CHAPTER 15

Process Improvement Using Control Charts

Learning Objectives

After mastering the material in this chapter, you will be able to:

LO15-1 Discuss the principles and importance of quality improvement.

LO15-2 Distinguish between common causes and assignable causes of process variation.

LO15-3 Sample a process by using rational subgrouping.

LO15-4 Use $\bar{x}$ and R charts to establish process control.

LO15-5 Detect the presence of assignable causes through pattern analysis.

LO15-6 Decide whether a process is capable of meeting specifications.

LO15-7 Use p charts to monitor process quality.

LO15-8 Use diagrams to discern the causes of quality problems (Optional).

Chapter Outline

This chapter explains how to use **control charts** to improve business processes. Basically, a control chart is a graphical device that helps us determine when a process is not operating consistently and thus is "out of control." The information provided by a control chart helps us discover the causes of unusual process variations. When such causes have been identified, we attempt to remove them in order to reduce the amount of process variation. By doing so, we improve the process.

We begin this chapter by tracing the history of the U.S. quality movement. Then we study control charts for monitoring the level and variability of a process and for monitoring the fraction of nonconforming (or defective) units produced. We also discuss how to evaluate the *process capability.* That is, we show how to assess a process's ability to produce individual items that meet customer requirements (*specifications*). In particular, we explain the concept of six sigma capability, which was introduced by Motorola Inc. In an optional section we discuss cause-and-effect diagrams.

In order to demonstrate the ideas of this chapter, we employ three case studies:

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The Hole Location Case: A manufacturer of automobile air conditioner compressors uses control charts to reduce variation in the locations of a hose connection hole that is punched in the outer housing (or shell) of the compressor.

The Hot Chocolate Temperature Case: The food service staff at a university dining hall wishes to avoid possible litigation by making sure that it does not serve excessively hot beverages. The staff uses control charts to find and eliminate causes of unusual variations in hot chocolate temperatures.

The Camshaft Case: An automobile manufacturer wishes to improve the process it uses to harden a part in a camshaft assembly. The manufacturer uses control charts and process capability studies to reduce the sources of process variation that are responsible for a 12 percent rework rate and a 9 percent scrap rate. After the process variation is reduced, virtually all of the hardened parts meet specifications. (Note: This case is included in the supplementary exercises.)

15.1 Quality: Its Meaning and a Historical Perspective

LO15-1 Discuss the principles and importance of quality improvement.

What is quality? It is not easy to define quality, and a number of different definitions have been proposed. One definition that makes sense is **fitness for use.** Here the user of a product or service can be an individual, a manufacturer, a retailer, or the like. For instance, an individual who purchases a High Definition television set or a DVD recorder expects the unit to be defect free and to provide years of reliable, high-performance service. If the TV or DVD recorder performs as desired, it is fit for use. Another definition of quality that makes sense says that **quality is the extent to which customers feel that a product or service exceeds their needs and expectations.** For instance, if the DVD recorder's purchaser believes the unit exceeds all the needs and expectations he or she had for the recorder when it was purchased, then the customer is satisfied with the unit's quality.

Three types of quality can be considered: **quality of design, quality of conformance,** and **quality of performance. Quality of design** has to do with intentional differences between goods and services with the same basic purpose. For instance, all DVD recorders are built to perform the same function—record and play back DVDs. However, DVD recorders differ with respect to various design characteristics—picture sharpness, sound quality, digital effects, ease of use, and so forth. A given level of design quality may satisfy some consumers and may not satisfy others. The product design will specify a set of **tolerances (specifications)** that must be met. For example, the design of a DVD recorder sets forth many specifications regarding electronic and physical characteristics that must be met if the unit is to operate acceptably. **Quality of conformance** is the ability of a process to meet the specifications set forth by the design. **Quality of performance** is how well the product or service actually performs in the marketplace. Companies must find out how well customers' needs are met and how reliable products are by conducting after-sales research.

The marketing research arm of a company must determine what the customer seeks in each of these dimensions. Consumer research is used to develop a product or service concept—a combination of design characteristics that exceeds the expectations of a large number of consumers. This concept is translated into a design. The design includes specifications that, if met, will satisfy consumer wants and needs. A production process is then developed to meet the design specifications. In order to do this, variables that can control the process must be identified, and the relationships between input variables and final quality characteristics must be understood. The manufacturer expresses quality characteristics as measurable variables that can be tracked and used to monitor and improve the performance of the process. Service call analysis often leads to product or service redesigns in order to improve the product or service concept. It is extremely important that the initial design be a good one so that excessive redesigns and customer dissatisfaction can be avoided.

History of the quality movement In the 1700s and 1800s, master craftsmen and their apprentices were responsible for designing and building products. Quantities of goods produced were small, and product quality was controlled by expert workmanship. Master craftsmen had a great deal of pride in their work, and quality was not a problem. However, the introduction of mass production in the late 1800s and early 1900s changed things. Production processes became very complex, with many workers (rather than one skilled craftsman) responsible for the final product. Inevitably, product quality characteristics displayed variation. In particular, Henry Ford developed the moving assembly line at Ford Motor Company. As assembly line manufacturing spread, quality became a problem. Production managers were rewarded for meeting production quotas, and quality suffered. To make mass-produced products more consistent, inspectors were hired to check product quality. However, 100 percent inspection proved to be costly, and people started to look for alternatives.

Much of the early work in quality control was done at Bell Telephone (now known as American Telephone and Telegraph or AT&T). The Bell System and Western Electric, the manufacturing arm of Bell Telephone, formed the Inspection Engineering Department to deal with quality problems. In 1924 Walter Shewhart of Bell Telephone Laboratories introduced the concept of statistical quality control—controlling quality of mass-produced goods. Shewhart believed that variation always exists in manufactured products, and that the variation can be studied, monitored, and controlled using statistics. In particular, Shewhart developed a statistical tool called the **control chart.** Such a chart is a graph that can tell a company when a process needs to be adjusted and when the process should be left alone. In the late 1920s Harold F. Dodge and Harold G. Romig, also of Bell Telephone Laboratories, introduced **statistical acceptance sampling,** a statistical sampling technique that enables a company to accept or reject a quantity of goods (called a **lot**) without inspecting the entire lot. By the mid-1930s, Western Electric was heavily using **statistical quality control (SQC)** to improve quality, increase productivity, and reduce inspection costs. However, these statistical methods were not widely adopted outside Bell Telephone.

During World War II statistical quality control became widespread. Faced with the task of producing large quantities of high-quality war matériel, industry turned to statistical methods, failure analysis, vendor certification, and early product design. The U.S. War Department required that suppliers of war matériel employ acceptance sampling, and its use became commonplace. Statistical control charts were also used, although not as widely as acceptance sampling.

In 1946 the **American Society for Quality Control (ASQC)** was established to encourage the use of quality improvement methods. The organization sponsors training programs, seminars, and publications dealing with quality issues. In spite of the efforts of the ASQC, however, interest in quality in American industry diminished after the war. American business had little competition in the world market—Europe and Japan were rebuilding their shattered economies. Tremendous emphasis was placed on increased production because firms were often unable to meet the demand for their products. Profits were high, and the concern for quality waned. As a result, postwar American managers did not understand the importance of quality and process improvement, and they were not informed about quality improvement techniques.

However, events in Japan took a different turn. After the war, Japanese industrial capacity was crippled. Productivity was very low, and products were of notoriously bad quality. In those days, products stamped “Made in Japan” were generally considered to be “cheap junk.” The man credited with turning this situation around is W. Edwards Deming. Deming, born in 1900, earned a Ph.D. in mathematical physics from Yale University in 1927. He then went to work in a Department of Agriculture–affiliated laboratory. Deming, who had learned statistics while studying physics, applied statistics to experiments conducted at the laboratory. Through this work, Deming was introduced to Walter Shewhart, who explained his theories about using statistical control charts to improve quality and productivity. During World War II, Deming was largely responsible for teaching 35,000 American engineers and technical people how to use statistics to improve the quality of war matériel. After the war, the Allied command sent a group of these engineers to Japan. Their mission was to improve the Japanese communication system. In doing so, the engineers employed the statistical methods they had learned, and Deming's work was brought to the attention of the **Union of Japanese Scientists and Engineers (JUSE).** Deming, who had started his own consulting firm in 1946, was asked by the JUSE to help increase Japanese productivity. In July 1950 Deming traveled to Japan and gave a series of lectures titled "Elementary Principles of the Statistical Control of Quality" to a group of 230 Japanese managers. Deming taught the Japanese how to use statistics to determine how well a system can perform, and taught them how to design process improvements to make the system operate better and more efficiently. He also taught the Japanese that the more quality a producer builds into a product, the less it costs. Realizing the serious nature of their economic crisis, the Japanese adopted Deming's ideas as a philosophy of doing business. Through Deming, the Japanese found that by listening to the wants and needs of consumers and by using statistical methods for process improvement in production, they could export high-quality products to the world market.

