

34. $8ab - 24ab^2$

35. $6x^2 - 18x + 30$

36. $7a^2 + 21a - 42$

37. $3a^3 + 6a^2 - 12a$

38. $5x^3 - 15x^2 + 25x$

39. $6m + 9mn - 12mn^2$

40. $4s + 6st - 14st^2$

41. $10r^3s^2 + 25r^2s^2 - 15r^2s^3$

> Videos

42. $28x^2y^3 - 35x^2y^2 + 42x^3y$

43. $9a^5 - 15a^4 + 21a^3 - 27a$

44. $8p^6 - 40p^4 + 24p^3 - 16p^2$

Basic Skills | **Advanced Skills** | Vocational-Technical Applications | Calculator/Computer | Above and Beyond | Getting Ready

Complete each of the following statements with **never, sometimes,** *or* **always.**

45. The GCF for two numbers is ______________ a prime number.

46. The GCF of a polynomial ________________ includes variables.

47. Multiplying the result of factoring will _________________ result in the original polynomial.

48. Factoring a negative number from a negative term will _______________ result in a negative term.

< Objective 2 >

Factor out the binomial in each expression.

49. $a(a + 2) - 3(a + 2)$

50. $b(b - 5) + 2(b - 5)$

51. $x(x - 2) + 3(x - 2)$

> Videos

52. $y(y + 5) - 3(y + 5)$

Answers

29. ________
30. ________
31. ________
32. ________
33. ________
34. ________
35. ________
36. ________
37. ________
38. ________
39. ________
40. ________
41. ________
42. ________
43. ________
44. ________
45. ________
46. ________
47. ________
48. ________
49. ________
50. ________
51. ________
52. ________

Answers

53.
54.
55.
56.
57.
58.
59.
60.
61.
62.
63.
64.
65.
66.
67.
68.
69.
70.
71.
72.
73.

< Objective 3 >

Factor each polynomial by grouping the first two terms and the last two terms.

53. $x^3 - 4x^2 + 3x - 12$

54. $x^3 - 6x^2 + 2x - 12$

55. $a^3 - 3a^2 + 5a - 15$

56. $6x^3 - 2x^2 + 9x - 3$

57. $10x^3 + 5x^2 - 2x - 1$

58. $x^5 + x^3 - 2x^2 - 2$

59. $x^4 - 2x^3 + 3x - 6$ > Videos

60. $x^3 - 4x^2 + 2x - 8$

Factor each polynomial completely by factoring out any common factors and then factor by grouping. Do not combine like terms.

61. $3x - 6 + xy - 2y$

62. $2x - 10 + xy - 5y$

63. $ab - ac + b^2 - bc$ > Videos

64. $ax + 2a + bx + 2b$

65. $3x^2 - 2xy + 3x - 2y$

66. $xy - 5y^2 - x + 5y$

67. $5s^2 + 15st - 2st - 6t^2$

68. $3a^3 + 3ab^2 + 2a^2b + 2b^3$

69. $3x^3 + 6x^2y - x^2y - 2xy^2$ > Videos

70. $2p^4 + 3p^3q - 2p^3q - 3p^2q^2$

Basic Skills | Advanced Skills | **Vocational-Technical Applications** | Calculator/Computer | Above and Beyond | Getting Ready

71. **Allied Health** A patient's protein secretion amount, in milligrams per day, is recorded over several days. Based on these observations, lab technicians determine that the polynomial $-t^3 - 6t^2 + 11t + 66$ provides a good approximation of the patient's protein secretion amounts t days after testing begins. Factor this polynomial.

72. **Allied Health** The concentration, in micrograms per milliliter $\frac{\mu g}{mL}$ of the antibiotic chloramphenicol is given by $8t^2 - 2t^3$, in which t is the number of hours after the drug is taken. Factor this polynomial.

73. **Manufacturing Technology** Polymer pellets need to be as perfectly round as possible. In order to avoid flat spots from forming during the hardening process, the pellets are kept off a surface by blasts of air. The height of a pellet above the surface t seconds after a blast is given by $v_0t - 4.9t^2$. Factor this expression.

74. **Information Technology** The total time to transmit a packet is given by the expression $d + 2p$, in which d is the quotient of the distance and the propagation velocity and p is the quotient of the size of the packet and the information transfer rate. How long will it take to transmit a 1,500-byte packet 10 meters on an Ethernet if the information transfer rate is 100 MB per second and the propagation velocity is 2×10^8 m/s?

Basic Skills | Advanced Skills | Vocational-Technical Applications | Calculator/Computer | **Above and Beyond** | Getting Ready

75. The GCF of $2x - 6$ is 2. The GCF of $5x + 10$ is 5. Find the GCF of the product $(2x - 6)(5x + 10)$.

76. The GCF of $3z + 12$ is 3. The GCF of $4z + 8$ is 4. Find the GCF of the product $(3z + 12)(4z + 8)$.

77. The GCF of $2x^3 - 4x$ is $2x$. The GCF of $3x + 6$ is 3. Find the GCF of the product $(2x^3 - 4x)(3x + 6)$.

78. State, in a sentence, the rule that exercises 75 to 77 illustrated.

Find the GCF of each product.

79. $(2a + 8)(3a - 6)$

80. $(5b - 10)(2b + 4)$

81. $(2x^2 + 5x)(7x - 14)$

82. $(6y^2 - 3y)(y + 7)$

83. **Geometry** The area of a rectangle with width t is given by $33t - t^2$. Factor the expression and determine the length of the rectangle in terms of t.

84. **Geometry** The area of a rectangle of length x is given by $3x^2 + 5x$. Find the width of the rectangle.

85. **Number Problem** For centuries, mathematicians have found factoring numbers into prime factors a fascinating subject. A prime number is a number that cannot be written as a product of any numbers but 1 and itself. The list of primes begins with 2 because 1 is not considered a prime number and then goes on: 3, 5, 7, 11, . . . What are the first 10 primes? What are the primes less than 100? If you list the numbers from 1 to 100 and then cross out all numbers that are multiples of 2, 3, 5, and 7, what is left? Are all the numbers not crossed out prime? Write a paragraph to explain why this might be so. You might want to investigate the Sieve of Eratosthenes, a system from 230 B.C.E. for finding prime numbers.

Answers

74. ______

75. ______

76. ______

77. ______

78. ______

79. ______

80. ______

81. ______

82. ______

83. ______

84. ______

85. ______

Answers

86. ______

87. ______

(a) ______

(b) ______

(c) ______

(d) ______

(e) ______

(f) ______

86. **Number Problem** If we could make a list of all the prime numbers, what number would be at the end of the list? Because there are an infinite number of prime numbers, there is no "largest prime number." But is there some formula that will give us all the primes? Here are some formulas proposed over the centuries:

$$n^2 + n + 17 \qquad 2n^2 + 29 \qquad n^2 - n + 11$$

In all these expressions, $n = +1, 2, 3, 4, \ldots$, that is, a positive integer beginning with 1. Investigate these expressions with a partner. Do the expressions give prime numbers when they are evaluated for these values of n? Do the expressions give *every* prime in the range of resulting numbers? Can you put in *any* positive number for n?

87. **Number Problem** How are primes used in coding messages and for security? Work together to decode the messages. The messages are coded using this code: After the numbers are factored into prime factors, the power of 2 gives the number of the letter in the alphabet. This code would be easy for a code breaker to figure out. Can you make up code that would be more difficult to break?

chapter 4 > Make the Connection

(a) 1310720, 229376, 1572864, 1760, 460, 2097152, 336

(b) 786432, 286, 4608, 278528, 1344, 98304, 1835008, 352, 4718592, 5242880

(c) Code a message using this rule. Exchange your message with a partner to decode it.

Basic Skills | Advanced Skills | Vocational-Technical Applications | Calculator/Computer | Above and Beyond | **Getting Ready**

Multiply.

(a) $(a - 1)(a + 4)$ **(b)** $(x - 1)(x + 3)$

(c) $(x - 3)(x - 3)$ **(d)** $(y - 11)(y + 3)$

(e) $(x + 5)(x + 7)$ **(f)** $(y + 1)(y - 13)$

Answers

1. 2 **3.** 8 **5.** x^2 **7.** a^3 **9.** $5x^4$ **11.** $2a^4$ **13.** $3xy$ **15.** $5b$ **17.** $3abc^2$ **19.** $(x + y)^2$ **21.** $4(2a + 1)$ **23.** $8(3m - 4n)$ **25.** $4(3m + 2)$ **27.** $5s(2s + 1)$ **29.** $12x(x + 1)$ **31.** $5a^2(3a - 5)$ **33.** $6pq(1 + 3p)$ **35.** $6(x^2 - 3x + 5)$ **37.** $3a(a^2 + 2a - 4)$ **39.** $3m(2 + 3n - 4n^2)$ **41.** $5r^2s^2(2r + 5 - 3s)$ **43.** $3a(3a^4 - 5a^3 + 7a^2 - 9)$ **45.** sometimes **47.** always **49.** $(a - 3)(a + 2)$ **51.** $(x + 3)(x - 2)$ **53.** $(x - 4)(x^2 + 3)$ **55.** $(a - 3)(a^2 + 5)$ **57.** $(2x + 1)(5x^2 - 1)$ **59.** $(x - 2)(x^3 + 3)$ **61.** $(x - 2)(3 + y)$ **63.** $(b - c)(a + b)$ **65.** $(x + 1)(3x - 2y)$ **67.** $(s + 3t)(5s - 2t)$ **69.** $x(x + 2y)(3x - y)$ **71.** $(t + 6)(-t^2 + 11)$ **73.** $t(v_0 - 4.9t)$ **75.** 10 **77.** $6x$ **79.** 6 **81.** $7x$ **83.** $t(33 - t)$; $33 - t$ **85.** Above and Beyond **87.** Above and Beyond **(a)** $a^2 + 3a - 4$ **(b)** $x^2 + 2x - 3$ **(c)** $x^2 - 6x + 9$ **(d)** $y^2 - 8y - 33$ **(e)** $x^2 + 12x + 35$ **(f)** $y^2 - 12y - 13$

4.2 Factoring Trinomials of the Form $x^2 + bx + c$

< 4.2 Objectives >

1 > Factor a trinomial of the form $x^2 + bx + c$

2 > Factor a trinomial containing a common factor

NOTE

The process used to factor here is frequently called the *trial-and-error method.* You'll see the reason for the name as you work through this section.

You learned how to find the product of any two binomials by using the FOIL method in Section 3.4. Because factoring is the reverse of multiplication, we now want to use that pattern to find the factors of certain trinomials.

Recall that when we multiply the binomials $x + 2$ and $x + 3$, our result is

$$(x + 2)(x + 3) = x^2 + 5x + 6$$

- x^2: The product of the first terms ($x \cdot x$).
- $5x$: The sum of the products of the outer and inner terms ($3x$ and $2x$).
- 6: The product of the last terms ($2 \cdot 3$).

> **CAUTION**
>
> Not every trinomial can be written as the product of two binomials.

Suppose now that you are given $x^2 + 5x + 6$ and want to find its factors. First, you know that the factors of a trinomial may be two binomials. So write

$$x^2 + 5x + 6 = (\qquad)(\qquad)$$

Because the first term of the trinomial is x^2, the first terms of the binomial factors must be x and x. We now have

$$x^2 + 5x + 6 = (x \qquad)(x \qquad)$$

The product of the last terms must be 6. Because 6 is positive, the factors must have *like* signs. Here are the possibilities:

$$\begin{aligned} 6 &= 1 \cdot 6 \\ &= 2 \cdot 3 \\ &= (-1)(-6) \\ &= (-2)(-3) \end{aligned}$$

This means that the possible factors of the trinomial are

$$(x + 1)(x + 6)$$
$$(x + 2)(x + 3)$$
$$(x - 1)(x - 6)$$
$$(x - 2)(x - 3)$$

How do we tell which is the correct pair? From the FOIL pattern we know that the sum of the outer and inner products must equal the middle term of the trinomial, in this case $5x$. This is the crucial step!

Possible Factorizations	Middle Terms
$(x + 1)(x + 6)$	$7x$
$(x + 2)(x + 3)$	$5x$
$(x - 1)(x - 6)$	$-7x$
$(x - 2)(x - 3)$	$-5x$

The correct middle term!

So we know that the correct factorization is

$$x^2 + 5x + 6 = (x + 2)(x + 3)$$

Are there any clues so far that will make this process quicker? Yes, there is an important one that you may have spotted. We started with a trinomial that had a positive middle term and a positive last term. The negative pairs of factors for 6 led to negative middle terms. So you don't need to bother with the negative factors if the middle term and the last term of the trinomial are both positive.

Example 1 Factoring a Trinomial

< Objective 1 >

(a) Factor $x^2 + 9x + 8$.

Because the middle term and the last term of the trinomial are both positive, consider only the positive factors of 8, that is, $8 = 1 \cdot 8$ or $8 = 2 \cdot 4$.

NOTE

If you are wondering why we didn't list $(x + 8)(x + 1)$ as a possibility, remember that multiplication is commutative. The order doesn't matter!

Possible Factorizations	Middle Terms
$(x + 1)(x + 8)$	$9x$
$(x + 2)(x + 4)$	$6x$

Because the first pair gives the correct middle term,

$$x^2 + 9x + 8 = (x + 1)(x + 8)$$

(b) Factor $x^2 + 12x + 20$.

NOTE

The factor-pairs of 20 are

$20 = 1 \cdot 20$
$= 2 \cdot 10$
$= 4 \cdot 5$

Possible Factorizations	Middle Terms
$(x + 1)(x + 20)$	$21x$
$(x + 2)(x + 10)$	$12x$
$(x + 4)(x + 5)$	$9x$

So

$$x^2 + 12x + 20 = (x + 2)(x + 10)$$

Check Yourself 1

Factor.

(a) $x^2 + 6x + 5$ **(b)** $x^2 + 10x + 16$

What if the middle term of the trinomial is negative but the first and last terms are still positive? Consider

Positive Positive

$$x^2 - 11x + 18$$

Negative

Because we want a negative middle term ($-11x$) and a positive last term, we use *two negative factors* for 18. Recall that the product of two negative numbers is positive.

Example 2 Factoring a Trinomial

(a) Factor $x^2 - 11x + 18$.

NOTE

The negative factors of 18 are

$18 = (-1)(-18)$

$= (-2)(-9)$

$= (-3)(-6)$

Possible Factorizations	Middle Terms
$(x - 1)(x - 18)$	$-19x$
$(x - 2)(x - 9)$	$-11x$
$(x - 3)(x - 6)$	$-9x$

So

$$x^2 - 11x + 18 = (x - 2)(x - 9)$$

(b) Factor $x^2 - 13x + 12$.

NOTE

The negative factors of 12 are

$12 = (-1)(-12)$

$= (-2)(-6)$

$= (-3)(-4)$

Possible Factorizations	Middle Terms
$(x - 1)(x - 12)$	$-13x$
$(x - 2)(x - 6)$	$-8x$
$(x - 3)(x - 4)$	$-7x$

So

$$x^2 - 13x + 12 = (x - 1)(x - 12)$$

A few more clues: We have listed all the possible factors in the above examples. It really isn't necessary. Just work until you find the right pair. Also, with practice much of this work can be done mentally.

Check Yourself 2

Factor.

(a) $x^2 - 10x + 9$ **(b)** $x^2 - 10x + 21$

Now let's look at the process of factoring a trinomial whose last term is negative. For instance, to factor $x^2 + 2x - 15$, we can start as before:

$$x^2 + 2x - 15 = (x \quad ?)(x \quad ?)$$

Note that the product of the last terms must be negative (−15 here). So we must choose factors that have different signs.

What are our choices for the factors of −15?

$$\begin{aligned} -15 &= (1)(-15) \\ &= (-1)(15) \\ &= (3)(-5) \\ &= (-3)(5) \end{aligned}$$

This means that the possible factors and the resulting middle terms are

Possible Factorizations	Middle Terms
$(x + 1)(x - 15)$	$-14x$
$(x - 1)(x + 15)$	$14x$
$(x + 3)(x - 5)$	$-2x$
$(x - 3)(x + 5)$	$2x$

NOTE

Another clue: Some students prefer to look at the list of numerical factors rather than looking at the actual algebraic factors. Here you want the pair whose sum is 2, the coefficient of the middle term of the trinomial. That pair is −3 and 5, which leads us to the correct factors.

So $x^2 + 2x - 15 = (x - 3)(x + 5)$.

Let's work through some examples in which the constant term is negative.

Example 3 Factoring a Trinomial

NOTE

You may be able to pick the factors directly from this list. You want the pair whose sum is −5 (the coefficient of the middle term).

(a) Factor $x^2 - 5x - 6$.

First, list the factors of −6. Of course, one factor will be positive, and one will be negative.

$$\begin{aligned} -6 &= (1)(-6) \\ &= (-1)(6) \\ &= (2)(-3) \\ &= (-2)(3) \end{aligned}$$

For the trinomial, then, we have

Possible Factorizations	Middle Terms
$(x + 1)(x - 6)$	$-5x$
$(x - 1)(x + 6)$	$5x$
$(x + 2)(x - 3)$	$-x$
$(x - 2)(x + 3)$	x

So $x^2 - 5x - 6 = (x + 1)(x - 6)$.

(b) Factor $x^2 + 8xy - 9y^2$.

The process is similar if two variables are involved in the trinomial. Start with

$$x^2 + 8xy - 9y^2 = (x \quad ?)(x \quad ?).$$

The product of the last terms must be $-9y^2$.

$$-9y^2 = (-y)(9y)$$
$$= (y)(-9y)$$
$$= (3y)(-3y)$$

Possible Factorizations	Middle Terms
$(x - y)(x + 9y)$	$8xy$
$(x + y)(x - 9y)$	$-8xy$
$(x + 3y)(x - 3y)$	0

So $x^2 + 8xy - 9y^2 = (x - y)(x + 9y)$.

Check Yourself 3

Factor.

(a) $x^2 + 7x - 30$ **(b)** $x^2 - 3xy - 10y^2$

As we pointed out in Section 4.1, any time that there is a common factor, that factor should be factored out *before* we try any other factoring technique. Consider Example 4.

Example 4 Factoring a Trinomial

< Objective 2 >

(a) Factor $3x^2 - 21x + 18$.

$3x^2 - 21x + 18 = 3(x^2 - 7x + 6)$ Factor out the common factor of 3.

We now factor the remaining trinomial. For $x^2 - 7x + 6$:

Possible Factorizations	Middle Terms
$(x - 1)(x - 6)$	$-7x$
$(x - 2)(x - 3)$	$-5x$

The correct middle term

> **CAUTION**
>
> A common mistake is to forget to write the 3 that was factored out as the first step.

So $3x^2 - 21x + 18 = 3(x - 1)(x - 6)$.

(b) Factor $2x^3 + 16x^2 - 40x$.

$2x^3 + 16x^2 - 40x = 2x(x^2 + 8x - 20)$ Factor out the common factor of $2x$.

To factor the remaining trinomial, which is $x^2 + 8x - 20$, we have

NOTE

Once we have found the desired middle term, there is no need to continue.

Possible Factorizations	Middle Terms
$(x + 2)(x - 10)$	$-8x$
$(x - 2)(x + 10)$	$8x$

The correct middle term

So $2x^3 + 16x^2 - 40x = 2x(x - 2)(x + 10)$.

Check Yourself 4

Factor.

(a) $3x^2 - 3x - 36$ **(b)** $4x^3 + 24x^2 + 32x$

One further comment: Have you wondered whether all trinomials are factorable? Look at the trinomial

$x^2 + 2x + 6$

The only possible factors are $(x + 1)(x + 6)$ and $(x + 2)(x + 3)$. Neither pair is correct (you should check the middle terms), and so this trinomial does not have factors with integer coefficients. Of course, there are many other trinomials that cannot be factored. Can you find one?

Check Yourself ANSWERS

1. **(a)** $(x + 1)(x + 5)$; **(b)** $(x + 2)(x + 8)$ **2.** **(a)** $(x - 9)(x - 1)$; **(b)** $(x - 3)(x - 7)$ **3.** **(a)** $(x + 10)(x - 3)$; **(b)** $(x + 2y)(x - 5y)$ **4.** **(a)** $3(x - 4)(x + 3)$; **(b)** $4x(x + 2)(x + 4)$

Reading Your Text

The following fill-in-the-blank exercises are designed to ensure that you understand some of the key vocabulary used in this section.

SECTION 4.2

(a) Factoring is the reverse of ______________.

(b) From the FOIL pattern, we know that the sum of the inner and outer products must equal the ______________ term of the trinomial.

(c) The product of two negative factors is always ______________.

(d) Some trinomials do not have ______________ with integer coefficients.

4.2 exercises

Basic Skills | Advanced Skills | Vocational-Technical Applications | Calculator/Computer | Above and Beyond | Getting Ready

MathZone

Boost your grade at mathzone.com!
- Practice Problems
- NetTutor
- Self-Tests
- e-Professors
- Videos

Name ______

Section ______ Date ______

Answers

1. ______ 2. ______
3. ______ 4. ______
5. ______ 6. ______
7. ______ 8. ______
9. ______ 10. ______
11. ______
12. ______
13. ______
14. ______
15. ______
16. ______
17. ______
18. ______
19. ______
20. ______
21. ______
22. ______
23. ______
24. ______

< Objective 1 >

Complete each of the following statements.

1. $x^2 - 8x + 15 = (x - 3)(\quad)$
2. $y^2 - 3y - 18 = (y - 6)(\quad)$
3. $m^2 + 8m + 12 = (m + 2)(\quad)$
4. $x^2 - 10x + 24 = (x - 6)(\quad)$
5. $p^2 - 8p - 20 = (p + 2)(\quad)$
6. $a^2 + 9a - 36 = (a + 12)(\quad)$
7. $x^2 - 16x + 64 = (x - 8)(\quad)$
8. $w^2 - 12w - 45 = (w + 3)(\quad)$
9. $x^2 - 7xy + 10y^2 = (x - 2y)(\quad)$ > Videos
10. $a^2 + 18ab + 81b^2 = (a + 9b)(\quad)$

Factor each of the following trinomials completely.

11. $x^2 + 8x + 15$
12. $x^2 - 11x + 24$
13. $x^2 - 11x + 28$
14. $y^2 - y - 20$
15. $s^2 + 13s + 30$
16. $b^2 + 14b + 33$
17. $a^2 - 2a - 48$
18. $x^2 - 17x + 60$
19. $x^2 - 8x + 7$
20. $x^2 + 7x - 18$
21. $m^2 + 3m - 28$ > Videos
22. $a^2 + 10a + 25$
23. $x^2 - 6x - 40$
24. $x^2 - 11x + 10$

25. $x^2 - 14x + 49$

26. $s^2 - 4s - 32$

27. $p^2 - 10p - 24$

28. $x^2 - 11x - 60$

29. $x^2 + 5x - 66$

30. $a^2 + 2a - 80$

31. $c^2 + 19c + 60$

32. $t^2 - 4t - 60$

33. $x^2 + 7xy + 10y^2$

34. $x^2 - 8xy + 12y^2$

35. $a^2 - ab - 42b^2$ > Videos

36. $m^2 - 8mn + 16n^2$

37. $x^2 - 13xy + 40y^2$

38. $r^2 - 9rs - 36s^2$

39. $x^2 - 2xy - 8y^2$

40. $u^2 + 6uv - 55v^2$

41. $25m^2 + 10mn + n^2$

42. $64m^2 - 16mn + n^2$

Basic Skills | **Advanced Skills** | Vocational-Technical Applications | Calculator/Computer | Above and Beyond | Getting Ready

Determine whether each of the following statements is **true** *or* **false.**

43. Factoring is the reverse of division.

44. From the FOIL pattern, we know that the sum of the inner and outer products must equal the middle term of the trinomial.

45. The sum of two negative factors is always negative.

46. Every trinomial has integer coefficients.

< Objective 2 >

Factor each of the following trinomials completely.

47. $3a^2 - 3a - 126$

48. $2c^2 + 2c - 60$

49. $r^3 + 7r^2 - 18r$

50. $m^3 + 5m^2 - 14m$

Answers

25. ____
26. ____
27. ____
28. ____
29. ____
30. ____
31. ____
32. ____
33. ____
34. ____
35. ____
36. ____
37. ____
38. ____
39. ____
40. ____
41. ____
42. ____
43. ____ 44. ____
45. ____ 46. ____
47. ____
48. ____
49. ____
50. ____

Answers

51. ____
52. ____
53. ____
54. ____
55. ____
56. ____
57. ____
58. ____
59. ____
60. ____
61. ____
62. ____
63. ____
64. ____
65. ____
66. ____
(a) ____
(b) ____
(c) ____
(d) ____
(e) ____
(f) ____
(g) ____
(h) ____

51. $2x^3 - 20x^2 - 48x$

52. $3p^3 + 48p^2 - 108p$

53. $x^2y - 9xy^2 - 36y^3$ > Videos

54. $4s^4 - 20s^3t - 96s^2t^2$

55. $m^3 - 29m^2n + 120mn^2$

56. $2a^3 - 52a^2b + 96ab^2$

Basic Skills | Advanced Skills | **Vocational-Technical Applications** | Calculator/Computer | Above and Beyond | Getting Ready

57. **Manufacturing Technology** The shape of a beam loaded with a single concentrated load is described by the expression $\frac{x^2 - 64}{200}$. Factor the numerator, $(x^2 - 64)$.

58. **Allied Health** The concentration, in micrograms per milliliter (mcg/mL), of Vancocin, an antibiotic used to treat peritonitis, is given by the negative of the polynomial $t^2 - 8t - 20$, where t is the number of hours since the drug was administered via intravenous injection. Write this given polynomial in factored form.

Basic Skills | Advanced Skills | Vocational-Technical Applications | Calculator/Computer | **Above and Beyond** | Getting Ready

Find all positive values for k for which each of the following can be factored.

59. $x^2 - kx + 16$ > Videos

60. $x^2 - kx + 17$

61. $x^2 - kx - 5$

62. $x^2 - kx - 7$

63. $x^2 + 3x + k$

64. $x^2 + 5x + k$

65. $x^2 + 2x - k$

66. $x^2 + x - k$

Basic Skills | Advanced Skills | Vocational-Technical Applications | Calculator/Computer | Above and Beyond | **Getting Ready**

Multiply.

(a) $(2x - 1)(2x + 3)$
(b) $(3a - 1)(a + 4)$
(c) $(x - 4)(2x - 3)$
(d) $(2w - 11)(w + 2)$
(e) $(y + 5)(2y + 9)$
(f) $(2x + 1)(x - 12)$
(g) $(p + 9)(2p + 5)$
(h) $(3a - 5)(2a + 4)$

Answers

1. $x - 5$ **3.** $m + 6$ **5.** $p - 10$ **7.** $x - 8$ **9.** $x - 5y$
11. $(x + 3)(x + 5)$ **13.** $(x - 4)(x - 7)$ **15.** $(s + 3)(s + 10)$
17. $(a - 8)(a + 6)$ **19.** $(x - 1)(x - 7)$ **21.** $(m + 7)(m - 4)$
23. $(x + 4)(x - 10)$ **25.** $(x - 7)(x - 7)$ **27.** $(p - 12)(p + 2)$
29. $(x + 11)(x - 6)$ **31.** $(c + 4)(c + 15)$ **33.** $(x + 2y)(x + 5y)$
35. $(a + 6b)(a - 7b)$ **37.** $(x - 5y)(x - 8y)$ **39.** $(x + 2y)(x - 4y)$
41. $(5m + n)(5m + n)$ **43.** False **45.** True **47.** $3(a + 6)(a - 7)$
49. $r(r - 2)(r + 9)$ **51.** $2x(x - 12)(x + 2)$ **53.** $y(x + 3y)(x - 12y)$
55. $m(m - 5n)(m - 24n)$ **57.** $(x + 8)(x - 8)$ **59.** 8, 10, or 17
61. 4 **63.** 2 **65.** 3, 8, 15, 24, . . . **(a)** $4x^2 + 4x - 3$
(b) $3a^2 + 11a - 4$ **(c)** $2x^2 - 11x + 12$ **(d)** $2w^2 - 7w - 22$
(e) $2y^2 + 19y + 45$ **(f)** $2x^2 - 23x - 12$ **(g)** $2p^2 + 23p + 45$
(h) $6a^2 + 2a - 20$

4.3 Factoring Trinomials of the Form $ax^2 + bx + c$

< 4.3 Objectives >

1 > Factor a trinomial of the form $ax^2 + bx + c$

2 > Completely factor a trinomial

3 > Use the *ac* test to determine factorability

4 > Use the results of the *ac* test to factor a trinomial

Factoring trinomials is more time-consuming when the coefficient of the first term is not 1. Look at the following multiplication.

$$(5x + 2)(2x + 3) = 10x^2 + 19x + 6$$

Factors of $10x^2$ — Factors of 6

Do you see the additional problem? We must consider all possible factors of the first coefficient (10 in our example) as well as those of the third term (6 in our example).

There is no easy way out! You need to form all possible combinations of factors and then check the middle term until the proper pair is found. If this seems a bit like guesswork, it is. In fact, some call this process factoring by *trial and error.*

We can simplify the work a bit by reviewing the sign patterns found in Section 4.2.

Property

Sign Patterns for Factoring Trinomials

1. If all terms of a trinomial are positive, the signs between the terms in the binomial factors are both plus signs.
2. If the third term of the trinomial is positive and the middle term is negative, the signs between the terms in the binomial factors are both minus signs.
3. If the third term of the trinomial is negative, the signs between the terms in the binomial factors are opposite (one is + and one is −).

Example 1 — **Factoring a Trinomial**

< Objective 1 >

Factor $3x^2 + 14x + 15$.

First, list the possible factors of 3, the coefficient of the first term.

$$3 = 1 \cdot 3$$

Now list the factors of 15, the last term.

$$15 = 1 \cdot 15$$
$$= 3 \cdot 5$$

Because the signs of the trinomial are all positive, we know any factors will have the form

$(\_x + \_)(\_x + \_)$

The product of the numbers in the last blanks must be 15.

The product of the numbers in the first blanks must be 3.

So the following are the possible factors and the corresponding middle terms:

Possible Factorizations	Middle Terms
$(x + 1)(3x + 15)$	$18x$
$(x + 15)(3x + 1)$	$46x$
$(3x + 3)(x + 5)$	$18x$
$(3x + 5)(x + 3)$	$14x$

The correct middle term

NOTE

Take the time to multiply the binomial factors. This habit will ensure that you have an expression equivalent to the original problem.

So

$3x^2 + 14x + 15 = (3x + 5)(x + 3)$

Check Yourself 1

Factor.

(a) $5x^2 + 14x + 8$ **(b)** $3x^2 + 20x + 12$

Example 2 Factoring a Trinomial

Factor $4x^2 - 11x + 6$.