Although American business was making only feeble attempts to improve product quality in the 1950s and 1960s, it was able to maintain a dominant competitive position. Many U.S. companies focused more on marketing and financial strategies than on product and production. But the Japanese and other foreign competitors were making inroads. By the 1970s, the quality of many Japanese and European products (for instance, automobiles, television sets, and electronic equipment) became far superior to their American-made counterparts. Also, rising prices made consumers more quality conscious—people expected high quality if they were going to pay high prices. As a result, the market shares of U.S. firms rapidly decreased. Many U.S. firms were severely injured or went out of business.

Meanwhile, Deming continued teaching and preaching quality improvement. While Deming was famous in Japan, he was relatively unknown in the United States until 1980. In June 1980 Deming was featured in an NBC television documentary titled "If Japan Can, Why Can't We?" This program, written and narrated by then–NBC correspondent Lloyd Dobyns, compared Japanese and American industrial productivity and credited Deming for Japan's success. Within days, demand for Deming's consulting services skyrocketed. Deming consulted with many major U.S. firms. Among these firms are The Ford Motor Company, General Motors Corporation, and The Procter & Gamble Company. Ford, for instance, began consulting with Deming in 1981. Donald Petersen, who was Ford's chairman and chief executive officer at the time, became a Deming disciple. By following the Deming philosophy, Ford, which was losing 2 billion dollars yearly in 1980, attempted to create a quality culture. Quality of Ford products was greatly improved, and the company again became profitable. The 1980s saw many U.S. companies adopt a philosophy of continuous improvement of quality and productivity in all areas of their businesses—manufacturing, accounting, sales, finance, personnel, marketing, customer service, maintenance, and so forth. This overall approach of applying quality principles to all company activities is called **total quality management (TQM)** or **total quality control (TQC).** It is becoming an important management strategy in American business. Dr. Deming taught seminars on quality improvement for managers and statisticians until his death on December 20, 1993. Deming's work resulted in widespread changes in both the structure of the world economy and the ways in which American businesses are managed.

The fundamental ideas behind Deming's approach to quality and productivity improvement are contained in his "**14 points.**" These are a set of managerial principles that, if followed, Deming believed would enable a company to improve quality and productivity, reduce costs, and compete effectively in the world market. We briefly summarize the 14 points in Table 15.1 on the next page. For more complete discussions of these points, see Bowerman and O'Connell (1996), Deming (1986), Walton (1986), Scherkenbach (1987), or Gitlow, Gitlow, Oppenheim, and Oppenheim (1989). Deming stressed that implementation of the 14 points requires both changes in management philosophy and the use of statistical methods. In addition, Deming believed that it is necessary to follow all of the points, not just some of them.

In 1988 the first **Malcolm Baldrige National Quality Awards** were presented. These awards, presented by the U.S. Commerce Department, are named for the late Malcolm Baldrige, who was Commerce Secretary during the Reagan administration. The awards were established to promote

TABLE 15.1 W. Edwards Deming's 14 Points

1. **Create constancy of purpose toward improvement of product and service with a plan to become competitive, stay in business, and provide jobs.**
   Devise a plan for the long-term success of the company based on quality improvement.
2. **Adopt a new philosophy.**
   Do not tolerate commonly accepted mistakes, delays, defective materials, and defective workmanship.
3. **Cease dependence on mass inspection.**
   Quality cannot be inspected into a product. It must be built into the product through process improvement.
4. **End the practice of awarding business on the basis of price tag.**
   Do not buy from the lowest bidder without taking the quality of goods purchased into account. Purchasing should be based on lowest total cost (including the cost of bad quality).
5. **Improve constantly and forever the system of production and service to improve quality and productivity, and thus constantly decrease costs.**
   Constantly seek to improve every aspect of the business.
6. **Institute training.**
   Workers should know how to do their jobs and should know how their jobs affect quality and the success of the company.
7. **Institute leadership.**
   The job of management is leadership, not mere supervision. Leadership involves understanding the work that needs to be done and fostering process improvement.
8. **Drive out fear, so that everyone may work more effectively for the company.**
   Workers should not be afraid to express ideas, to ask questions, or to take appropriate action.
9. **Break down organizational barriers.**
   Barriers that damage the company performance (such as competition between staff areas, poor communication, disputes between labor and management, and so on) must be removed so that everyone can work for the good of the company.
10. **Eliminate slogans, exhortations, and arbitrary numerical goals and targets for the workforce that urge the workers to achieve new levels of productivity and quality without providing methods.**
    Slogans and numerical goals (such as production quotas) are counterproductive unless management provides methods for achieving them.
11. **Eliminate work standards and numerical quotas.**
    Work standards and numerical quotas that specify the quantity of goods to be produced while quality is ignored are counterproductive and should be eliminated.
12. **Remove barriers that rob employees of their pride of workmanship.**
    While workers want to do a good job and have pride in their work, bad management practices often rob workers of their pride. Barriers that rob workers of pride (such as inadequate instructions, cheap materials, poor maintenance, and so on) must be removed.
13. **Institute a vigorous program of education and self-improvement.**
    Education and training are necessary for everyone if continuous improvement is to be achieved.
14. **Take action to accomplish the transformation.**
    A management structure that is committed to continuous improvement must be put in place.

Source: W. Edwards Deming, "Deming's 14 Points, condensed version" from *Out of Crisis.* Copyright © MIT Press. Used with permission.

quality awareness, to recognize quality achievements by U.S. companies, and to publicize successful quality strategies. The Malcolm Baldrige National Quality Award Consortium, formed by the ASQC (now known as the ASQ) and the American Productivity and Quality Center, administers the award. The Baldrige award has become one of the most prestigious honors in American business. Annual awards are given in three categories—manufacturing, service, and small business. Winners include companies such as Motorola Inc., Xerox Corporation Business Products and Systems, the Commercial Nuclear Fuel Division of Westinghouse Electric Corporation, Milliken and Company, Cadillac Division, General Motors Corporation, Ritz Carlton Hotels, and AT&T Consumer Communications.

Finally, the 1990s saw the adoption of an international quality standards system called **ISO 9000.** More than 90 countries around the globe have adopted the ISO 9000 series of standards for their companies, as have many multinational corporations (including AT&T, 3M, IBM, Motorola, and DuPont). As a brief introduction to ISO 9000, we quote "Is ISO 9000 for You?" published by CEEM Information Systems:

> **What Is ISO 9000?**
> ISO 9000 is a series of international standards for quality assurance management systems. It establishes the organizational structure and processes for assuring that the production of goods or services meets a consistent and agreed-upon level of quality for a company's customers.
>
> The ISO 9000 series is unique in that it applies to a very wide range of organizations and industries encompassing both the manufacturing and service sectors.

Why Is ISO 9000 Important?

ISO 9000 is important for two reasons. First . . . the discipline imposed by the standard for processes influencing your quality management systems can enhance your company's quality consistency.