Because only the middle term is negative, we know the factors have the form

$(\_x - \_)(\_x - \_)$

Both signs are negative.

Now look at the factors of the first coefficient and the last term.

$4 = 1 \cdot 4$ $\quad 6 = 1 \cdot 6$

$\;\;= 2 \cdot 2$ $\quad \;\;= 2 \cdot 3$

This gives us the possible factors:

Possible Factorizations	Middle Terms
$(x - 1)(4x - 6)$	$-10x$
$(x - 6)(4x - 1)$	$-25x$
$(x - 2)(4x - 3)$	$-11x$

The correct middle term

RECALL

Again, at least mentally, check your work by multiplying the factors.

Note that, in this example, we *stopped* as soon as the correct pair of factors was found. So

$$4x^2 - 11x + 6 = (x - 2)(4x - 3)$$

Check Yourself 2

Factor.

(a) $2x^2 - 9x + 9$ (b) $6x^2 - 17x + 10$

Next, we will factor a trinomial whose last term is negative.

Example 3 Factoring a Trinomial

Factor $5x^2 + 6x - 8$.

Because the last term is negative, the factors have the form

$$(\_x + \_)(\_x - \_)$$

Consider the factors of the first coefficient and the last term.

$$5 = 1 \cdot 5 \qquad 8 = 1 \cdot 8$$
$$= 2 \cdot 4$$

The possible factors are then

Possible Factorizations	Middle Terms
$(x + 1)(5x - 8)$	$-3x$
$(x + 8)(5x - 1)$	$39x$
$(5x + 1)(x - 8)$	$-39x$
$(5x + 8)(x - 1)$	$3x$
$(x + 2)(5x - 4)$	$6x$

Again we stop as soon as the correct pair of factors is found.

$$5x^2 + 6x - 8 = (x + 2)(5x - 4)$$

Check Yourself 3

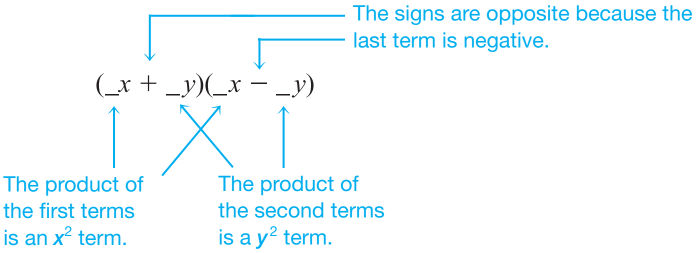
Factor $4x^2 + 5x - 6$.

The same process is used to factor a trinomial with more than one variable.

Example 4 Factoring a Trinomial

Factor $6x^2 + 7xy - 10y^2$.

The form of the factors must be



Again look at the factors of the first and last coefficients.

$6 = 1 \cdot 6 \qquad 10 = 1 \cdot 10$
$\;\; = 2 \cdot 3 \qquad\;\;\; = 2 \cdot 5$

NOTE

Be certain that you have a pattern that matches up every possible pair of coefficients.

Possible Factorizations	Middle Terms
$(x + y)(6x - 10y)$	$-4xy$
$(x + 10y)(6x - y)$	$59xy$
$(6x + y)(x - 10y)$	$-59xy$
$(6x + 10y)(x - y)$	$4xy$
$(x + 2y)(6x - 5y)$	$7xy$

Once more, we stop as soon as the correct factors are found.

$6x^2 + 7xy - 10y^2 = (x + 2y)(6x - 5y)$

Check Yourself 4

Factor $15x^2 - 4xy - 4y^2$.

Example 5 illustrates a special kind of trinomial called a *perfect square trinomial.*

Example 5 Factoring a Trinomial

Factor $9x^2 + 12xy + 4y^2$.

Because all terms are positive, the form of the factors must be

$(\_x + \_y)(\_x + \_y)$

Consider the factors of the first and last coefficients.

$9 = 9 \cdot 1 \qquad 4 = 4 \cdot 1$
$\;\; = 3 \cdot 3 \qquad\;\; = 2 \cdot 2$

Possible Factorizations	Middle Terms
$(x + y)(9x + 4y)$	$13xy$
$(x + 4y)(9x + y)$	$37xy$
$(3x + 2y)(3x + 2y)$	$12xy$

NOTE

Perfect square trinomials can be factored by using previous methods. Recognizing the special pattern simply saves time.

So

$$9x^2 + 12xy + 4y^2 = (3x + 2y)(3x + 2y)$$
$$= (3x + 2y)^2$$

Square of $3x$ — $2(3x)(2y)$ — Square of $2y$

This trinomial is the result of squaring a binomial, thus the special name of perfect square trinomial.

Check Yourself 5

Factor.

(a) $4x^2 + 28x + 49$ (b) $16x^2 - 40xy + 25y^2$

Before looking at Example 6, review one important point from Section 4.2. Recall that when you factor trinomials, you should not forget to look for a common factor as the first step. If there is a common factor, factor it out and then factor the remaining trinomial as before.

Example 6 Factoring a Trinomial

< Objective 2 >

Factor $18x^2 - 18x + 4$.

First look for a common factor in all three terms. Here that factor is 2, so write

$$18x^2 - 18x + 4 = 2(9x^2 - 9x + 2)$$

By our earlier methods, we can factor the remaining trinomial as

$$9x^2 - 9x + 2 = (3x - 1)(3x - 2)$$

So

$$18x^2 - 18x + 4 = 2(3x - 1)(3x - 2)$$

Don't forget the 2 that was factored out!

NOTE

If you don't see why this is true, use your pencil to work it out before you move on!

Check Yourself 6

Factor $16x^2 + 44x - 12$.

Now look at an example in which the common factor includes a variable.

Example 7 Factoring a Trinomial

Factor

$$6x^3 + 10x^2 - 4x$$

The common factor is $2x$.

So

$$6x^3 + 10x^2 - 4x = 2x(3x^2 + 5x - 2)$$

Because

$$3x^2 + 5x - 2 = (3x - 1)(x + 2)$$

we have

$$6x^3 + 10x^2 - 4x = 2x(3x - 1)(x + 2)$$

RECALL

Be certain to include the monomial factor.

Check Yourself 7

Factor $6x^3 - 27x^2 + 30x$.

You have now had a chance to work with a variety of factoring techniques. Your success in factoring polynomials depends on your ability to recognize when to use which technique. Here are some guidelines to help you apply the factoring methods you have studied in this chapter.

Step by Step

Factoring Polynomials

Step 1 Look for a greatest common factor other than 1. If such a factor exists, factor out the GCF.

Step 2 If the polynomial that remains is a *trinomial,* try to factor the trinomial by the trial-and-error methods of Sections 4.2 and 4.3.

Example 8 illustrates the use of this strategy.

Example 8 Factoring a Trinomial

NOTE

$m^2 + 4$ cannot be factored any further.

(a) Factor $5m^2n + 20n$.

First, we see that the GCF is $5n$. Factoring it out gives

$$5m^2n + 20n = 5n(m^2 + 4)$$

(b) Factor $3x^3 - 24x^2 + 48x$.

First, we see that the GCF is $3x$. Factoring out $3x$ yields

$$3x^3 - 24x^2 + 48x = 3x(x^2 - 8x + 16)$$
$$= 3x(x - 4)(x - 4) \quad \text{or} \quad 3x(x - 4)^2$$

(c) Factor $8r^2s + 20rs^2 - 12s^3$.

First, the GCF is $4s$, and we can write the original polynomial as

$$8r^2s + 20rs^2 - 12s^3 = 4s(2r^2 + 5rs - 3s^2)$$

Because the remaining polynomial is a trinomial, we can use the trial-and-error method to complete the factoring.

$$8r^2s + 20rs^2 - 12s^3 = 4s(2r - s)(r + 3s)$$

Check Yourself 8

Factor the following polynomials.

(a) $8a^3 + 32a^2b + 32ab^2$ **(b)** $7x^3 + 7x^2y - 42xy^2$
(c) $5m^4 + 15m^3 + 5m^2$

To this point we have used the trial-and-error method to factor trinomials. We have also learned that not all trinomials can be factored. In the remainder of this section we will look at the same kinds of trinomials, but in a slightly different context. We first determine whether a trinomial is factorable. We then use the results of that analysis to factor the trinomial.

Some students prefer the trial-and-error method for factoring because it is generally faster and more intuitive. Other students prefer the method used in the remainder of this section (called the *ac* method) because it yields the answer in a systematic way. We let you determine which method you prefer.

We begin by looking at some factored trinomials.

Example 9 Matching Trinomials and Their Factors

Determine which of the following are true statements.

(a) $x^2 - 2x - 8 = (x - 4)(x + 2)$

This is a true statement. Using the FOIL method, we see that

$$(x - 4)(x + 2) = x^2 + 2x - 4x - 8$$
$$= x^2 - 2x - 8$$

(b) $x^2 + 6x + 5 = (x + 2)(x + 3)$

This is not a true statement.

$$(x + 2)(x + 3) = x^2 + 3x + 2x + 6 = x^2 + 5x + 6$$

(c) $x^2 + 5x - 14 = (x - 2)(x + 7)$

This is true: $(x - 2)(x + 7) = x^2 + 7x - 2x - 14 = x^2 + 5x - 14$

(d) $x^2 - 8x - 15 = (x - 5)(x - 3)$

This is false: $(x - 5)(x - 3) = x^2 - 3x - 5x + 15 = x^2 - 8x + 15$

Check Yourself 9

Determine which of the following are true statements.

(a) $2x^2 - 2x - 3 = (2x - 3)(x + 1)$
(b) $3x^2 + 11x - 4 = (3x - 1)(x + 4)$
(c) $2x^2 - 7x + 3 = (x - 3)(2x - 1)$

The first step in learning to factor a trinomial is to identify its coefficients. So that we are consistent, we first write the trinomial in standard form, $ax^2 + bx + c$, and then label the three coefficients as a, b, and c.

Example 10 Identifying the Coefficients of $ax^2 + bx + c$

First, when necessary, rewrite the trinomial in $ax^2 + bx + c$ form. Then give the values for a, b, and c, in which a is the coefficient of the x^2 term, b is the coefficient of the x term, and c is the constant.

NOTE

The negative sign is attached to the coefficient.

(a) $x^2 - 3x - 18$

$a = 1 \qquad b = -3 \qquad c = -18$

(b) $x^2 - 24x + 23$

$a = 1 \qquad b = -24 \qquad c = 23$

(c) $x^2 + 8 - 11x$

First rewrite the trinomial in descending-exponent order:

$x^2 - 11x + 8$

$a = 1 \qquad b = -11 \qquad c = 8$

Check Yourself 10

First, when necessary, rewrite the trinomials in $ax^2 + bx + c$ form. Then label a, b, and c, in which a is the coefficient of the x^2 term, b is the coefficient of the x term, and c is the constant.

(a) $x^2 + 5x - 14$ **(b)** $x^2 - 18x + 17$ **(c)** $x - 6 + 2x^2$

Not all trinomials can be factored. To discover whether a trinomial is factorable, we try the ***ac* test.**

Definition

The *ac* Test

A trinomial of the form $ax^2 + bx + c$ is factorable if (and only if) there are two integers, m and n, such that

$ac = mn \qquad \text{and} \qquad b = m + n$

In Example 11 we will look for m and n to determine whether each trinomial is factorable.

Example 11 Using the *ac* Test

< Objective 3 >

Use the ac test to determine which of the following trinomials can be factored. Find the values of m and n for each trinomial that can be factored.

(a) $x^2 - 3x - 18$

First, we find the values of a, b, and c, so that we can find ac.

$a = 1 \qquad b = -3 \qquad c = -18$

$ac = 1(-18) = -18 \qquad \text{and} \qquad b = -3$

Then, we look for two numbers, m and n, such that their product is ac and their sum is b. In this case, that means

$mn = -18$ and $m + n = -3$

We now look at all pairs of integers with a product of -18. We then look at the sum of each pair of integers, looking for a sum of -3.

mn	$m + n$
$1(-18) = -18$	$1 + (-18) = -17$
$2(-9) = -18$	$2 + (-9) = -7$
$3(-6) = -18$	$3 + (-6) = -3$
$6(-3) = -18$	
$9(-2) = -18$	
$18(-1) = -18$	

We need to look no further than 3 and -6.

NOTE

We could have chosen $m = -6$ and $n = 3$ as well.

3 and -6 are the two integers with a product of ac and a sum of b. We can say that

$m = 3$ and $n = -6$

Because we found values for m and n, we know that $x^2 - 3x - 18$ is factorable.

(b) $x^2 - 24x + 23$

We find that

$a = 1 \quad b = -24 \quad c = 23$

$ac = 1(23) = 23$ and $b = -24$

So

$mn = 23$ and $m + n = -24$

We now calculate integer pairs, looking for two numbers with a product of 23 and a sum of -24.

mn	$m + n$
$1(23) = 23$	$1 + 23 = 24$
$-1(-23) = 23$	$-1 + (-23) = -24$

$m = -1$ and $n = -23$

So, $x^2 - 24x + 23$ is factorable.

(c) $x^2 - 11x + 8$

We find that $a = 1$, $b = -11$, and $c = 8$. Therefore, $ac = 8$ and $b = -11$. Thus $mn = 8$ and $m + n = -11$. We calculate integer pairs:

mn	$m + n$
$1(8) = 8$	$1 + 8 = 9$
$2(4) = 8$	$2 + 4 = 6$
$-1(-8) = 8$	$-1 + (-8) = -9$
$-2(-4) = 8$	$-2 + (-4) = -6$

There are no other pairs of integers with a product of 8, and none of these pairs has a sum of -11. The trinomial $x^2 - 11x + 8$ is not factorable.

(d) $2x^2 + 7x - 15$

We find that $a = 2$, $b = 7$, and $c = -15$. Therefore, $ac = 2(-15) = -30$ and $b = 7$. Thus $mn = -30$ and $m + n = 7$. We calculate integer pairs:

mn	$m + n$
$1(-30) = -30$	$1 + (-30) = -29$
$2(-15) = -30$	$2 + (-15) = -13$
$3(-10) = -30$	$3 + (-10) = -7$
$5(-6) = -30$	$5 + (-6) = -1$
$6(-5) = -30$	$6 + (-5) = 1$
$10(-3) = -30$	$10 + (-3) = 7$

There is no need to go any further. We see that 10 and -3 have a product of -30 and a sum of 7, so

$m = 10$ and $n = -3$

Therefore, $2x^2 + 7x - 15$ is factorable.

Check Yourself 11

Use the *ac* test to determine which of the following trinomials can be factored. Find the values of *m* and *n* for each trinomial that can be factored.

(a) $x^2 - 7x + 12$ **(b)** $x^2 + 5x - 14$
(c) $3x^2 - 6x + 7$ **(d)** $2x^2 + x - 6$

It is not always necessary to evaluate all the products and sums to determine whether a trinomial is factorable. You may have noticed patterns and shortcuts that make it easier to find *m* and *n*. By all means, use them to help you find *m* and *n*. This is essential in mathematical thinking. You are taught a mathematical process that will always work for solving a problem. Such a process is called an **algorithm.** It is very easy to teach a computer to use an algorithm. It is very difficult (some would say impossible) for a computer to have insight. Shortcuts that you discover are *insights.* They may be the most important part of your mathematical education.

So far we have used the results of the *ac* test to determine whether a trinomial is factorable. The results can also be used to help factor the trinomial.

Example 12 Using the Results of the *ac* Test to Factor

< Objective 4 >

Rewrite the middle term as the sum of two terms and then factor by grouping.

(a) $x^2 - 3x - 18$

We find that $a = 1$, $b = -3$, and $c = -18$, so $ac = -18$ and $b = -3$. We are looking for two numbers, *m* and *n*, where $mn = -18$ and $m + n = -3$. In Example 11, part **(a),** we looked at every pair of integers whose product (mn) was -18, to find a pair that had a sum ($m + n$) of -3. We found the two integers to be 3 and -6, because $3(-6) = -18$ and $3 + (-6) = -3$, so $m = 3$ and $n = -6$. We now use that result to rewrite the middle term as the sum of $3x$ and $-6x$.

$$x^2 + 3x - 6x - 18$$

We then factor by grouping:

$$\begin{aligned} x^2 + 3x - 6x - 18 &= (x^2 + 3x) - (6x + 18) \\ &= x(x + 3) - 6(x + 3) \\ &= (x + 3)(x - 6) \end{aligned}$$

(b) $x^2 - 24x + 23$

We use the results from Example 11, part **(b),** in which we found $m = -1$ and $n = -23$, to rewrite the middle term of the equation.

$$x^2 - 24x + 23 = x^2 - x - 23x + 23$$

Then we factor by grouping:

$$\begin{aligned} x^2 - x - 23x + 23 &= (x^2 - x) - (23x - 23) \\ &= x(x - 1) - 23(x - 1) \\ &= (x - 1)(x - 23) \end{aligned}$$

(c) $2x^2 + 7x - 15$

From Example 11, part **(d),** we know that this trinomial is factorable, and $m = 10$ and $n = -3$. We use that result to rewrite the middle term of the trinomial.

$$\begin{aligned} 2x^2 + 7x - 15 &= 2x^2 + 10x - 3x - 15 \\ &= (2x^2 + 10x) - (3x + 15) \\ &= 2x(x + 5) - 3(x + 5) \\ &= (x + 5)(2x - 3) \end{aligned}$$

Note that we did not factor the trinomial in Example 11, part **(c),** $x^2 - 11x + 8$. Recall that, by the *ac* method, we determined that this trinomial is not factorable.

Check Yourself 12

Use the results of Check Yourself 11 to rewrite the middle term as the sum of two terms and then factor by grouping.

(a) $x^2 - 7x + 12$ **(b)** $x^2 + 5x - 14$ **(c)** $2x^2 + x - 6$

Next, we look at some examples that require us to first find m and n and then factor the trinomial.

Example 13 Rewriting Middle Terms to Factor

Rewrite the middle term as the sum of two terms and then factor by grouping.

(a) $2x^2 - 13x - 7$

We find $a = 2$, $b = -13$, and $c = -7$, so $mn = ac = -14$ and $m + n = b = -13$. Therefore,

mn	$m + n$
$1(-14) = -14$	$1 + (-14) = -13$

So, $m = 1$ and $n = -14$. We rewrite the middle term of the trinomial as follows:

$$\begin{aligned} 2x^2 - 13x - 7 &= 2x^2 + x - 14x - 7 \\ &= (2x^2 + x) - (14x + 7) \\ &= x(2x + 1) - 7(2x + 1) \\ &= (2x + 1)(x - 7) \end{aligned}$$

(b) $6x^2 - 5x - 6$

We find that $a = 6$, $b = -5$, and $c = -6$, so $mn = ac = -36$ and $m + n = b = -5$.

mn	$m + n$
$1(-36) = -36$	$1 + (-36) = -35$
$2(-18) = -36$	$2 + (-18) = -16$
$3(-12) = -36$	$3 + (-12) = -9$
$4(-9) = -36$	$4 + (-9) = -5$

So, $m = 4$ and $n = -9$. We rewrite the middle term of the trinomial:

$$\begin{aligned} 6x^2 - 5x - 6 &= 6x^2 + 4x - 9x - 6 \\ &= (6x^2 + 4x) - (9x + 6) \\ &= 2x(3x + 2) - 3(3x + 2) \\ &= (3x + 2)(2x - 3) \end{aligned}$$

Check Yourself 13

Rewrite the middle term as the sum of two terms and then factor by grouping.

(a) $2x^2 - 7x - 15$ **(b)** $6x^2 - 5x - 4$

Be certain to check trinomials and binomial factors for any common monomial factor. (There is no common factor in the binomial unless it is also a common factor in the original trinomial.) Example 14 shows the factoring out of monomial factors.

Example 14 Factoring Out Common Factors

Completely factor the trinomial.

$3x^2 + 12x - 15$

We could first factor out the common factor of 3:

$3x^2 + 12x - 15 = 3(x^2 + 4x - 5)$

Finding m and n for the trinomial $x^2 + 4x - 5$ yields $mn = -5$ and $m + n = 4$.

mn	$m + n$
$1(-5) = -5$	$1 + (-5) = -4$
$5(-1) = -5$	$-1 + (5) = 4$

So, $m = 5$ and $n = -1$. This gives us

$$\begin{aligned} 3x^2 + 12x - 15 &= 3(x^2 + 4x - 5) \\ &= 3(x^2 + 5x - x - 5) \\ &= 3[(x^2 + 5x) - (x + 5)] \\ &= 3[x(x + 5) - (x + 5)] \\ &= 3[(x + 5)(x - 1)] \\ &= 3(x + 5)(x - 1) \end{aligned}$$

Check Yourself 14

Completely factor the trinomial.

$6x^3 + 3x^2 - 18x$

Not all possible product pairs need to be tried to find m and n. A look at the sign pattern of the trinomial will eliminate many of the possibilities. Assuming the leading coefficient is positive, there are four possible sign patterns.

Pattern	Example	Conclusion
1. b and c are both positive.	$2x^2 + 13x + 15$	m and n must both be positive.
2. b is negative and c is positive.	$x^2 - 7x + 12$	m and n must both be negative.
3. b is positive and c is negative.	$x^2 + 3x - 10$	m and n are of opposite signs. (The value with the larger absolute value is positive.)
4. b and c are both negative.	$x^2 - 3x - 10$	m and n are of opposite signs. (The value with the larger absolute value is negative.)

Check Yourself ANSWERS

1. **(a)** $(5x + 4)(x + 2)$; **(b)** $(3x + 2)(x + 6)$ **2.** **(a)** $(2x - 3)(x - 3)$; **(b)** $(6x - 5)(x - 2)$ **3.** $(4x - 3)(x + 2)$ **4.** $(3x - 2y)(5x + 2y)$ **5.** **(a)** $(2x + 7)^2$; **(b)** $(4x - 5y)^2$ **6.** $4(4x - 1)(x + 3)$ **7.** $3x(2x - 5)(x - 2)$ **8.** **(a)** $8a(a + 2b)(a + 2b)$; **(b)** $7x(x + 3y)(x - 2y)$; **(c)** $5m^2(m^2 + 3m + 1)$ **9.** **(a)** False; **(b)** true; **(c)** true **10.** **(a)** $a = 1, b = 5, c = -14$; **(b)** $a = 1, b = -18, c = 17$; **(c)** $a = 2, b = 1, c = -6$ **11.** **(a)** Factorable, $m = -3, n = -4$; **(b)** factorable, $m = 7, n = -2$; **(c)** not factorable; **(d)** factorable, $m = 4, n = -3$ **12.** **(a)** $x^2 - 3x - 4x + 12 = (x - 3)(x - 4)$; **(b)** $x^2 + 7x - 2x - 14 = (x + 7)(x - 2)$; **(c)** $2x^2 + 4x - 3x - 6 = (x + 2)(2x - 3)$ **13.** **(a)** $2x^2 - 10x + 3x - 15 = (x - 5)(2x + 3)$; **(b)** $6x^2 - 8x + 3x - 4 = (3x - 4)(2x + 1)$ **14.** $3x(2x - 3)(x + 2)$

Reading Your Text

The following fill-in-the-blank exercises are designed to ensure that you understand some of the key vocabulary used in this section.

SECTION 4.3

(a) If all the terms of a trinomial are positive, the signs between the terms in the binomial factors are both ______________ signs.

(b) If the third term of a trinomial is negative, the signs between the terms in the binomial factors are ______________.

(c) The first step in factoring a polynomial is to factor out the __________.

(d) A mathematical process that will always work for solving a problem is called an ______________.

4.3 exercises

Basic Skills | Advanced Skills | Vocational-Technical Applications | Calculator/Computer | Above and Beyond | Getting Ready

MathZone

Boost your grade at mathzone.com!
> Practice Problems
> NetTutor
> Self-Tests
> e-Professors
> Videos

Name ____________

Section ________ Date ________

Answers

1. ____________
2. ____________
3. ____________
4. ____________
5. ____________
6. ____________
7. ____________
8. ____________
9. ____________
10. ____________
11. ____________
12. ____________
13. ____________
14. ____________
15. ____________
16. ____________
17. ____________

< Objective 1 >

Complete each of the following statements.

1. $4x^2 - 4x - 3 = (2x + 1)(\quad)$
2. $3w^2 + 11w - 4 = (w + 4)(\quad)$
3. $6a^2 + 13a + 6 = (2a + 3)(\quad)$
4. $25y^2 - 10y + 1 = (5y - 1)(\quad)$
5. $15x^2 - 16x + 4 = (3x - 2)(\quad)$
6. $6m^2 + 5m - 4 = (3m + 4)(\quad)$
7. $16a^2 + 8ab + b^2 = (4a + b)(\quad)$
8. $6x^2 + 5xy - 4y^2 = (3x + 4y)(\quad)$ > Videos
9. $4m^2 + 5mn - 6n^2 = (m + 2n)(\quad)$
10. $10p^2 - pq - 3q^2 = (5p - 3q)(\quad)$

State whether each of the following is **true** *or* **false.**

11. $x^2 + 2x - 3 = (x + 3)(x - 1)$
12. $y^2 - 3y - 18 = (y - 6)(y + 3)$
13. $x^2 - 10x - 24 = (x - 6)(x + 4)$
14. $a^2 + 9a - 36 = (a - 12)(a + 4)$
15. $x^2 - 16x + 64 = (x - 8)(x - 8)$
16. $w^2 - 12w - 45 = (w - 9)(w - 5)$
17. $25y^2 - 10y + 1 = (5y - 1)(5y + 1)$ > Videos

18. $6x^2 + 5xy - 4y^2 = (6x - 2y)(x + 2y)$

19. $10p^2 - pq - 3q^2 = (5p - 3q)(2p + q)$

20. $6a^2 + 13a + 6 = (2a + 3)(3a + 2)$

For each of the following trinomials, label a, b, and c.

21. $x^2 + 4x - 9$

22. $x^2 + 5x + 11$

23. $x^2 - 3x + 8$

24. $x^2 + 7x - 15$

25. $3x^2 + 5x - 8$

26. $2x^2 + 7x - 9$

27. $4x^2 + 11 + 8x$

28. $5x^2 - 9 + 7x$

29. $5x - 3x^2 - 10$

30. $9x - 7x^2 - 18$

< Objective 3 >

Use the ac test to determine which of the following trinomials can be factored. Find the values of m and n for each trinomial that can be factored.

31. $x^2 + x - 6$

32. $x^2 + 2x - 15$

33. $x^2 + x + 2$

34. $x^2 - 3x + 7$

35. $x^2 - 5x + 6$

36. $x^2 - x + 2$

37. $2x^2 + 5x - 3$

38. $3x^2 - 14x - 5$

39. $6x^2 - 19x + 10$ > Videos

40. $4x^2 + 5x + 6$

Basic Skills | **Advanced Skills** | Vocational-Technical Applications | Calculator/Computer | Above and Beyond | Getting Ready

Complete each of the following statements with **never, sometimes,** *or* **always.**

41. A trinomial with integer coefficients is ____________________ factorable.

42. If a trinomial with all positive terms is factored, the signs between the terms in the binomial factors will ______________ be positive.

Answers

18. ____________
19. ____________
20. ____________
21. ____________
22. ____________
23. ____________
24. ____________
25. ____________
26. ____________
27. ____________
28. ____________
29. ____________
30. ____________
31. ____________
32. ____________
33. ____________
34. ____________
35. ____________
36. ____________
37. ____________
38. ____________
39. ____________
40. ____________
41. ____________
42. ____________

Answers

43. ______
44. ______
45. ______
46. ______
47. ______
48. ______
49. ______
50. ______
51. ______
52. ______
53. ______
54. ______
55. ______
56. ______
57. ______
58. ______
59. ______
60. ______
61. ______
62. ______
63. ______
64. ______
65. ______
66. ______
67. ______
68. ______

43. The product of two binomials ________________ results in a trinomial.

44. If the GCF for the terms in a polynomial is not 1, it should ____________ be factored out first.

< Objectives 2–4 >

By any method, factor each polynomial completely.

45. $x^2 + 8x + 15$

46. $x^2 - 11x + 24$

47. $s^2 + 13s + 30$

48. $b^2 + 14b + 33$

49. $x^2 - 6x - 40$

50. $x^2 - 11x + 10$

51. $p^2 - 10p - 24$

52. $x^2 - 11x - 60$

53. $x^2 + 5x - 66$

54. $a^2 + 2a - 80$

55. $c^2 + 19c + 60$

56. $t^2 - 4t - 60$

57. $n^2 + 5n - 50$

58. $x^2 - 16x + 63$

59. $x^2 + 7xy + 10y^2$

60. $x^2 - 8xy + 12y^2$

61. $a^2 - ab - 42b^2$

62. $m^2 - 8mn + 16n^2$

63. $x^2 - 13xy + 40y^2$

64. $r^2 - 9rs - 36s^2$

65. $6x^2 + 19x + 10$

66. $6x^2 - 7x - 3$

67. $15x^2 + x - 6$

68. $12w^2 + 19w + 4$

69. $6m^2 + 25m - 25$

70. $8x^2 - 6x - 9$

71. $9x^2 - 12x + 4$

72. $20x^2 - 23x + 6$

73. $12x^2 - 8x - 15$

74. $16a^2 + 40a + 25$

75. $3y^2 + 7y - 6$

76. $12x^2 + 11x - 15$

77. $8x^2 - 27x - 20$ > Videos

78. $24v^2 + 5v - 36$

79. $2x^2 + 3xy + y^2$

80. $3x^2 - 5xy + 2y^2$

81. $5a^2 - 8ab - 4b^2$

82. $5x^2 + 7xy - 6y^2$

83. $9x^2 + 4xy - 5y^2$

84. $16x^2 + 32xy + 15y^2$

85. $6m^2 - 17mn + 12n^2$

86. $15x^2 - xy - 6y^2$

87. $36a^2 - 3ab - 5b^2$

88. $3q^2 - 17qr - 6r^2$

89. $x^2 + 4xy + 4y^2$

90. $25b^2 - 80bc + 64c^2$

91. $20x^2 - 20x - 15$

92. $24x^2 - 18x - 6$

93. $8m^2 + 12m + 4$

94. $14x^2 - 20x + 6$

95. $15r^2 - 21rs + 6s^2$

96. $10x^2 + 5xy - 30y^2$

Answers

69. ______
70. ______
71. ______
72. ______
73. ______
74. ______
75. ______
76. ______
77. ______
78. ______
79. ______
80. ______
81. ______
82. ______
83. ______
84. ______
85. ______
86. ______
87. ______
88. ______
89. ______
90. ______
91. ______
92. ______
93. ______
94. ______
95. ______
96. ______

Answers

97. ______

98. ______

99. ______

100. ______

101. ______

102. ______

103. ______

104. ______

105. ______

106. ______

107. ______

108. ______

109. ______

110. ______

111. ______

112. ______

97. $2x^3 - 2x^2 - 4x$

98. $2y^3 + y^2 - 3y$

99. $2y^4 + 5y^3 + 3y^2$

> Videos

100. $4z^3 - 18z^2 - 10z$

101. $36a^3 - 66a^2 + 18a$

Basic Skills | Advanced Skills | **Vocational-Technical Applications** | Calculator/Computer | Above and Beyond | Getting Ready

102. **Allied Health** The number of people who are sick t days after the outbreak of a flu epidemic is given by the polynomial

$$50 + 25t - 3t^2$$

Write this polynomial in factored form.