Second . . . more and more companies, both here at home and internationally, are requiring their suppliers to be ISO 9000 registered.[1]

Clearly, quality has finally become a crucially important issue in American business. The quality revolution now affects every area in business. But the Japanese continue to mount new challenges. For years, the Japanese have used **designed statistical experiments** to develop new processes, find and remedy process problems, improve product performance, and improve process efficiency. Much of this work is based on the insights of Genichi Taguchi, a Japanese engineer. His methods of experimental design, the so-called **Taguchi methods,** have been heavily used in Japan since the 1960s. Although Taguchi's methodology is controversial in statistical circles, the use of experimental design gives the Japanese a considerable advantage over U.S. competitors because it enables them to design a high level of quality into a product before production begins. Some U.S. manufacturers have begun to use experimental design techniques to design quality into their products. It will be necessary for many more U.S. companies to do so in order to remain competitive in the future—a challenge for the 21st century.

15.2 Statistical Process Control and Causes of Process Variation

LO15-2 Distinguish between common causes and assignable causes of process variation.

Statistical process control **Statistical process control (SPC)** is a systematic method for analyzing process data (quality characteristics) in which we monitor and study the **process variation.** The goal is to stabilize the process and to reduce the amount of process variation. When a process has been stabilized, we say that the process is in *statistical control.* That is, more formally:

A process is in **statistical control** when the process measurements display a constant amount of variation around a constant mean (or *level*).

The ultimate goal of SPC is **continuous process improvement.** While we often use SPC to monitor and improve manufacturing processes, SPC is also commonly used to improve service quality. For instance, we might use SPC to reduce the time it takes to process a loan application, or to improve the accuracy of an order entry system.

Before the widespread use of SPC, quality control was based on an **inspection** approach. Here the product is first made, and then the final product is inspected to eliminate defective items. This is called **action on the output** of the process. The emphasis here is on detecting defective product that has already been produced. This is costly and wasteful because, if defective product is produced, the bad items must be (1) **scrapped,** (2) **reworked or reprocessed** (that is, fixed), or (3) **downgraded** (sold off at a lower price). In fact, the cost of bad quality (scrap, rework, and so on) can be tremendously high. It is not unusual for this cost to be as high as 10 to 30 percent or more of a company's dollar sales.

In contrast to the inspection approach, SPC emphasizes integrating quality improvement into the process. Here the goal is **preventing bad quality by taking appropriate action on the process.** In order to accomplish this goal, we must decide when actions on the process are needed. The focus of much of this chapter is to show how such decisions can be made.

Causes of process variation In order to understand SPC methodology, we must realize that the variations we observe in quality characteristics are caused by different sources. These sources include factors such as equipment (machines or the like), materials, people, methods and procedures, the environment, and so forth. Here we must distinguish between **usual process variation** and **unusual process variation.** Usual process variation results from what we call **common causes of process variation.**

Common causes are sources of variation that have the potential to influence all process observations. That is, these sources of variation are inherent to the current process design.

[1]CEEM Information Services, "Is ISO 9000 for You?" 1993.

Common cause variation can be substantial. For instance, obsolete or poorly maintained equipment, a poorly designed process, and inadequate instructions for workers are examples of common causes that might significantly influence all process output. As an example, suppose that we are filling 16-ounce jars with grape jelly. A 25-year-old, obsolete filler machine might be a common cause of process variation that influences all the jar fills. While (in theory) it might be possible to replace the filler machine with a new model, we might have chosen not to do so, and the obsolete filler causes all the jar fills to exhibit substantial variation.

Common causes also include small influences that would cause slight variation even if all conditions are held as constant as humanly possible. For example, in the jar fill situation, small variations in the speed at which jars move under the filler valves, slight floor vibrations, and small differences between filler valve settings would always influence the jar fills even when conditions are held as constant as possible. Sometimes these small variations are described as being due to "chance."

Together, the important and unimportant common causes of variation determine the **usual process variability.** That is, these causes determine the amount of variation that exists when the process is operating routinely. We can reduce the amount of common cause variation by removing some of the important common causes. **Reducing common cause variation is usually a management responsibility.** For instance, replacing obsolete equipment, redesigning a plant or process, or improving plant maintenance would require management action.

In addition to common cause variation, processes are affected by a different kind of variation called **assignable cause variation** (sometimes also called **special cause** or **specific cause variation).**

Assignable causes are sources of **unusual process variation.** These are intermittent or permanent changes in the process that are not common to all process observations and that may cause important process variation. Assignable causes are usually of short duration, but they can be persistent or recurring conditions.

For example, in the jar filling situation, one of the filler valves may become clogged so that some jars are being substantially underfilled (or perhaps are not filled at all). Or a relief operator might incorrectly set the filler so that all jars are being substantially overfilled for a short period of time. As another example, suppose that a bank wishes to study the length of time customers must wait before being served by a teller. If a customer fills out a banking form incorrectly, this might cause a temporary delay that increases the waiting time for other customers. Notice that **assignable causes** such as these can often be remedied by local supervision—for instance, by a production line foreman, a machine operator, a head bank teller, or the like. **One objective of SPC is to detect and eliminate assignable causes of process variation.** By doing this, we reduce the amount of process variation. This results in improved quality.

It is important to point out that an assignable cause could be beneficial—that is, it could be an unusual process variation resulting in unusually good process performance. In such a situation, we wish to discover the root cause of the variation, and then we wish to incorporate this condition into the process if possible. For instance, suppose we find that a process performs unusually well when a raw material purchased from a particular supplier is used. It might be desirable to purchase as much of the raw material as possible from this supplier.

When a process exhibits only common cause variation, it will operate in a stable, or consistent, fashion. That is, in the absence of any unusual process variations, **the process will display a constant amount of variation around a constant mean.** On the other hand, if assignable causes are affecting the process, then the process will not be stable—unusual variations will cause the process mean or variability to change over time. It follows that

1. When a process is influenced only by **common cause variation, the process will be in statistical control.**
2. When a process is influenced by **one or more assignable causes, the process will not be in statistical control.**

In general, in order to bring a process into statistical control, we must find and eliminate undesirable assignable causes of process variation, and we should (if feasible) build desirable assignable causes into the process. When we have done these things, the process is what we call a **stable, common cause system.** This means that the process operates in a **consistent** fashion and is **predictable.** Because there are no unusual process variations, the process (as currently configured) is doing all it can be expected to do.

When a process is in statistical control, management can evaluate the **process capability.** That is, it can assess whether the process can produce output meeting customer or producer requirements. If it does not, action by local supervision will not remedy the situation—remember, the assignable causes (the sources of process variation that can be dealt with by local supervision) have already been removed. Rather, some fundamental change will be needed in order to reduce common cause variation. For instance, perhaps a new, more modern filler machine must be purchased and installed. This will require action by management.

Finally, the SPC approach is really a philosophy of doing business. It is an entire firm or organization that is focused on a single goal: continuous quality and productivity improvement. The impetus for this philosophy must come from management. Unless management is supportive and directly involved in the ongoing quality improvement process, the SPC approach will not be successful.

Exercises for Sections 15.1 and 15.2

CONCEPTS

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15.1 Write an essay comparing the management philosophy that Dr. Deming advocated in his 14 points to the management styles you have been exposed to in your personal work experiences. Do you think Dr. Deming's philosophy is preferable to the management styles you have seen in practice? Which of the 14 points do you agree with? Which do you disagree with?

15.2 Write a paragraph explaining how common causes of process variation differ from assignable causes of process variation.

METHODS AND APPLICATIONS

15.3 In this exercise we consider several familiar processes. In each case, describe several common causes and several assignable causes that might result in variation of the given quality characteristic.

- **a** Process: getting ready for school or work in the morning.
  Quality characteristic: the time it takes to get ready.
- **b** Process: driving, walking, or otherwise commuting from your home or apartment to school or work.
  Quality characteristic: the time it takes to commute.
- **c** Process: studying for and taking a statistics exam.
  Quality characteristic: the score received on the exam.
- **d** Process: starting your car in the morning.
  Quality characteristic: the time it takes to start your car.

15.4 Form a group of three or four students in your class. As a group project, select a familiar process and determine a variable that measures the quality of some aspect of the output of this process. Then list some common causes and assignable causes that might result in variation of the variable you have selected for the process. Discuss your lists in class.

15.3 Sampling a Process, Rational Subgrouping, and Control Charts

LO15-3 Sample a process by using rational subgrouping.