103. **Manufacturing Technology** The bending moment in an overhanging beam is described by the expression

$$218(x^2 - 20x + 36)$$

Factor the $x^2 - 20x + 36$ portion of the expression.

104. **Manufacturing Technology** The flow rate through a hydraulic hose can be found from the equation

$$2Q^2 + Q - 21 = 0$$

Factor the left side of this equation.

Basic Skills | Advanced Skills | Vocational-Technical Applications | Calculator/Computer | **Above and Beyond** | Getting Ready

Find a positive value for k for which each of the following can be factored.

105. $x^2 + kx + 8$

106. $x^2 + kx + 9$

107. $x^2 - kx + 16$

108. $x^2 - kx + 17$

Factor each of the following polynomials completely.

109. $10(x + y)^2 - 11(x + y) - 6$

> Videos

110. $8(a - b)^2 + 14(a - b) - 15$

111. $5(x - 1)^2 - 15(x - 1) - 350$

112. $3(x + 1)^2 - 6(x + 1) - 45$

113. $15 + 29x - 48x^2$

114. $12 + 4a - 21a^2$

115. $-6x^2 + 19x - 15$

116. $-3s^2 - 10s + 8$

Basic Skills | Advanced Skills | Vocational-Technical Applications | Calculator/Computer | Above and Beyond | **Getting Ready**

Multiply.

(a) $(x - 1)(x + 1)$

(b) $(a + 7)(a - 7)$

(c) $(x - y)(x + y)$

(d) $(2x - 5)(2x + 5)$

(e) $(3a - b)(3a + b)$

(f) $(5a - 4b)(5a + 4b)$

Answers

1. $2x - 3$ **3.** $3a + 2$ **5.** $5x - 2$ **7.** $4a + b$ **9.** $4m - 3n$
11. True **13.** False **15.** True **17.** False **19.** True
21. $a = 1, b = 4, c = -9$ **23.** $a = 1, b = -3, c = 8$
25. $a = 3, b = 5, c = -8$ **27.** $a = 4, b = 8, c = 11$
29. $a = -3, b = 5, c = -10$ **31.** Factorable; $3, -2$ **33.** Not factorable
35. Factorable; $-3, -2$ **37.** Factorable; $6, -1$ **39.** Factorable; $-15, -4$
41. sometimes **43.** sometimes **45.** $(x + 3)(x + 5)$
47. $(s + 10)(s + 3)$ **49.** $(x - 10)(x + 4)$ **51.** $(p - 12)(p + 2)$
53. $(x + 11)(x - 6)$ **55.** $(c + 4)(c + 15)$ **57.** $(n + 10)(n - 5)$
59. $(x + 2y)(x + 5y)$ **61.** $(a - 7b)(a + 6b)$ **63.** $(x - 5y)(x - 8y)$
65. $(3x + 2)(2x + 5)$ **67.** $(5x - 3)(3x + 2)$ **69.** $(6m - 5)(m + 5)$
71. $(3x - 2)(3x - 2) = (3x - 2)^2$ **73.** $(6x + 5)(2x - 3)$
75. $(3y - 2)(y + 3)$ **77.** $(8x + 5)(x - 4)$ **79.** $(2x + y)(x + y)$
81. $(5a + 2b)(a - 2b)$ **83.** $(9x - 5y)(x + y)$ **85.** $(3m - 4n)(2m - 3n)$
87. $(12a - 5b)(3a + b)$ **89.** $(x + 2y)^2$ **91.** $5(2x - 3)(2x + 1)$
93. $4(2m + 1)(m + 1)$ **95.** $3(5r - 2s)(r - s)$ **97.** $2x(x - 2)(x + 1)$
99. $y^2(2y + 3)(y + 1)$ **101.** $6a(3a - 1)(2a - 3)$ **103.** $(x - 18)(x - 2)$
105. 6 or 9 **107.** 8 or 10 or 17 **109.** $(5x + 5y + 2)(2x + 2y - 3)$
111. $5(x - 11)(x + 6)$ **113.** $(1 + 3x)(15 - 16x)$ **115.** $(3x - 5)(-2x + 3)$
(a) $x^2 - 1$ **(b)** $a^2 - 49$ **(c)** $x^2 - y^2$ **(d)** $4x^2 - 25$ **(e)** $9a^2 - b^2$
(f) $25a^2 - 16b^2$

Answers

113. ______

114. ______

115. ______

116. ______

(a) ______

(b) ______

(c) ______

(d) ______

(e) ______

(f) ______

4.4 Difference of Squares and Perfect Square Trinomials

< 4.4 Objectives >

1 > Factor a binomial that is the difference of two squares

2 > Factor a perfect square trinomial

In Section 3.4, we introduced some special products. Recall the following formula for the product of a sum and difference of two terms:

$$(a + b)(a - b) = a^2 - b^2$$

This also means that a binomial of the form $a^2 - b^2$, called a **difference of two squares,** has as its factors $a + b$ and $a - b$.

To use this idea for factoring, we can write

$$a^2 - b^2 = (a + b)(a - b)$$

A **perfect square** term has a coefficient that is a square (1, 4, 9, 16, 25, 36, etc.), and any variables have exponents that are multiples of 2 (x^2, y^4, z^6, etc.).

Example 1 Identifying Perfect Square Terms

< Objective 1 >

For each of the following, decide whether it is a perfect square term. If it is, rewrite the expression as an expression squared.

(a) $36x$

(b) $24x^6$

(c) $9x^4$

(d) $64x^6$

(e) $16x^9$

Only parts **(c)** and **(d)** are perfect square terms.

$9x^4 = (3x^2)^2$

$64x^6 = (8x^3)^2$

Check Yourself 1

For each of the following, decide whether it is a perfect square term. If it is, rewrite the expression as an expression squared.

(a) $36x^{12}$ **(b)** $4x^6$ **(c)** $9x^7$ **(d)** $25x^8$ **(e)** $16x^{25}$

In Example 2, we will factor the difference between two perfect square terms.

Example 2 Factoring the Difference of Two Squares

NOTE

You could also write $(x - 4)(x + 4)$. The order doesn't matter because multiplication is commutative.

Factor $x^2 - 16$.

Think $x^2 - 4^2$.

Because $x^2 - 16$ is a difference of squares, we have

$x^2 - 16 = (x + 4)(x - 4)$

Check Yourself 2

Factor $m^2 - 49$.

Any time an expression is a difference of two squares, it can be factored.

Example 3 Factoring the Difference of Two Squares

Factor $4a^2 - 9$.

Think $(2a)^2 - 3^2$.

So

$$4a^2 - 9 = (2a)^2 - (3)^2$$
$$= (2a + 3)(2a - 3)$$

Check Yourself 3

Factor $9b^2 - 25$.

The process for factoring a difference of squares does not change when more than one variable is involved.

Example 4 Factoring the Difference of Two Squares

NOTE

Think $(5a)^2 - (4b^2)^2$.

Factor $25a^2 - 16b^4$.

$25a^2 - 16b^4 = (5a + 4b^2)(5a - 4b^2)$

Check Yourself 4

Factor $49c^4 - 9d^2$.

We will now consider an example that combines common-term factoring with difference-of-squares factoring. Note that the common factor is always factored out as the *first step.*

Example 5 Removing the GCF First

Factor $32x^2y - 18y^3$.

Note that $2y$ is a common factor, so

$$32x^2y - 18y^3 = 2y\underbrace{(16x^2 - 9y^2)}_{\text{Difference of squares}}$$

$$= 2y(4x + 3y)(4x - 3y)$$

NOTE

Step 1
Factor out the GCF.
Step 2
Factor the remaining binomial.

Check Yourself 5

Factor $50a^3 - 8ab^2$.

> CAUTION

Note that this is different from the sum of two squares (such as $x^2 + y^2$), which never has real factors.

Recall the following multiplication pattern.

$$(a + b)^2 = a^2 + 2ab + b^2$$

For example,

$$(x + 2)^2 = x^2 + 4x + 4$$

$$(x + 5)^2 = x^2 + 10x + 25$$

$$(2x + 1)^2 = 4x^2 + 4x + 1$$

Recognizing this pattern can simplify the process of factoring perfect square trinomials.

Example 6 Factoring a Perfect Square Trinomial

< Objective 2 >

Factor the trinomial $4x^2 + 12xy + 9y^2$.

Note that this is a perfect square trinomial in which

$a = 2x$ and $b = 3y$.

In factored form, we have

$$4x^2 + 12xy + 9y^2 = (2x + 3y)^2$$

Check Yourself 6

Factor the trinomial $16u^2 + 24uv + 9v^2$.

Recognizing the same pattern can simplify the process of factoring perfect square trinomials in which the second term is negative.

Example 7 Factoring a Perfect Square Trinomial

Factor the trinomial $25x^2 - 10xy + y^2$.

This is also a perfect square trinomial, in which

$a = 5x$ and $b = -y$.

In factored form, we have

$$25x^2 - 10xy + y^2 = (5x + (-y))^2 = (5x - y)^2$$

Check Yourself 7

Factor the trinomial $4u^2 - 12uv + 9v^2$.

Check Yourself ANSWERS

1. **(a)** $(6x^6)^2$; **(b)** $(2x^3)^2$; **(d)** $(5x^4)^2$ **2.** $(m + 7)(m - 7)$
3. $(3b + 5)(3b - 5)$ **4.** $(7c^2 + 3d)(7c^2 - 3d)$
5. $2a(5a + 2b)(5a - 2b)$ **6.** $(4u + 3v)^2$ **7.** $(2u - 3v)^2$

Reading Your Text

The following fill-in-the-blank exercises are designed to ensure that you understand some of the key vocabulary used in this section.

SECTION 4.4

(a) A perfect square term has a coefficient that is a square and any variables have exponents that are ________ of 2.

(b) Any time an expression is the difference of two squares, it can be ________.

(c) When factoring, the ________ factor is always factored out as the first step.

(d) Although the difference of two squares can be factored, the ________ of two squares cannot.

4.4 exercises

Basic Skills | Advanced Skills | Vocational-Technical Applications | Calculator/Computer | Above and Beyond | Getting Ready

MathZone

Boost your grade at mathzone.com!

> Practice Problems
> NetTutor
> Self-Tests
> e-Professors
> Videos

Name ____________

Section ________ Date ________

Answers

1. ____ 2. ____
3. ____ 4. ____
5. ____ 6. ____
7. ____ 8. ____
9. ____ 10. ____
11. ____
12. ____
13. ____
14. ____
15. ____
16. ____
17. ____
18. ____
19. ____
20. ____
21. ____
22. ____
23. ____
24. ____

< Objective 1 >

For each of the following binomials, is the binomial a difference of squares?

1. $3x^2 + 2y^2$

2. $5x^2 - 7y^2$

3. $16a^2 - 25b^2$

4. $9n^2 - 16m^2$

5. $16r^2 + 4$

6. $p^2 - 45$

7. $16a^2 - 12b^3$

8. $9a^2b^2 - 16c^2d^2$

9. $a^2b^2 - 25$ > Videos

10. $4a^3 - b^3$

Factor the following binomials.

11. $m^2 - n^2$

12. $r^2 - 9$

13. $x^2 - 49$

14. $c^2 - d^2$

15. $49 - y^2$

16. $81 - b^2$

17. $9b^2 - 16$

18. $36 - x^2$

19. $16w^2 - 49$

20. $4x^2 - 25$

21. $4s^2 - 9r^2$

22. $64y^2 - x^2$

23. $9w^2 - 49z^2$ > Videos

24. $25x^2 - 81y^2$

25. $16a^2 - 49b^2$

26. $64m^2 - 9n^2$

27. $x^4 - 36$

28. $y^6 - 49$

29. $x^2y^2 - 16$

30. $m^2n^2 - 64$

31. $25 - a^2b^2$

32. $49 - w^2z^2$

33. $81a^2 - 100b^6$

34. $64x^4 - 25y^4$

35. $18x^3 - 2xy^2$ > Videos

36. $50a^2b - 2b^3$

37. $12m^3n - 75mn^3$

38. $63p^4 - 7p^2q^2$

Determine whether each of the following statements is **true** *or* **false.**

39. A perfect square term has a coefficient that is a square and any variables have exponents that are factors of 2.

40. Any time an expression is the difference of two squares, it can be factored.

41. Although the difference of two squares can be factored, the sum of two squares cannot.

42. When factoring, the middle factor is always factored out as the first step.

< Objective 2 >

Determine whether each of the following trinomials is a perfect square. If it is, factor the trinomial.

43. $x^2 - 14x + 49$

44. $x^2 + 9x + 16$

45. $x^2 - 18x - 81$

46. $x^2 + 10x + 25$

47. $x^2 - 18x + 81$

48. $x^2 - 24x + 48$

Answers

25. ________
26. ________
27. ________
28. ________
29. ________
30. ________
31. ________
32. ________
33. ________
34. ________
35. ________
36. ________
37. ________
38. ________
39. ________
40. ________
41. ________
42. ________
43. ________
44. ________
45. ________
46. ________
47. ________
48. ________

Answers

49. ____

50. ____

51. ____

52. ____

53. ____

54. ____

55. ____

56. ____

57. ____

58. ____

59. ____

60. ____

61. ____

62. ____

Factor the following trinomials.

49. $x^2 + 4x + 4$

50. $x^2 + 6x + 9$

51. $x^2 - 10x + 25$

52. $x^2 - 8x + 16$

Basic Skills | **Advanced Skills** | Vocational-Technical Applications | Calculator/Computer | Above and Beyond | Getting Ready

53. $4x^2 + 12xy + 9y^2$

54. $16x^2 + 40xy + 25y^2$

55. $9x^2 - 24xy + 16y^2$ > Videos

56. $9w^2 - 30wv + 25v^2$

57. $y^3 - 10y^2 + 25y$

58. $12b^3 - 12b^2 + 3b$

Basic Skills | Advanced Skills | **Vocational-Technical Applications** | Calculator/Computer | Above and Beyond | Getting Ready

59. **Manufacturing Technology** The difference d in the calculated maximum deflection between two similar cantilevered beams is given by the formula

$$d = \left(\frac{w}{8EI}\right)(l_1^2 - l_2^2)(l_1^2 + l_2^2)$$

Rewrite the formula in its completely factored form.

60. **Manufacturing Technology** The work done W by a steam turbine is given by the formula

$$W = \frac{1}{2}m(v_1^2 - v_2^2)$$

Factor the right-hand side of this equation.

61. **Allied Health** A toxic chemical is introduced into a protozoan culture. The number of deaths per hour is given by the polynomial $338 - 2t^2$, in which t is the number of hours after the chemical is introduced. Factor this expression.

62. **Allied Health** Radiation therapy is one technique used to control cancer. After treatment, the total number of cancerous cells, in thousands, can be estimated by $144 - 4t^2$, in which t is the number of days of treatment. Factor this expression.

Basic Skills | Advanced Skills | Vocational-Technical Applications | Calculator/Computer | **Above and Beyond** | Getting Ready

Factor each expression.

63. $x^2(x + y) - y^2(x + y)$

> Videos

64. $a^2(b - c) - 16b^2(b - c)$

65. $2m^2(m - 2n) - 18n^2(m - 2n)$

66. $3a^3(2a + b) - 27ab^2(2a + b)$

67. Find a value for k so that $kx^2 - 25$ will have the factors $2x + 5$ and $2x - 5$.

68. Find a value for k so that $9m^2 - kn^2$ will have the factors $3m + 7n$ and $3m - 7n$.

69. Find a value for k so that $2x^3 - kxy^2$ will have the factors $2x$, $x - 3y$, and $x + 3y$.

70. Find a value for k so that $20a^3b - kab^3$ will have the factors $5ab$, $2a - 3b$, and $2a + 3b$.

71. Complete the following statement in complete sentences: "To factor a number, you. . . ."

72. Complete this statement: "To factor an algebraic expression into prime factors means. . . ."

Basic Skills | Advanced Skills | Vocational-Technical Applications | Calculator/Computer | Above and Beyond | **Getting Ready**

Factor.

(a) $2x(3x + 2) - 5(3x + 2)$

(b) $3y(y - 4) + 5(y - 4)$

(c) $3x(x + 2y) + y(x + 2y)$

(d) $5x(2x - y) - 3(2x - y)$

(e) $4x(2x - 5y) - 3y(2x - 5y)$

Answers

1. No **3.** Yes **5.** No **7.** No **9.** Yes **11.** $(m + n)(m - n)$
13. $(x + 7)(x - 7)$ **15.** $(7 + y)(7 - y)$ **17.** $(3b + 4)(3b - 4)$
19. $(4w + 7)(4w - 7)$ **21.** $(2s + 3r)(2s - 3r)$ **23.** $(3w + 7z)(3w - 7z)$

Answers

63. ____

64. ____

65. ____

66. ____

67. ____

68. ____

69. ____

70. ____

71. ____

72. ____

(a) ____

(b) ____

(c) ____

(d) ____

(e) ____

25. $(4a + 7b)(4a - 7b)$ **27.** $(x^2 + 6)(x^2 - 6)$ **29.** $(xy + 4)(xy - 4)$
31. $(5 + ab)(5 - ab)$ **33.** $(9a + 10b^3)(9a - 10b^3)$
35. $2x(3x + y)(3x - y)$ **37.** $3mn(2m + 5n)(2m - 5n)$ **39.** False
41. True **43.** Yes; $(x - 7)^2$ **45.** No **47.** Yes; $(x - 9)^2$
49. $(x + 2)^2$ **51.** $(x - 5)^2$ **53.** $(2x + 3y)^2$ **55.** $(3x - 4y)^2$
57. $y(y - 5)^2$ **59.** $d = \left(\frac{w}{8EI}\right)(l_1 + l_2)(l_1 - l_2)(l_1^2 + l_2^2)$
61. $2(13 - t)(13 + t)$ **63.** $(x + y)^2(x - y)$
65. $2(m - 2n)(m + 3n)(m - 3n)$ **67.** 4 **69.** 18
71. Above and Beyond **(a)** $(3x + 2)(2x - 5)$ **(b)** $(y - 4)(3y + 5)$
(c) $(x + 2y)(3x + y)$ **(d)** $(2x - y)(5x - 3)$ **(e)** $(2x - 5y)(4x - 3y)$

4.5 Strategies in Factoring

< 4.5 Objectives >

1 > Recognize factoring patterns

2 > Apply appropriate factoring strategies

In Sections 4.1 to 4.4 you have seen a variety of techniques for factoring polynomials. This section reviews those techniques and presents some guidelines for choosing an appropriate strategy or combination of strategies.

1. Always look for a greatest common factor. If you find a GCF (other than 1), factor out the GCF as your first step.

To factor $5x^2y - 10xy + 25xy^2$, the GCF is $5xy$, so

$$5x^2y - 10xy + 25xy^2 = 5xy(x - 2 + 5y)$$

2. Now look at the number of terms in the polynomial you are trying to factor.

(a) If the polynomial is a *binomial,* consider the formula for the difference of two squares. Recall that a sum of squares does not factor over the real numbers.

(i) To factor $x^2 - 49y^2$, recognize the difference of squares, so

$$x^2 - 49y^2 = (x + 7y)(x - 7y)$$

(ii) The binomial

$$x^2 + 64$$

cannot be further factored.

(b) If the polynomial is a *trinomial,* try to factor the trinomial as a product of two binomials, using trial and error.

To factor $2x^2 - x - 6$, a consideration of possible factors of the first and last terms of the trinomial will lead to

$$2x^2 - x - 6 = (2x + 3)(x - 2)$$

NOTE

You may prefer to use the *ac* method shown in Section 4.3.

(c) If the polynomial has *more than three terms,* try factoring by grouping.

To factor $2x^2 - 3xy + 10x - 15y$, group the first two terms, and then the last two, and factor out common factors.

$$2x^2 - 3xy + 10x - 15y = x(2x - 3y) + 5(2x - 3y)$$

Now factor out the common factor $(2x - 3y)$.

$$2x^2 - 3xy + 10x - 15y = (2x - 3y)(x + 5)$$

3. You should always factor the given polynomial completely. So after you apply one of the techniques given in part 2, another one may be necessary.

(a) To factor

$$6x^3 + 22x^2 - 40x$$

first factor out the common factor of $2x$. So

$$6x^3 + 22x^2 - 40x = 2x(3x^2 + 11x - 20)$$

Now continue to factor the trinomial as before and

$$6x^3 + 22x^2 - 40x = 2x(3x - 4)(x + 5)$$

(b) To factor

$$x^3 - x^2y - 4x + 4y$$

first we proceed by grouping:

$$x^3 - x^2y - 4x + 4y = x^2(x - y) - 4(x - y)$$
$$= (x - y)(x^2 - 4)$$

Now because $x^2 - 4$ is a difference of two squares, we continue to factor and obtain

$$x^3 - x^2y - 4x + 4y = (x - y)(x + 2)(x - 2)$$

Example 1 Recognizing Factoring Patterns

< Objective 1 >

For each of the following expressions, state the appropriate first step for factoring the polynomial.

(a) $9x^2 - 18x - 72$

Find the GCF.

(b) $x^2 - 3x + 2xy - 6y$

Group the terms.

(c) $x^4 - 81y^4$

Factor the difference of squares.

(d) $3x^2 + 7x + 2$

Use the *ac* method (or trial and error).

Check Yourself 1

For each of the following expressions, state the appropriate first step for factoring the polynomial.

(a) $5x^2 + 2x - 3$
(b) $a^4b^4 - 16$
(c) $3x^2 + 3x - 60$
(d) $2a^2 - 5a + 4ab - 10b$

Example 2 Factoring Polynomials

< Objective 2 >

For each of the following expressions, completely factor the polynomial.

(a) $9x^2 - 18x - 72$

The GCF is 9.

$$9x^2 - 18x - 72 = 9(x^2 - 2x - 8)$$
$$= 9(x - 4)(x + 2)$$

(b) $x^2 - 3x + 2xy - 6y$

Grouping the terms, we have

$$\begin{aligned} x^2 - 3x + 2xy - 6y &= (x^2 - 3x) + (2xy - 6y) \\ &= x(x - 3) + 2y(x - 3) \\ &= (x - 3)(x + 2y) \end{aligned}$$

(c) $x^4 - 81y^4$

Factoring the difference of squares, we find

$$\begin{aligned} x^4 - 81y^4 &= (x^2 + 9y^2)(x^2 - 9y^2) \\ &= (x^2 + 9y^2)(x - 3y)(x + 3y) \end{aligned}$$

(d) $3x^2 + 7x + 2$

Using the *ac* method, we find $m = 1$ and $n = 6$.

$$\begin{aligned} 3x^2 + 7x + 2 &= 3x^2 + x + 6x + 2 \\ &= (3x^2 + x) + (6x + 2) \\ &= x(3x + 1) + 2(3x + 1) \\ &= (3x + 1)(x + 2) \end{aligned}$$

Check Yourself 2

For each of the following expressions, completely factor the polynomial.

(a) $5x^2 + 2x - 3$

(b) $a^4b^4 - 16$

(c) $3x^2 + 3x - 60$

(d) $2a^2 - 5a + 4ab - 10b$

There are other patterns that sometimes occur when factoring. Several of these relate to the factoring of expressions that contain terms that are perfect cubes. The most common are the sum or difference of cubes, shown here.

Factoring the sum of two perfect cubes

$$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$$

Factoring the difference of two perfect cubes

$$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$$

Example 3 Factoring Expressions Involving Perfect Cube Terms

Factor each of the following expressions.

(a) $8x^3 + 27y^3$.

$$\begin{aligned} 8x^3 + 27y^3 &= (2x)^3 + (3y)^3 && \text{Substitute these values into the given patterns} \\ &= [(2x) + (3y)][(2x)^2 - (2x)(3y) + (3y)^2] && \text{Simplify} \\ &= (2x + 3y)(4x^2 - 6xy + 9y^2) \end{aligned}$$

(b) $a^3b^3 - 64c^3$

$$a^3b^3 - 64c^3 = (ab)^3 - (4c)^3$$
$$= [(ab) - (4c)][(ab)^2 + (ab)(4c) + (4c)^2]$$
$$= (ab - 4c)(a^2b^2 + 4abc + 16c^2)$$

Check Yourself 3

Factor each of the following expressions

(a) $a^3 + 64b^3c^3$ **(b)** $27x^3 - 8y^3$

Check Yourself ANSWERS

1. (a) *ac* method (or trial and error); **(b)** factor the difference of squares; **(c)** find the GCF; **(d)** group the terms
2. (a) $(5x - 3)(x + 1)$; **(b)** $(a^2b^2 + 4)(ab - 2)(ab + 2)$; **(c)** $3(x + 5)(x - 4)$; **(d)** $(2a - 5)(a + 2b)$ **3. (a)** $(a + 4bc)(a^2 - 4abc + 16b^2c^2)$; **(b)** $(3x - 2y)(9x^2 - 6xy + 4y^2)$

Reading Your Text

The following fill-in-the-blank exercises are designed to ensure that you understand some of the key vocabulary used in this section.

SECTION 4.5

(a) The first step in factoring requires that we find the ______________ of all the terms.

(b) If a polynomial is a ______________ , we can consider the formula for the difference of two squares.

(c) You should always factor a given polynomial ______________.

(d) If a polynomial has more than three terms, we can try factoring by ______________.

Basic Skills | Advanced Skills | Vocational-Technical Applications | Calculator/Computer | Above and Beyond | Getting Ready

4.5 exercises

MathZone

Boost your grade at mathzone.com!
> Practice Problems
> NetTutor
> Self-Tests
> e-Professors
> Videos

Name ____________

Section ________ Date ________

< Objectives 1–2 >

Factor each polynomial completely. To begin, state which method should be applied as the first step, given the guidelines of this section. Then continue the exercise and factor each polynomial completely.

1. $x^2 - 3x$

2. $4y^2 - 9$

3. $x^2 - 5x - 24$

4. $8x^3 + 10x$

5. $x(x - y) + 2(x - y)$

6. $5a^2 - 10a + 25$

7. $2x^2y - 6xy + 8y^2$ > Videos

8. $2p - 6q + pq - 3q^2$

9. $y^2 - 13y + 40$

10. $m^3 + 27m^2n$

11. $3b^2 + 17b - 28$ > Videos

12. $3x^2 + 6x - 5xy - 10y$ > Videos

13. $3x^2 - 14xy - 24y^2$

14. $16c^2 - 49d^2$

15. $2a^2 + 11a + 12$

16. $m^3n^3 - mn$

17. $125r^3 + r^2$

18. $(x - y)^2 - 16$

Answers

1. ____________
2. ____________
3. ____________
4. ____________
5. ____________
6. ____________
7. ____________
8. ____________
9. ____________
10. ____________
11. ____________
12. ____________
13. ____________
14. ____________
15. ____________
16. ____________
17. ____________
18. ____________

Answers

19. ________
20. ________
21. ________
22. ________
23. ________
24. ________
25. ________
26. ________
27. ________
28. ________
29. ________
30. ________
31. ________
32. ________
33. ________
34. ________
35. ________
36. ________

(a) ____ (b) ____
(c) ____ (d) ____
(e) ____ (f) ____

19. $3x^2 - 30x + 63$ **20.** $3a^2 - 108$

21. $40a^2 + 5$ **22.** $4p^2 - 8p - 60$

Basic Skills | **Advanced Skills** | Vocational-Technical Applications | Calculator/Computer | Above and Beyond | Getting Ready

23. $2w^2 - 14w - 36$ **24.** $xy^3 - 9xy$

25. $3a^2b - 48b^3$ > Videos **26.** $12b^3 - 86b^2 + 14b$

27. $x^4 - 3x^2 - 10$ **28.** $m^4 - 9n^4$

29. $8p^3 - q^3r^3$ **30.** $27x^3 + 125y^3$

Basic Skills | Advanced Skills | Vocational-Technical Applications | Calculator/Computer | **Above and Beyond** | Getting Ready

Factor completely.

31. $(x - 5)^2 - 169$ > Videos **32.** $(x - 7)^2 - 81$

33. $x^2 + 4xy + 4y^2 - 16$ **34.** $9x^2 + 12xy + 4y^2 - 25$

35. $6(x - 2)^2 + 7(x - 2) - 5$ **36.** $12(x + 1)^2 - 17(x + 1) + 6$

Basic Skills | Advanced Skills | Vocational-Technical Applications | Calculator/Computer | Above and Beyond | **Getting Ready**

Solve.

(a) $x - 5 = 0$ **(b)** $2x - 1 = 0$ **(c)** $3x + 2 = 0$
(d) $x + 4 = 0$ **(e)** $7 - x = 0$ **(f)** $9 - 4x = 0$

Answers

1. GCF, $x(x - 3)$ **3.** Trial and error, $(x - 8)(x + 3)$ **5.** GCF, $(x + 2)(x - y)$ **7.** GCF, $2y(x^2 - 3x + 4y)$ **9.** Trial and error, $(y - 5)(y - 8)$ **11.** Trial and error, $(b + 7)(3b - 4)$ **13.** Trial and error, $(3x + 4y)(x - 6y)$ **15.** Trial and error, $(2a + 3)(a + 4)$ **17.** GCF, $r^2(125r + 1)$ **19.** GCF, then trial and error, $3(x - 3)(x - 7)$ **21.** GCF, $5(8a^2 + 1)$ **23.** GCF, then trial and error, $2(w - 9)(w + 2)$ **25.** GCF, then difference of squares, $3b(a + 4b)(a - 4b)$ **27.** Trial and error, $(x^2 - 5)(x^2 + 2)$ **29.** $(2p - qr)(4p^2 - 2pqr + q^2r^2)$ **31.** $(x + 8)(x - 18)$ **33.** $(x + 2y + 4)(x + 2y - 4)$ **35.** $(2x - 5)(3x - 1)$ **(a)** $x = 5$ **(b)** $x = \frac{1}{2}$ **(c)** $x = -\frac{2}{3}$ **(d)** $x = -4$ **(e)** $x = 7$ **(f)** $x = \frac{9}{4}$

4.6 Solving Quadratic Equations by Factoring

< 4.6 Objectives >

1 > Solve quadratic equations by factoring

2 > Solve applications of quadratic equations

The factoring techniques you have learned provide us with tools for solving equations that can be written in the form

$$ax^2 + bx + c = 0 \qquad a \neq 0$$

This is a quadratic equation in one variable, here x. You can recognize such a quadratic equation by the fact that the highest power of the variable x is the second power.

in which a, b, and c are constants.