In order to find and eliminate assignable causes of process variation, we sample output from the process. To do this, we first decide which **process variables**—that is, which process characteristics—will be studied. Several graphical techniques (sometimes called *prestatistical tools*) are used here. Pareto charts (see Section 2.1 on page 38) help identify problem areas and opportunities for improvement. Cause-and-effect diagrams (see optional Section 15.7 on page 15-42) help uncover sources of process variation and potentially important process variables. The goal is to identify process variables that can be studied in order to decrease the gap between customer expectations and process performance.

Whenever possible and economical, it is best to study a **quantitative,** rather than a **categorical,** process variable. For example, suppose we are filling 16-ounce jars with grape jelly, and suppose specifications state that each jar should contain between 15.95 and 16.05 ounces of jelly. If we record the fill of each sampled jar by simply noting that the jar either "meets specifications" (the fill is between 15.95 and 16.05 ounces) or "does not meet the specifications," then we are studying a **categorical process variable.** However, if we measure and record the amount of grape jelly contained in the jar (say, to the nearest one-hundredth of an ounce), then we are studying a **quantitative process variable.** Actually measuring the fill is best because this tells us **how close** we are to the specification limits and thus provides more information. As we will soon see, this additional information often allows us to decide whether to take action on a process by using a relatively small number of measurements.

When we study a quantitative process variable, we say that we are employing **measurement data.** To analyze such data, we take a series of samples (usually called **subgroups**) over time. Each subgroup consists of a set of several measurements; subgroup sizes between 2 and 6 are often used. Summary statistics (for example, means and ranges) for each subgroup are calculated and are plotted versus time. By comparing plot points, we hope to discover when unusual process variations are taking place.

Each subgroup is typically observed over a short period of time—a period of time in which the process operating characteristics do not change much. That is, we employ **rational subgroups.**

Rational Subgroups

Rational subgroups are selected so that, **if process changes of practical importance exist, the chance that these changes will occur between subgroups is maximized and the chance that these changes will occur within subgroups is minimized.**

In order to obtain rational subgroups, we must determine the frequency with which subgroups will be selected. For example, we might select a subgroup once every 15 minutes, once an hour, or once a day. In general, we should observe subgroups often enough to detect important process changes. For instance, suppose we wish to study a process, and suppose we feel that workers' shift changes (that take place every eight hours) may be an important source of process variation. In this case, rational subgroups can be obtained by selecting a subgroup during each eight-hour shift. Here shift changes will occur **between** subgroups. Therefore, if shift changes are an important source of variation, the rational subgroups will enable us to observe the effects of these changes by comparing plot points for different subgroups (shifts). However, in addition, suppose hourly machine resets are made, and we feel that these resets may also be an important source of process variation. In this case, rational subgroups can be obtained by selecting a subgroup during each hour. Here machine resets will occur **between** subgroups, and we will be able to observe their effects by comparing plot points for different subgroups (hours). If in this situation we selected one subgroup each eight-hour shift, we would not obtain rational subgroups. This is because hourly machine resets would occur **within** subgroups, and we would not be able to observe the effects of these resets by comparing plot points for different shifts. In general, it is very important to try to identify important sources of variation (potential assignable causes such as shift changes, resets, and so on) before deciding how subgroups will be selected. As previously stated, constructing a cause-and-effect diagram helps uncover these sources of variation (see optional Section 15.7 on page 15-42).

Once we determine the sampling frequency, we need to determine the **subgroup size**—that is, the number of measurements that will be included in each subgroup—and how we will actually select the measurements in each subgroup. It is recommended that the **subgroup size be held constant.** Denoting this constant subgroup size as n, we typically choose n to be from 2 to 6, with $n = 4$ or 5 being a frequent choice. To illustrate how we can actually select the subgroup measurements, suppose we select a subgroup of 5 units every hour from the output of a machine that produces 100 units per hour. We can select these units by using a **consecutive, periodic,** or **random** sampling process. If we employ consecutive sampling, we would select 5 consecutive units produced by the machine at the beginning of (or at some time during) each hour. Here **production conditions**—machine operator, machine setting, raw material batch, and so forth—**will be as constant as possible within the subgroup.** Such a subgroup provides a "freeze-frame picture" of the process at a particular point in time. Thus the **chance of variations occurring within the subgroups is minimized.** If we use periodic sampling, we would select 5 units periodically through each hour. For example, because the machine produces 100 units per hour, we could select the 1st, 21st, 41st, 61st, and 81st units produced. If we use random sampling, we would use a random number table to randomly select 5 of the 100 units produced during each hour. If production conditions are really held fairly constant during each hour, then consecutive, periodic, and random sampling will each provide a similar representation of the process. If production conditions vary considerably during each hour, and if we are able to recognize this variation by using a periodic or random sampling procedure, this would tell us that we should be sampling the process more often than once an hour. Of course, if we are using periodic or random sampling every hour, we might not realize that the process operates with considerably less variation during shorter periods (perhaps because we have not used a consecutive sampling procedure). We therefore might not recognize the extent of the hourly variation.

Lastly, it is important to point out that we **must also take subgroups for a period of time that is long enough to give potential sources of variation a chance to show up.** If, for instance, different batches of raw materials are suspected to be a significant source of process variation, and if we receive new batches every few days, we may need to collect subgroups for several weeks in order to assess the effects of the batch-to-batch variation. **A statistical rule of thumb says that we require at least 20 subgroups of size 4 or 5 in order to judge statistical control and in order to obtain reasonable estimates of the process mean and variability.** However, practical considerations may require the collection of much more data.

We now look at a more concrete example of subgrouped data.

EXAMPLE 15.1 The Hole Location Case:[2] Subgrouped Process Data

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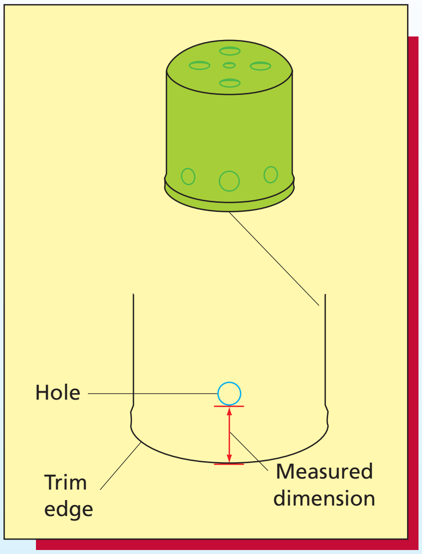
A manufacturer produces automobile air conditioner compressor shells. The compressor shell is basically the outer metal housing of the compressor. Several holes of various sizes must be punched into the shell to accommodate hose connections that must be made to the compressor. If any one of these holes is punched in the wrong location, the compressor shell becomes a piece of scrap metal (at considerable cost to the manufacturer). Figure 15.1(a) illustrates a compressor shell (note the holes that have been punched in the housing). Experience with the hole-punching process suggests that substantial changes (machine resets, equipment lubrication, and so forth)

FIGURE 15.1 The Compressor Shell and the Hole Location Data DS HoleLoc

(a) Holes punched in a compressor shell for hose connections



(b) Twenty subgroups of 5 hole location measurements (measurement from trim edge to the bottom of hole; target value is 3.00 inches)

		Measurement						
Time	Subgroup	1	2	3	4	5	Mean	Range
8:00 AM	1	3.05	3.02	3.04	3.09	3.05	3.05	0.07
8:20 AM	2	3.00	3.04	2.98	2.99	2.99	3.00	0.06
8:40 AM	3	3.07	3.06	2.94	2.97	3.01	3.01	0.13
9:00 AM	4	3.02	2.96	3.01	2.98	3.02	2.998	0.06
9:20 AM	5	3.01	2.98	3.04	3.01	3.01	3.01	0.06
9:40 AM	6	3.01	3.02	2.99	2.97	2.96	2.99	0.06
10:00 AM	7	3.03	2.98	2.92	3.17	2.96	3.012	0.25
10:20 AM	8	3.05	3.03	2.96	3.01	2.97	3.004	0.09
10:40 AM	9	2.99	2.96	3.01	3.00	2.95	2.982	0.06
11:00 AM	10	3.02	3.02	2.98	3.03	3.02	3.014	0.05
11:20 AM	11	2.97	2.96	2.96	3.00	3.04	2.986	0.08
11:40 AM	12	3.06	3.04	3.02	3.10	3.05	3.054	0.08
12:00 PM	13	2.99	3.00	3.04	2.96	3.02	3.002	0.08
12:20 PM	14	3.00	3.01	2.99	3.00	3.01	3.002	0.02
12:40 PM	15	3.02	2.96	3.04	2.95	2.97	2.988	0.09
1:00 PM	16	3.02	3.02	3.04	2.98	3.03	3.018	0.06
1:20 PM	17	3.01	2.87	3.09	3.02	3.00	2.998	0.22
1:40 PM	18	3.05	2.96	3.01	2.97	2.98	2.994	0.09
2:00 PM	19	3.02	2.99	3.00	2.98	3.00	2.998	0.04
2:20 PM	20	3.00	3.00	3.01	3.05	3.01	3.014	0.05