An equation written in the form $ax^2 + bx + c = 0$ is called a **quadratic equation in standard form.** Using factoring to solve quadratic equations requires the **zero-product principle,** which says that if the product of two factors is 0, then one or both of the factors must be equal to 0. In symbols:

Definition

Zero-Product Principle

If $a \cdot b = 0$, then $a = 0$ or $b = 0$ or $a = b = 0$.

We can now apply this principle to solve quadratic equations.

Example 1 Solving Equations by Factoring

< Objective 1 >

Solve.

$x^2 - 3x - 18 = 0$

Factoring on the left, we have

$(x - 6)(x + 3) = 0$

By the zero-product principle, we know that one or both of the factors must be zero. We can then write

$x - 6 = 0 \qquad \text{or} \qquad x + 3 = 0$

Solving each equation gives

$x = 6 \qquad \text{or} \qquad x = -3$

NOTE

To use the zero-product principle, 0 must be on one side of the equation.

The two solutions are 6 and -3.

Quadratic equations can be checked in the same way as linear equations were checked: by substitution. For instance, if $x = 6$, we have

$$
\begin{aligned}
6^2 - 3 \cdot 6 - 18 &\stackrel{?}{=} 0 \\
36 - 18 - 18 &\stackrel{?}{=} 0 \\
0 &= 0
\end{aligned}
$$

which is a true statement. We leave it to you to check the solution -3.

Check Yourself 1

Solve $x^2 - 9x + 20 = 0$.

Other factoring techniques are also used in solving quadratic equations. Example 2 illustrates this.

Example 2 Solving Equations by Factoring

(a) Solve $x^2 - 5x = 0$.

Again, factor the left side of the equation and apply the zero-product principle.

$x(x - 5) = 0$

Now

$x = 0$ or $x - 5 = 0$

$x = 5$

The two solutions are 0 and 5.

(b) Solve $x^2 - 9 = 0$.

Factoring yields

$(x + 3)(x - 3) = 0$

$x + 3 = 0$ or $x - 3 = 0$

$x = -3$ $x = 3$

The solutions may be written as $x = \pm 3$.

> **CAUTION**
>
> A *common mistake* is to forget the statement $x = 0$ when you are solving equations of this type. Be sure to include the *two statements* obtained.

> **NOTE**
>
> The symbol $\pm$ is read "plus or minus."

Check Yourself 2

Solve by factoring.

(a) $x^2 + 8x = 0$ **(b)** $x^2 - 16 = 0$

Example 3 illustrates a crucial point. Our solution technique depends on the zero-product principle, which means that the product of factors *must be equal to* 0. The importance of this is shown now.

Example 3 Solving Equations by Factoring

> **> CAUTION**
>
> Consider the equation
>
> $x(2x - 1) = 3$
>
> Students are sometimes tempted to write
>
> $x = 3 \quad \text{or} \quad 2x - 1 = 3$
>
> This is *not correct.* Instead, subtract 3 from both sides of the equation *as the first step* to write
>
> $x(2x - 1) - 3 = 0$
>
> Then proceed to write the equation in standard form. Only then can you factor and proceed as before.

Solve $2x^2 - x = 3$.

The first step in the solution is to write the equation in standard form (that is, write it so that one side of the equation is 0). So start by adding -3 to both sides of the equation.
Then,

$$2x^2 - x - 3 = 0$$

Make sure all terms are on one side of the equation. The other side will be 0.

You can now factor and solve by using the zero-product principle.

$$(2x - 3)(x + 1) = 0$$

$$2x - 3 = 0 \quad \text{or} \quad x + 1 = 0$$

$$2x = 3 \qquad\qquad x = -1$$

$$x = \frac{3}{2}$$

The solutions are $\frac{3}{2}$ and -1.

Check Yourself 3

Solve $3x^2 = 5x + 2$.

In all the previous examples, the quadratic equations had two distinct real-number solutions. That may not always be the case, as we shall see.

Example 4 Solving Equations by Factoring

Solve $x^2 - 6x + 9 = 0$.

Factoring, we have

$$(x - 3)(x - 3) = 0$$

and

$$x - 3 = 0 \quad \text{or} \quad x - 3 = 0$$

$$x = 3 \qquad\qquad x = 3$$

The solution is 3.

A quadratic (or second-degree) equation always has *two* solutions. When an equation such as this one has two solutions that are the same number, we call 3 the **repeated** (or **double**) **solution** of the equation.

Although a quadratic equation always has two solutions, they may not always be real numbers. You will learn more about this in a later course.

Check Yourself 4

Solve $x^2 + 6x + 9 = 0$.

Always examine the quadratic member of an equation for common factors. It will make your work much easier, as Example 5 illustrates.

Example 5 Solving Equations by Factoring

Solve $3x^2 - 3x - 60 = 0$.

First, note the common factor 3 in the quadratic member of the equation. Factoring out the 3, we have

$$3(x^2 - x - 20) = 0$$

Now, because the common factor has no variables, we can divide both sides of the equation by 3.

$$\frac{3(x^2 - x - 20)}{3} = \frac{0}{3}$$

or

$$x^2 - x - 20 = 0$$

We can now factor and solve as before.

$$(x - 5)(x + 4) = 0$$

$$x - 5 = 0 \quad \text{or} \quad x + 4 = 0$$

$$x = 5 \qquad\qquad x = -4$$

NOTE

Notice the advantage of dividing both sides of the equation by 3. The coefficients in the quadratic expression become smaller and are easier to factor.

Check Yourself 5

Solve $2x^2 - 10x - 48 = 0$.

Many applications can be solved with quadratic equations.

Example 6 Solving an Application

< Objective 2 >

The Microhard Corporation has found that the equation

$$P = x^2 - 7x - 94$$

describes the profit P, in thousands of dollars, for every x hundred computers sold. How many computers were sold if the profit was \$50,000?

If the profit was \$50,000, then $P = 50$. We now set up and solve the equation.

$$50 = x^2 - 7x - 94$$

$$0 = x^2 - 7x - 144$$

$$0 = (x + 9)(x - 16)$$

$$x = -9 \quad \text{or} \quad x = 16$$

They cannot sell a negative number of computers, so $x = 16$. They sold 1,600 computers.

NOTE

P is expressed in thousands so the value 50 is substituted for P, not 50,000.

Check Yourself 6

The Gerbil Babyfood Corporation has found that the equation

$P = x^2 - 6x - 7$

describes the profit P, in hundreds of dollars, for every x thousand jars sold. How many jars were sold if the profit was $20,000?

Check Yourself ANSWERS

1. 4, 5 **2.** **(a)** 0, −8; **(b)** 4, −4 **3.** $-\frac{1}{3}$, 2 **4.** −3 **5.** −3, 8
6. 9,000 jars

Reading Your Text

The following fill-in-the-blank exercises are designed to ensure that you understand some of the key vocabulary used in this section.

SECTION 4.6

(a) An equation written in the form $ax^2 + bx + c = 0$ is called a ____________ equation in standard form.

(b) Using factoring to solve quadratic equations requires the ____________ principle.

(c) To use the zero-product principle, it is important that the product of factors be equal to ____________.

(d) When an equation has two solutions that are the same number, we call it a ____________ solution.

4.6 exercises

Basic Skills | Advanced Skills | Vocational-Technical Applications | Calculator/Computer | Above and Beyond | Getting Ready

MathZone

Boost your grade at mathzone.com!
- Practice Problems
- NetTutor
- Self-Tests
- e-Professors
- Videos

Name ____________

Section ________ Date ________

Answers

1. ______ 2. ______
3. ______ 4. ______
5. ______ 6. ______
7. ______ 8. ______
9. ______ 10. ______
11. ______ 12. ______
13. ______ 14. ______
15. ______ 16. ______
17. ______ 18. ______
19. ______ 20. ______
21. ______ 22. ______
23. ______ 24. ______
25. ______ 26. ______
27. ______ 28. ______
29. ______ 30. ______
31. ______ 32. ______

< Objective 1 >

Solve each of the following quadratic equations.

1. $(x - 3)(x - 4) = 0$

2. $(x - 7)(x + 1) = 0$

3. $(3x + 1)(x - 6) = 0$

4. $(5x - 4)(x - 6) = 0$

5. $x^2 - 2x - 3 = 0$

6. $x^2 + 5x + 4 = 0$

7. $x^2 - 7x + 6 = 0$

8. $x^2 + 3x - 10 = 0$

9. $x^2 + 8x + 15 = 0$

10. $x^2 - 3x - 18 = 0$

11. $x^2 + 4x - 21 = 0$

12. $x^2 - 12x + 32 = 0$

13. $x^2 - 4x = 12$ > Videos

14. $x^2 + 8x = -15$

15. $x^2 + 5x = 14$

16. $x^2 = 11x - 24$

17. $2x^2 + 5x - 3 = 0$

18. $3x^2 + 7x + 2 = 0$

19. $4x^2 - 24x + 35 = 0$

20. $6x^2 + 11x - 10 = 0$

21. $4x^2 + 11x = -6$

22. $5x^2 + 2x = 3$

23. $5x^2 + 13x = 6$

24. $4x^2 = 13x + 12$ > Videos

25. $x^2 - 2x = 0$

26. $x^2 + 5x = 0$

27. $x^2 = -8x$

28. $x^2 = 7x$

29. $5x^2 - 15x = 0$ > Videos

30. $4x^2 + 20x = 0$

31. $x^2 - 25 = 0$

32. $x^2 = 49$

33. $x^2 = 81$

34. $x^2 = 64$

35. $2x^2 - 18 = 0$

36. $3x^2 - 75 = 0$

37. $3x^2 + 24x + 45 = 0$

38. $4x^2 - 4x = 24$

Basic Skills | **Advanced Skills** | Vocational-Technical Applications | Calculator/Computer | Above and Beyond | Getting Ready

39. $2x(3x + 14) = 10$ > Videos

40. $3x(5x + 9) = 6$

41. $(x + 3)(x - 2) = 14$

42. $(x - 5)(x + 2) = 18$

< Objective 2 >

Solve the following problems.

43. Number Problem The product of two consecutive integers is 132. Find the two integers.

44. Number Problem The product of two consecutive positive even integers is 120. Find the two integers. > Videos

45. Number Problem The sum of an integer and its square is 72. What is the integer?

46. Number Problem The square of an integer is 56 more than the integer. Find the integer.

47. Geometry If the sides of a square are increased by 3 in., the area is increased by 39 in.^{2}. What were the dimensions of the original square?

48. Geometry If the sides of a square are decreased by 2 cm, the area is decreased by 36 cm^2. What were the dimensions of the original square?

49. Business and Finance The profit on a small appliance is given by $P = x^2 - 3x - 60$, in which x is the number of appliances sold per day. How many appliances were sold on a day when there was a \$20 loss?

50. Business and Finance The relationship between the number of calculators x that a company can sell per month and the price of each calculator p is given by $x = 1{,}700 - 100p$. Find the price at which a calculator should be sold to produce a monthly revenue of \$7,000. (*Hint:* Revenue $= xp$.)

Answers

33. ______
34. ______
35. ______
36. ______
37. ______
38. ______
39. ______
40. ______
41. ______
42. ______
43. ______
44. ______
45. ______
46. ______
47. ______
48. ______
49. ______
50. ______

Answers

51. ______

52. ______

53. ______

54. ______

55. ______

Basic Skills | Advanced Skills | **Vocational-Technical Applications** | Calculator/Computer | Above and Beyond | Getting Ready

51. **Allied Health** The concentration, C, in micrograms per milliliter (mcg/mL), of Tobrex, an antibiotic prescribed for burn patients, is given by the equation $C = 12 + t - t^2$, where t is the number of hours since the drug was administered via intravenous injection. Find the value of t when the concentration is $C = 0$.

52. **Allied Health** The number of people who are sick t days after the outbreak of a flu epidemic is given by the equation $P = 50 + 25t - 3t^2$. Write the polynomial in factored form. Find the value of t when the number of people is $P = 0$.

53. **Manufacturing Technology** The maximum stress for a given allowable strain (deformation) for a certain material is given by the polynomial

$$S = 85.8x - 0.6x^2 - 1{,}537.2$$

in which x is the allowable strain in micrometers. Find the allowable strain in micrometers when the stress is $S = 0$.

Hint: Rearrange the polynomial and factor out a common factor of -0.6 first.

Basic Skills | Advanced Skills | Vocational-Technical Applications | Calculator/Computer | **Above and Beyond** | Getting Ready

54. Write a short comparison that explains the difference between $ax^2 + bx + c$ and $ax^2 + bx + c = 0$.

55. When solving quadratic equations, some people try to solve an equation in the manner shown below, but this does not work! Write a paragraph to explain what is wrong with this approach.

$$2x^2 + 7x + 3 = 52$$
$$(2x + 1)(x + 3) = 52$$
$$2x + 1 = 52 \quad \text{or} \quad x + 3 = 52$$
$$x = \frac{51}{2} \quad \text{or} \quad x = 49$$

Answers

1. 3, 4 **3.** $-\frac{1}{3}, 6$ **5.** $-1, 3$ **7.** 1, 6 **9.** $-3, -5$ **11.** $-7, 3$ **13.** $-2, 6$ **15.** $-7, 2$ **17.** $-3, \frac{1}{2}$ **19.** $\frac{5}{2}, \frac{7}{2}$ **21.** $-\frac{3}{4}, -2$ **23.** $-3, \frac{2}{5}$ **25.** 0, 2 **27.** $0, -8$ **29.** 0, 3 **31.** $-5, 5$ **33.** $-9, 9$ **35.** $-3, 3$ **37.** $-5, -3$ **39.** $-5, \frac{1}{3}$ **41.** $4, -5$ **43.** 11, 12 or $-12, -11$ **45.** -9 or 8 **47.** 5 in. by 5 in. **49.** 8 **51.** $t = 4$ hours **53.** $x = 21$ or $x = 122$ micrometers **55.** Above and Beyond

Definition/Procedure	Example	Reference
An Introduction to Factoring		**Section 4.1**
Common Monomial Factor		
A single term that is a factor of every term of the polynomial. The greatest common factor (GCF) of a polynomial is the factor that is a product of (a) the largest common numerical factor and (b) each variable with the smallest exponent in any term.	$4x^2$ is the greatest common monomial factor of $8x^4 - 12x^3 + 16x^2$.	*p.* 376
Factoring a Monomial from a Polynomial **1.** Determine the GCF for all terms. **2.** Use the GCF to factor each term and then apply the distributive property in the form $ab + ac = a(b + c)$ ↑ The greatest common factor **3.** Mentally check by multiplication.	$8x^4 - 12x^3 + 16x^2$ $= 4x^2(2x^2 - 3x + 4)$	*p.* 377
Factoring Trinomials		**Sections 4.2 and 4.3**
Factoring by Grouping When there are four terms of a polynomial, factor the first pair and factor the last pair. If these two pairs have a common binomial factor, factor that out. The result will be the product of two binomials.	$4x^2 - 6x + 10x - 15$ $= 2x(2x - 3) + 5(2x - 3)$ $= (2x - 3)(2x + 5)$	*p.* 379
Trial and Error To factor a trinomial, find the appropriate sign pattern and then find integer values that yield the appropriate coefficients for the trinomial.	$x^2 - 5x - 24$ $= (x - \quad)(x + \quad)$ $= (x - 8)(x + 3)$	*p.* 387
Using the ac Method to Factor		
To factor a trinomial, first use the *ac* test to determine factorability. If the trinomial is factorable, the *ac* test will yield two terms (which have as their sum the middle term) that allow the factoring to be completed by using the grouping method.	$x^2 + 3x - 28$ $ac = -28; b = 3$ $mn = -28; m + n = 3$ $m = 7, n = -4$ $x^2 + 7x - 4x - 28$ $= x(x + 7) - 4(x + 7)$ $= (x - 4)(x + 7)$	*p.* 405