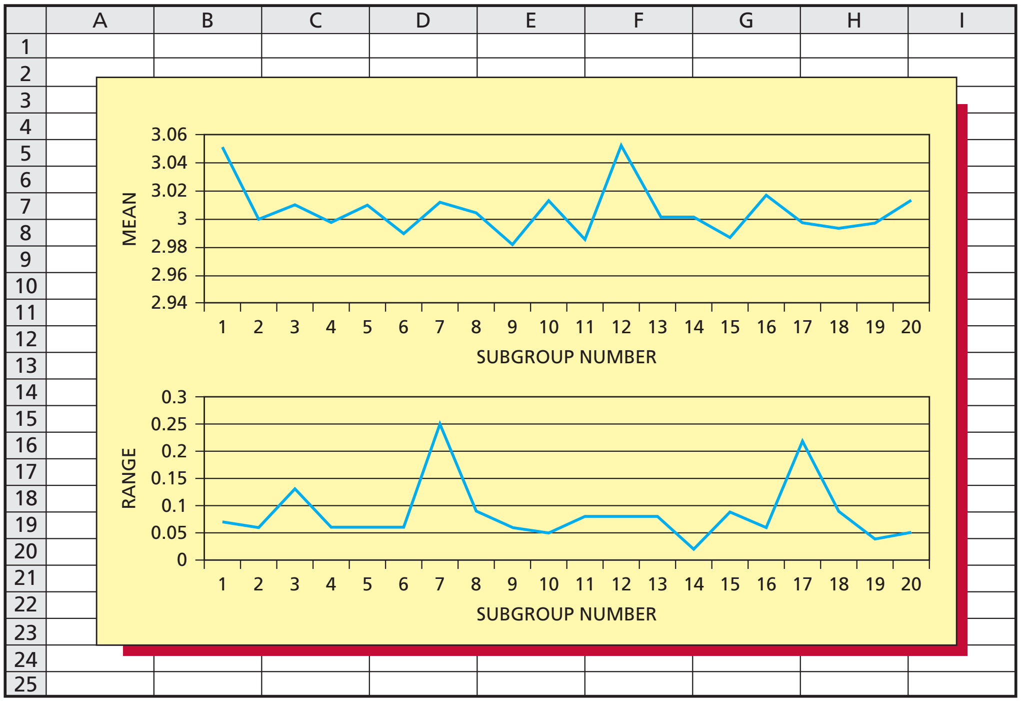
[2] The data for this case were obtained from a metal fabrication plant located in the Cincinnati, Ohio, area. For confidentiality, we have agreed to withhold the company's name.

can occur quite frequently—as often as two or three times an hour. Because we wish to observe the impact of these changes if and when they occur, rational subgroups are obtained by selecting a subgroup every 20 minutes or so. Specifically, about every 20 minutes five compressor shells are consecutively selected from the process output. For each shell selected, a measurement that helps to specify the location of a particular hole in the compressor shell is made. The measurement is taken by measuring from one of the edges of the compressor shell (called the trim edge) to the bottom of the hole [see Figure 15.1(a)]. Obviously, it is not possible to measure to the center of the hole because you cannot tell where it is! The target value for the measured dimension is 3.00 inches. Of course, the manufacturer would like as little variation around the target as possible. Figure 15.1(b) gives the measurements obtained for 20 subgroups that were selected between 8 A.M. and 2:20 P.M. on a particular day. Here a subgroup consists of the five measurements labeled 1 through 5 in a single row in the table. Notice that Figure 15.1(b) also gives the mean, $\bar{x}$, and the range, R, of the measurements in each subgroup. In the next section we will see how to use the subgroup means and ranges to detect when unusual process variations have taken place.

Subgrouped data are used to determine when assignable causes of process variation exist. Typically, we analyze subgrouped data by plotting summary statistics for the subgroups versus time. The resulting plots are often called **graphs of process performance.** For example, the subgroup means and the subgroup ranges of the hole location measurements in Figure 15.1(b) are plotted in time order on graphs of process performance in the Excel output of Figure 15.2. The subgroup means ($\bar{x}$ values) and ranges (R values) are plotted on the vertical axis, while the time sequence (in this case, the subgroup number) is plotted on the horizontal axis. The $\bar{x}$ values and R values for corresponding subgroups are lined up vertically. The plot points on each graph are connected by line segments as a visual aid. However, the lines between the plot points do not really say anything about the process performance between the observed subgroups. Notice that the subgroup means and ranges vary over time.

If we consider the plot of subgroup means, very high and very low points are undesirable—they represent large deviations from the target hole location dimension (3 inches). If we consider the plot of subgroup ranges, very high points are undesirable (high variation in the hole location dimensions), while very low points are desirable (little variation in the hole location dimensions).

FIGURE 15.2 Excel Output of Graphs of Performance (Subgroup Means and Ranges) for the Hole Location Data in Figure 15.1(b)



We now wish to answer a very basic question. **Is the variation that we see on the graphs of performance due to the usual process variation (that is, due to common causes), or is the variation due to one or more assignable causes (unusual variations)?** It is possible that unusual variations have occurred and that action should be taken to reduce the variation in production conditions. It is also possible that the variation in the plot points is caused by common causes and that (given the current configuration of the process) production conditions have been held as constant as possible. For example, do the high points on the $\bar{x}$ plot in Figure 15.2 suggest that one or more assignable causes have increased the hole location dimensions enough to warrant corrective action? As another example, do the high points on the R plot suggest that excess variability in the hole location dimensions exists and that corrective action is needed? Or does the lowest point on the R plot indicate that an improvement in process performance (reduction in variation) has occurred due to an assignable cause?

We can answer these questions by converting the graphs of performance shown in Figure 15.2 into **control charts.** In general, by converting graphs of performance into control charts, we can (with only a small chance of being wrong) determine whether observed process variations are unusual (due to assignable causes). That is, the purpose of a control chart is to monitor a process so we can take corrective action in response to assignable causes when it is needed. This is called **statistical process monitoring.** The use of "seat of the pants intuition" is not a particularly effective way to decide whether observed process performance is unusual. By using a control chart, we can reduce our chances of making two possible errors—(1) taking action when none is needed and (2) not taking action when action is needed.

A control chart employs a **center line** (denoted **CNL**) and two control limits—an **upper control limit** (denoted **UCL**) and a **lower control limit** (denoted **LCL**). The center line represents the average performance of the process when it is in a state of statistical control—that is, when only common cause variation exists. The upper and lower control limits are horizontal lines situated above and below the center line. These control limits are established so that, when the process is in control, almost all plot points will be between the upper and lower limits. In practice, the control limits are used as follows:

1. If all observed plot points are **between the LCL and UCL** (and if no unusual patterns of points exist—this will be explained later), we have no evidence that assignable causes exist and we assume that the process is in **statistical control.** In this case, **only common causes of process variation exist,** and **no action to remove assignable causes is taken on the process.** If we were to take such action, we would be **unnecessarily tampering** with the process.
2. If we observe one or more plot points **outside the control limits,** then we have evidence that the process is **out of control due to one or more assignable causes.** Here **we must take action on the process** to remove these assignable causes.

In the next section we begin to discuss how to construct control charts. Before doing this, however, we must emphasize the importance of **documenting** a process while the subgroups of data are being collected. The time at which each subgroup is taken is recorded, and the person who collected the data is also recorded. Any process changes (machine resets, adjustments, shift changes, operator changes, and so on) must be documented. Any potential sources of variation that may significantly affect the process output should be noted. If the process is not well documented, it will be very difficult to identify the root causes of unusual variations that may be detected when we analyze the subgroups of data.

15.4 $\bar{x}$ and R Charts

LO15-4 Use $\bar{x}$ and R charts to establish process control.

$\bar{x}$ and R charts are the most commonly used control charts for **measurement data** (such charts are often called **variables control charts**). **Subgroup means** are plotted versus time on the $\bar{x}$ chart, while **subgroup ranges** are plotted on the R chart. The $\bar{x}$ chart monitors the **process mean** or **level** (we wish to run near a desired target level). The R chart is used to monitor the amount of **variability** around the process level (we desire as little variability as possible around the target). Note here that we employ two control charts, and that **it is important to use the two charts together.** If we do not use both charts, we will not get all the information needed to improve the process.

Before seeing how to construct $\bar{x}$ and R charts, we should mention that it is also possible to monitor the process variability by using a chart for **subgroup standard deviations.** Such a chart is called an ***s* chart.** However, the overwhelming majority of practitioners use R charts rather than s charts. This is partly due to historical reasons. When control charts were developed, electronic calculators and computers did not exist. It was, therefore, much easier to compute a subgroup range than it was to compute a subgroup standard deviation. For this reason, the use of R charts has persisted. Some people also feel that it is easier for factory personnel (some of whom may have little mathematical background) to understand and relate to the subgroup range. In addition, while the standard deviation (which is computed using all the measurements in a subgroup) is a better measure of variability than the range (which is computed using only two measurements), the R chart usually suffices. This is because $\bar{x}$ and R charts usually employ small subgroups—as mentioned previously, subgroup sizes are often between 2 and 6. For such subgroup sizes, it can be shown that using subgroup ranges is almost as effective as using subgroup standard deviations.

To construct $\bar{x}$ and R charts, suppose we have observed rational subgroups of n measurements over successive time periods (hours, shifts, days, or the like). We first calculate the mean $\bar{x}$ and range R for each subgroup, and we construct graphs of performance for the $\bar{x}$ values and for the R values (as in Figure 15.2). In order to calculate center lines and control limits, let $\bar{x}$ denote the mean of the subgroup of n measurements that is selected in a particular time period. Furthermore, assume that the population of all process measurements that could be observed in any time period is normally distributed with mean μ and standard deviation σ, and also assume successive process measurements are statistically independent.[3] Then, if μ and σ stay constant over time, the sampling distribution of subgroup means in any time period is normally distributed with mean μ and standard deviation $\sigma/\sqrt{n}$. It follows that (in any time period) 99.73 percent of all possible values of the subgroup mean $\bar{x}$ are in the interval

$$[\mu - 3(\sigma/\sqrt{n}),\ \mu + 3(\sigma/\sqrt{n})]$$

This fact is illustrated in Figure 15.3. It follows that we can set a center line and control limits for the $\bar{x}$ chart as

$$\text{Center line} = \mu$$
$$\text{Upper control limit} = \text{UCL}_{\bar{x}} = \mu + 3(\sigma/\sqrt{n})$$
$$\text{Lower control limit} = \text{LCL}_{\bar{x}} = \mu - 3(\sigma/\sqrt{n})$$

If an observed subgroup mean is inside these control limits, we have no evidence to suggest that the process is out of control. However, if the subgroup mean is outside these limits, we conclude that μ and/or σ have changed, and that the process is out of control. The $\bar{x}$ chart limits are illustrated in Figure 15.3.

If the process is in control, and thus μ and σ stay constant over time, it follows that μ and σ are the mean and standard deviation of all possible process measurements. For this reason, we call μ the **process mean** and σ the **process standard deviation.** Because in most real situations we do not know the true values of μ and σ, we must estimate these values. If the process is in control, an appropriate estimate of the process mean μ is

$$\bar{\bar{x}} = \text{the mean of all observed subgroup means}$$

($\bar{\bar{x}}$ is **pronounced "*x* double bar"**). It follows that the center line for the $\bar{x}$ chart is

$$\text{Center line}_{\bar{x}} = \bar{\bar{x}}$$

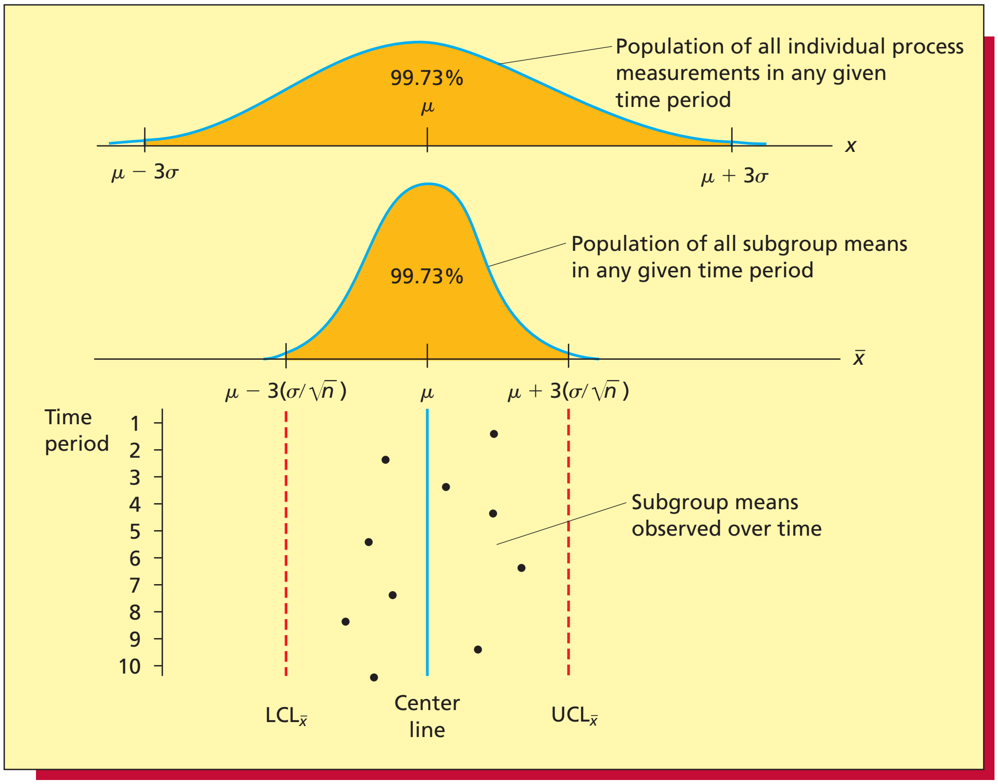
To obtain control limits for the $\bar{x}$ chart, we compute

$$\bar{R} = \text{the mean of all observed subgroup ranges}$$

It can be shown that an appropriate estimate of the process standard deviation σ is $(\bar{R}/d_2)$, where d_2 is a constant that depends on the subgroup size n. Although we do not present a development of d_2 here, it intuitively makes sense that, for a given subgroup size, our best estimate of the process standard deviation should be related to the average of the subgroup ranges ($\bar{R}$). The

[3]Basically, *statistical independence* here means that successive process measurements do not display any kind of pattern over time.

FIGURE 15.3 An Illustration of $\bar{x}$ Chart Control Limits with the Process Mean μ and Process Standard Deviation σ Known



number d_2 relates these quantities. Values of d_2 are given in Table 15.2 (on the next page) for subgroup sizes $n = 2$ through $n = 25$. At the end of this section we further discuss why we use $\bar{R}/d_2$ to estimate the process standard deviation.