Continued

Definition/Procedure	Example	Reference
Difference of Squares and Perfect Square Trinomials		**Section 4.4**
Factoring a Difference of Squares		
Use the following form: $a^2 - b^2 = (a + b)(a - b)$	To factor: $16x^2 - 25y^2$: Think: $(4x)^2 - (5y)^2$ so $16x^2 - 25y^2$ $= (4x + 5y)(4x - 5y)$	*p.* 418
Factoring a Perfect Square Trinomial		
Use the following form: $a^2 + 2ab + b^2 = (a + b)^2$	$4x^2 + 12xy + 9y^2$ $= (2x)^2 + 2(2x)(3y) + (3y)^2$ $= (2x + 3y)^2$	*p.* 420
Strategies in Factoring		**Section 4.5**
When factoring a polynomial, **1.** Look for the GCF. **2.** Consider the number of terms. **a.** If it is a binomial, look for a difference of squares. **b.** If it is trinomial, use the *ac* method or trial and error. **c.** If there are four or more terms, try grouping terms. **3.** Be certain that the polynomial is completely factored.	Given $12x^3 - 86x^2 + 14x$, factor out $2x$. $2x(6x^2 - 43x + 7)$ $= 2x(6x - 1)(x - 7)$	*p.* 427
Solving Quadratic Equations by Factoring		**Section 4.6**
1. Add or subtract the necessary terms on both sides of the equation so that the equation is in standard form (set equal to 0). **2.** Factor the quadratic expression. **3.** Set each factor equal to 0. **4.** Solve the resulting equations to find the solutions. **5.** Check each solution by substituting in the original equation.	To solve $x^2 + 7x = 30$ $x^2 + 7x - 30 = 0$ $(x + 10)(x - 3) = 0$ $x + 10 = 0$ or $x - 3 = 0$ $x = -10$ and $x = 3$ are solutions.	*p.* 433

This summary exercise set is provided to give you practice with each of the objectives of this chapter. Each exercise is keyed to the appropriate chapter section. When you are finished, you can check your answers to the odd-numbered exercises against those presented in the back of the text. If you have difficulty with any of these questions, go back and reread the examples from that section. Your instructor will give you guidelines on how best to use these exercises in your instructional setting.

4.1 *Factor each of the following polynomials.*

1. $18a + 24$

2. $9m^2 - 21m$

3. $24s^2t - 16s^2$

4. $18a^2b + 36ab^2$

5. $35s^3 - 28s^2$

6. $3x^3 - 6x^2 + 15x$

7. $18m^2n^2 - 27m^2n + 18m^2n^3$

8. $121x^8y^3 + 77x^6y^3$

9. $8a^2b + 24ab - 16ab^2$

10. $3x^2y - 6xy^3 + 9x^3y - 12xy^2$

11. $x(2x - y) + y(2x - y)$

12. $5(w - 3z) - w(w - 3z)$

4.2 *Factor each of the following trinomials completely.*

13. $x^2 + 9x + 20$

14. $x^2 - 10x + 24$

15. $a^2 - a - 12$

16. $w^2 - 13w + 40$

17. $x^2 + 12x + 36$

18. $r^2 - 9r - 36$

19. $b^2 - 4bc - 21c^2$

20. $m^2n + 4mn - 32n$

21. $m^3 + 2m^2 - 35m$

22. $2x^2 - 2x - 40$

23. $3y^3 - 48y^2 + 189y$

24. $3b^3 - 15b^2 - 42b$

4.3 *Factor each of the following trinomials completely.*

25. $3x^2 + 8x + 5$

26. $5w^2 + 13w - 6$

27. $2b^2 - 9b + 9$

28. $8x^2 + 2x - 3$

29. $10x^2 - 11x + 3$

30. $4a^2 + 7a - 15$

31. $9y^2 - 3yz - 20z^2$

32. $8x^2 + 14xy - 15y^2$

33. $8x^3 - 36x^2 - 20x$

34. $9x^2 - 15x - 6$

35. $6x^3 - 3x^2 - 9x$

36. $5w^2 - 25wz + 30z^2$

4.4 *Factor each of the following completely.*

37. $p^2 - 49$

38. $25a^2 - 16$

39. $m^2 - 9n^2$

40. $16r^2 - 49s^2$

41. $25 - z^2$

42. $a^4 - 16b^2$

43. $25a^2 - 36b^2$

44. $x^6 - 4y^2$

45. $3w^3 - 12wz^2$

46. $9a^4 - 49b^2$

47. $2m^2 - 72n^4$

48. $3w^3z - 12wz^3$

49. $x^2 + 8x + 16$

50. $x^2 - 18x + 81$

51. $4x^2 + 12x + 9$

52. $9x^2 - 12x + 4$

53. $16x^3 + 40x^2 + 25x$

54. $4x^3 - 4x^2 + x$

4.5 *Factor the following polynomials completely.*

55. $x^2 - 4x + 5x - 20$

56. $x^2 + 7x - 2x - 14$

57. $6x^2 + 4x - 15x - 10$

58. $12x^2 - 9x - 28x + 21$

59. $6x^3 + 9x^2 - 4x^2 - 6x$

60. $3x^4 + 6x^3 + 5x^3 + 10x^2$

4.6 *Solve each of the following quadratic equations.*

61. $(x - 1)(2x + 3) = 0$

62. $x^2 - 5x + 6 = 0$

63. $x^2 - 10x = 0$

64. $x^2 = 144$

65. $x^2 - 2x = 15$

66. $3x^2 - 5x - 2 = 0$

67. $4x^2 - 13x + 10 = 0$

68. $2x^2 - 3x = 5$

69. $3x^2 - 9x = 0$

70. $x^2 - 25 = 0$

71. $2x^2 - 32 = 0$

72. $2x^2 - x - 3 = 0$

self-test 4

Name ______________________

Section __________ Date __________

Answers

1. ______________________
2. ______________________
3. ______________________
4. ______________________
5. ______________________
6. ______________________
7. ______________________
8. ______________________
9. ______________________
10. ______________________
11. ______________________
12. ______________________
13. ______________________
14. ______________________
15. ______________________
16. ______________________
17. ______________________
18. ______________________

The purpose of this self-test is to help you check your progress and to review for the next in-class exam. Allow yourself about an hour to take this test. At the end of that hour check your answers against those given in the back of the text. Section references accompany the answers. If you missed any questions, go back to those sections and reread the examples until you master the concepts.

Factor each of the following polynomials.

1. $12b + 18$

2. $9p^3 - 12p^2$

3. $5x^2 - 10x + 20$

4. $6a^2b - 18ab + 12ab^2$

Factor each of the following polynomials completely.

5. $a^2 - 10a + 25$

6. $64m^2 - n^2$

7. $49x^2 - 16y^2$

8. $32a^2b - 50b^3$

Factor each of the following polynomials completely.

9. $a^2 - 5a - 14$

10. $b^2 + 8b + 15$

11. $x^2 - 11x + 28$

12. $y^2 + 12yz + 20z^2$

13. $x^2 + 2x - 5x - 10$

14. $6x^2 + 2x - 9x - 3$

Factor each of the following polynomials completely.

15. $2x^2 + 15x - 8$

16. $3w^2 + 10w + 7$

17. $8x^2 - 2xy - 3y^2$

18. $6x^3 + 3x^2 - 30x$

The Streeter/Hutchison Series in Mathematics Beginning Algebra

Answers

19. ____

20. ____

21. ____

22. ____

23. ____

24. ____

25. ____

Solve each of the following equations for x.

19. $x^2 - 8x + 15 = 0$

20. $x^2 - 3x = 4$

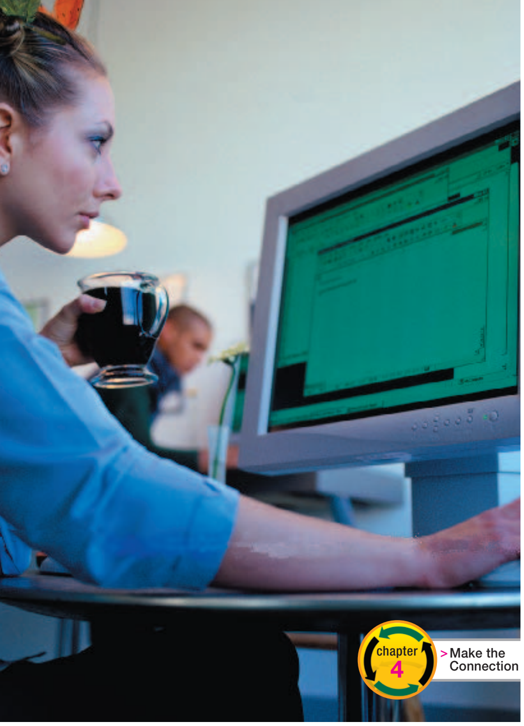
21. $3x^2 + x - 2 = 0$

22. $4x^2 - 12x = 0$

23. $x(x - 4) = 0$

24. $(x - 3)(x - 2) = 30$

25. $x^2 - 14x = -49$



Activity 4 ::
ISBNs and the Check Digit

Each activity in this text is designed to either enhance your understanding of the topics of the preceding chapter, or to provide you with a mathematical extension of those topics, or both. The activities can be undertaken by one student, but they are better suited for a small group project. Occasionally it is only through discussion that different facets of the activity become apparent. For material related to this activity, visit the text website at www.mhhe.com/streeter.

Have you ever noticed the long number, usually accompanied by a bar code, that can be found on the back of a book? This is called the International Standard Book Number (ISBN). Each book has a unique ISBN, which is 10 digits in length. A common form for an ISBN is X-XX-XXXXXX-X.

Each ISBN has four blocks of digits.

- The first block of digits on the left represents the language of the book (0 is used to represent English). This block is usually one digit in length.
- The second block of digits represents the publisher. This block is usually two or three digits in length.
- The third block of digits represents the number assigned to the book by the publishing company. This is usually five or six digits in length.
- The fourth block consists of the check digit.

For the purposes of this activity, we will consider the ISBN 0-07-229654-2, which is the ISBN for *Elementary and Intermediate Algebra* by Hutchison, Bergman, and Hoelzle.

The digit of most interest is the final digit, the **check digit.** When an ISBN is entered into a computer, the computer looks at the check digit to make certain that the numbers were properly entered. We will look at the algorithm used to generate the check digit.

Activity

I. To determine the check digit for the ISBN of a text, follow these steps.

1. Multiply each of the nine assigned digits by a weighted value. The weighted values are 1 for the first digit from the left, 2 for the second digit, 3 for third digit, etc. In our case, we have

$$1 \cdot 0 + 2 \cdot 0 + 3 \cdot 7 + 4 \cdot 2 + 5 \cdot 2 + 6 \cdot 9 + 7 \cdot 6 + 8 \cdot 5 + 9 \cdot 4 = 211$$

2. In mathematics, we occasionally are interested in only the remainder after we do division. We refer to the remainder as the **modular** (or mod) of the divisor. The ISBN uses (mod 11) to determine the check digit.

$211 \div 11 = 19$ with a remainder of 2. We say $211 = 2 \pmod{11}$

That is how we get the 2 in the ISBN 0-07-229654-2.

II. If we are given an ISBN and we want to check its validity, we can follow a similar algorithm.

1. Multiply each of the 10 digits by a weighted value. The weighted values are still 1 for the first digit from the left, 2 for the second digit, etc., but we also multiply 10 times the tenth (check) digit.

In our case, we now have

$$1 \cdot 0 + 2 \cdot 0 + 3 \cdot 7 + 4 \cdot 2 + 5 \cdot 2 + 6 \cdot 9 + 7 \cdot 6 + 8 \cdot 5 + 9 \cdot 4 + 10 \cdot 2 = 231$$

2. For any valid ISBN, the result will have a remainder of zero when divided by 11. In other words, we will have 0 (mod 11). Note that $231 \div 11 = 21$.

Determine whether each of the following is a valid ISBN.

0-07-038023-6
0-15-249584-2
0-553-34948-1
0-07-000317-3

For those numbers above that are valid, go online and find the books to which they refer.

cumulative review chapters 0-4

Name ______________________

Section __________ Date __________

The following exercises are presented to help you review concepts from earlier chapters. This is meant as a review and not as a comprehensive exam. The answers are presented in the back of the text. Section references accompany the answers. If you have difficulty with any of these exercises, be certain to at least read through the summary related to those sections.

Perform the indicated operations.

1. $7 - (-10)$

2. $(-34) \div (17)$

Perform each of the indicated operations.

3. $(7x^2 + 5x - 4) + (2x^2 - 6x - 1)$

4. $(3a^2 - 2a) - (7a^2 + 5)$

5. Subtract $4b^2 - 3b$ from the sum of $6b^2 + 5b$ and $4b^2 - 3$.

6. $3rs(5r^2s - 4rs + 6rs^2)$

7. $(2a - b)(3a^2 - ab + b^2)$

8. $\dfrac{7xy^3 - 21x^2y^2 + 14x^3y}{-7xy}$

9. $\dfrac{3a^2 - 10a - 8}{a - 4}$

10. $\dfrac{2x^3 - 8x + 5}{2x + 4}$

Solve the following equation for x.

11. $2 - 4(3x + 1) = 8 - 7x$

Solve the following inequality.

12. $4(x - 7) \le -(x - 5)$

Solve the following equation for the indicated variable.

13. $S = \dfrac{n}{2}(a + t)$ for t

Answers

1. ______________________
2. ______________________
3. ______________________
4. ______________________
5. ______________________
6. ______________________
7. ______________________
8. ______________________
9. ______________________
10. ______________________
11. ______________________
12. ______________________
13. ______________________

The Streeter/Hutchison Series in Mathematics Beginning Algebra

Answers

14. _____
15. _____
16. _____
17. _____
18. _____
19. _____
20. _____
21. _____
22. _____
23. _____
24. _____
25. _____
26. _____
27. _____
28. _____
29. _____
30. _____

Simplify the following expressions.

14. x^6x^{11}

15. $(3x^2y^3)(2x^3y^4)$

16. $(3x^2y^3)^2(-4x^3y^2)^0$

17. $\frac{16x^2y^5}{4xy^3}$

18. $(3x^2)^3(2x)^2$

Factor each of the following polynomials completely.

19. $36w^5 - 48w^4$

20. $5x^2y - 15xy + 10xy^2$

21. $25x^2 + 30xy + 9y^2$

22. $4p^3 - 144pq^2$

23. $a^2 + 4a + 3$

24. $2w^3 - 4w^2 - 24w$

25. $3x^2 + 11xy + 6y^2$

Solve each of the following equations.

26. $a^2 - 7a + 12 = 0$

27. $3w^2 - 48 = 0$

28. $15x^2 + 5x = 10$

Solve the following problems.

29. **Number Problem** Twice the square of a positive integer is 12 more than 10 times that integer. What is the integer?

30. **Geometry** The length of a rectangle is 1 in. more than 4 times its width. If the area of the rectangle is 105 in.^{2}, find the dimensions of the rectangle.