Substituting the estimate $\bar{\bar{x}}$ of μ and the estimate $\bar{R}/d_2$ of σ into the limits

$$\mu + 3(\sigma/\sqrt{n}) \qquad \text{and} \qquad \mu - 3(\sigma/\sqrt{n})$$

we obtain

$$UCL_{\bar{x}} = \bar{\bar{x}} + 3\left(\frac{\bar{R}/d_2}{\sqrt{n}}\right) = \bar{\bar{x}} + \left(\frac{3}{d_2\sqrt{n}}\right)\bar{R}$$

$$LCL_{\bar{x}} = \bar{\bar{x}} - 3\left(\frac{\bar{R}/d_2}{\sqrt{n}}\right) = \bar{\bar{x}} - \left(\frac{3}{d_2\sqrt{n}}\right)\bar{R}$$

Finally, we define

$$A_2 = \frac{3}{d_2\sqrt{n}}$$

and rewrite the control limits as

$$UCL_{\bar{x}} = \bar{\bar{x}} + A_2\bar{R} \qquad \text{and} \qquad LCL_{\bar{x}} = \bar{\bar{x}} - A_2\bar{R}$$

Here we call A_2 a **control chart constant.** As the formula for A_2 implies, this control chart constant depends on the subgroup size n. Values of A_2 are given in Table 15.2 for subgroup sizes $n = 2$ through $n = 25$.

The center line for the R chart is

$$\text{Center line}_R = \bar{R}$$

Furthermore, assuming normality, it can be shown that there are control chart constants D_4 and D_3 so that

$$UCL_R = D_4\bar{R} \qquad \text{and} \qquad LCL_R = D_3\bar{R}$$

TABLE 15.2 Control Chart Constants for $\bar{x}$ and R Charts

	Chart for Averages ($\bar{x}$)	Chart for Ranges (R)		
Subgroup Size, n	Factor for Control Limits, A_2	Divisor for Estimate of Standard Deviation, d_2	Factors for Control Limits D_3	D_4
2	1.880	1.128	—	3.267
3	1.023	1.693	—	2.574
4	0.729	2.059	—	2.282
5	0.577	2.326	—	2.114
6	0.483	2.534	—	2.004
7	0.419	2.704	0.076	1.924
8	0.373	2.847	0.136	1.864
9	0.337	2.970	0.184	1.816
10	0.308	3.078	0.223	1.777
11	0.285	3.173	0.256	1.744
12	0.266	3.258	0.283	1.717
13	0.249	3.336	0.307	1.693
14	0.235	3.407	0.328	1.672
15	0.223	3.472	0.347	1.653
16	0.212	3.532	0.363	1.637
17	0.203	3.588	0.378	1.622
18	0.194	3.640	0.391	1.608
19	0.187	3.689	0.403	1.597
20	0.180	3.735	0.415	1.585
21	0.173	3.778	0.425	1.575
22	0.167	3.819	0.434	1.566
23	0.162	3.858	0.443	1.557
24	0.157	3.895	0.451	1.548
25	0.153	3.931	0.459	1.541

Here the control chart constants D_4 and D_3 also depend on the subgroup size n. Values of D_4 and D_3 are given in Table 15.2 for subgroup sizes $n = 2$ through $n = 25$. We summarize the center lines and control limits for $\bar{x}$ and R charts in the following box:

$\bar{x}$ and R Chart Center Lines and Control Limits

Center line$_{\bar{x}} = \bar{\bar{x}}$ Center line$_R = \bar{R}$

$\text{UCL}_{\bar{x}} = \bar{\bar{x}} + A_2\bar{R}$ $\text{UCL}_R = D_4\bar{R}$

$\text{LCL}_{\bar{x}} = \bar{\bar{x}} - A_2\bar{R}$ $\text{LCL}_R = D_3\bar{R}$

where $\bar{\bar{x}}$ = the mean of all subgroup means
$\bar{R}$ = the mean of all subgroup ranges

and A_2, D_4, and D_3 are control chart constants that depend on the subgroup size (see Table 15.2). When D_3 is not listed, the R chart does not have a lower control limit.[4]

EXAMPLE 15.2 The Hole Location Case: Trial Control Limits

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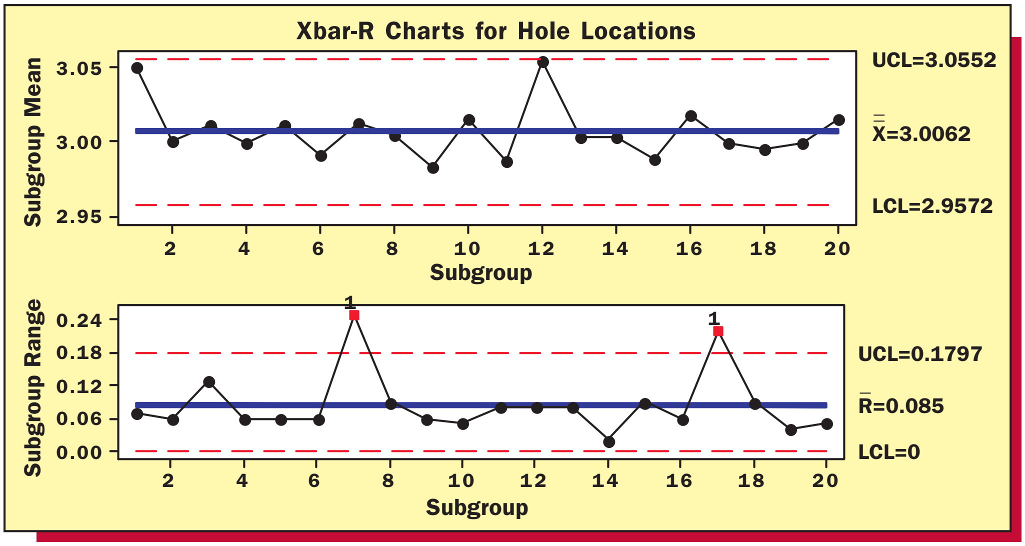
Consider the hole location data for air conditioner compressor shells that is given in Figure 15.1 (page 15-10). In order to calculate $\bar{x}$ and R chart control limits for this data, we compute

$$\bar{\bar{x}} = \text{the average of the 20 subgroup means}$$

$$= \frac{3.05 + 3.00 + \cdots + 3.014}{20} = 3.0062$$

[4]When D_3 is not listed, the theoretical lower control limit for the R chart is negative. In this case, some practitioners prefer to say that the LCL_R equals 0. Others prefer to say that the LCL_R does not exist because a range R equal to 0 does not indicate that an assignable cause exists and because it is impossible to observe a negative range below LCL_R. We prefer the second alternative. In practice, it makes no difference.

FIGURE 15.4 MINITAB Output of $\bar{x}$ and R Charts for the Hole Location Data



$$\bar{R} = \text{the average of the 20 subgroup ranges}$$
$$= \frac{.07 + .06 + \cdots + .05}{20} = 0.085$$

Looking at Table 15.2, we see that when the subgroup size is $n = 5$, the control chart constants needed for $\bar{x}$ and R charts are $A_2 = .577$ and $D_4 = 2.114$. It follows that center lines and control limits are

$$\text{Center line}_{\bar{x}} = \bar{\bar{x}} = 3.0062$$
$$\text{UCL}_{\bar{x}} = \bar{\bar{x}} + A_2\bar{R} = 3.0062 + .577(0.085) = 3.0552$$
$$\text{LCL}_{\bar{x}} = \bar{\bar{x}} - A_2\bar{R} = 3.0062 - .577(0.085) = 2.9572$$

$$\text{Center line}_R = \bar{R} = .085$$
$$\text{UCL}_R = D_4\bar{R} = 2.114(.085) = 0.1797$$

Because D_3 is not listed in Table 15.2 for a subgroup size of $n = 5$, the R chart does not have a lower control limit. Figure 15.4 presents the MINITAB output of the $\bar{x}$ and R charts for the hole location data. Note that the center lines and control limits that we have just calculated are shown on the $\bar{x}$ and R charts.

Control limits such as those computed in Example 15.2 are called **trial control limits.** Theoretically, control limits are supposed to be computed using subgroups collected while the process is in statistical control. However, it is impossible to know whether the process is in control until we have constructed the control charts. If, after we have set up the $\bar{x}$ and R charts, we find that the process is in control, we can use the charts to monitor the process.

If the charts show that the process is not in statistical control (for example, there are plot points outside the control limits), we must find and eliminate the assignable causes before we can calculate control limits for monitoring the process. In order to understand how to find and eliminate assignable causes, we must understand how changes in the process mean and the process variation show up on $\bar{x}$ and R charts. To do this, consider Figures 15.5 and 15.6 on the next page. These figures illustrate that, whereas a change in the process mean shows up only on the $\bar{x}$ chart, a change in the process variation shows up on both the $\bar{x}$ and R charts. Specifically, Figure 15.5 shows that, when the process mean increases, the sample means plotted on the $\bar{x}$ chart increase and go out of control. Figure 15.6 shows that, when the process variation

FIGURE 15.5 A Shift of the Process Mean

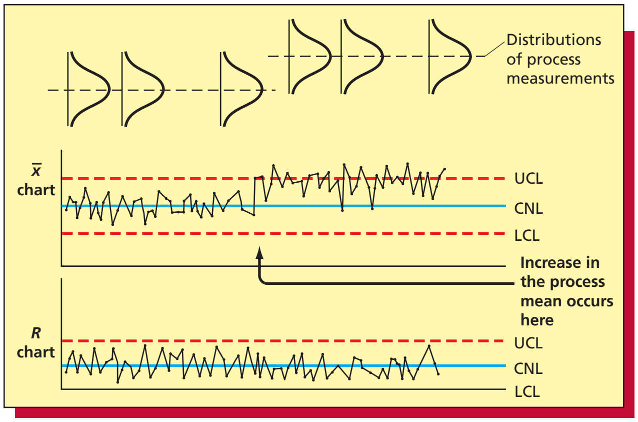
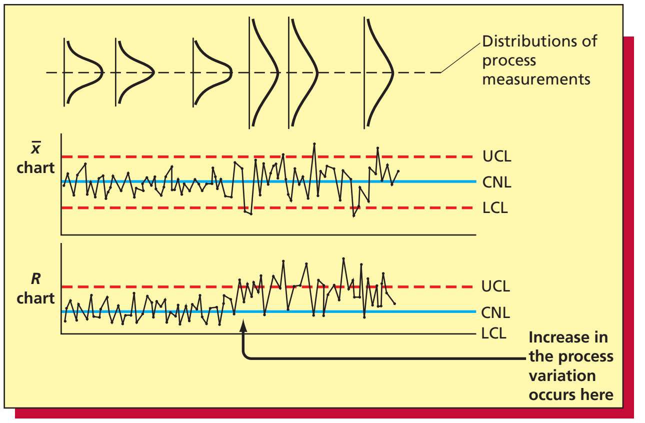


FIGURE 15.6 An Increase in the Process Variation



(standard deviation, σ) increases,

1 The sample ranges plotted on the R chart increase and go out of control.

2 The sample means plotted on the $\bar{x}$ chart become more variable (because, when σ increases, $\sigma_{\bar{x}} = \sigma/\sqrt{n}$ increases) and go out of control.

Because changes in the process mean and in the process variation show up on the $\bar{x}$ chart, we do not begin by analyzing the $\bar{x}$ chart. This is because, if there were out-of-control sample means on the $\bar{x}$ chart, we would not know whether the process mean or the process variation had changed. Therefore, it might be more difficult to identify the assignable causes of the out-of-control sample means because the assignable causes that would cause the process mean to shift could be very different from the assignable causes that would cause the process variation to increase. For instance, unwarranted frequent resetting of a machine might cause the process level to shift up and down, while improper lubrication of the machine might increase the process variation.

In order to simplify and better organize our analysis procedure, we begin by analyzing the R chart, which reflects only changes in the process variation. Then we analyze the $\bar{x}$ chart.

EXAMPLE 15.3 The Hole Location Case: Establishing Statistical Control

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Consider the $\bar{x}$ and R charts for the hole location data that are given in Figure 15.4 on page 15-16. To develop control limits that can be used for ongoing control, we first examine the R chart. We find two points above the UCL on the R chart. This indicates that excess within-subgroup variability exists at these points. We see that the out-of-control points correspond to subgroups 7 and 15. Investigation reveals that, when these subgroups were selected, an inexperienced, newly hired operator ran the operation while the regular operator was on break. We find that the inexperienced operator is not fully closing the clamps that fasten down the compressor shells during the hole punching operation. This is causing excess variability in the hole locations. This assignable cause can be eliminated by thoroughly retraining the newly hired operator.

Because we have identified and corrected the assignable cause associated with the points that are out of control on the R chart, we can drop subgroups 7 and 17 from the data set. We recalculate center lines and control limits by using the remaining 18 subgroups. We first recompute (omitting $\bar{x}$ and R values for subgroups 7 and 17):

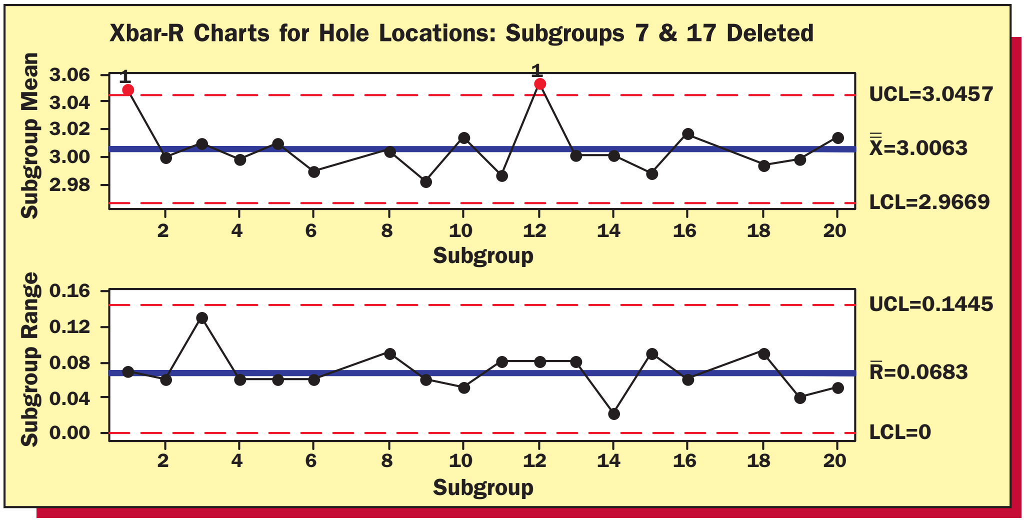
$$\bar{\bar{x}} = \frac{54.114}{18} = 3.0063 \qquad \text{and} \qquad \bar{R} = \frac{1.23}{18} = .0683$$

Notice here that $\bar{\bar{x}}$ has not changed much (see Figure 15.4), but $\bar{R}$ has been reduced from .085 to .0683. Using the new $\bar{\bar{x}}$ and $\bar{R}$ values, revised control limits for the $\bar{x}$ chart are

$$\text{UCL}_{\bar{x}} = \bar{\bar{x}} + A_2\bar{R} = 3.0063 + .577(.0683) = 3.0457$$

$$\text{LCL}_{\bar{x}} = \bar{\bar{x}} - A_2\bar{R} = 3.0063 - .577(.0683) = 2.9669$$

FIGURE 15.7 MINITAB Output of $\bar{x}$ and R Charts for the Hole Location Data: Subgroups 7 and 17 Omitted



The revised UCL for the R chart is

$$\text{UCL}_R = D_4\bar{R} = 2.114(.0683) = .1444$$

Because D_3 is not listed for subgroups of size 5, the R chart does not have a LCL. Here the reduction in $\bar{R}$ has reduced the UCL on the R chart from .1797 to .1444 and has also narrowed the control limits for the $\bar{x}$ chart. For instance, the UCL for the $\bar{x}$ chart has been reduced from 3.0552 to 3.0457. The MINITAB output of the $\bar{x}$ and R charts employing these revised center lines and control limits is shown in Figure 15.7.

We must now check the revised R chart for statistical control. We find that the chart shows good control: there are no other points outside the control limits or long runs of points on either side of the center line. Because the R chart is in good control, we can analyze the revised $\bar{x}$ chart. We see that two plot points are above the UCL on the $\bar{x}$ chart. Notice that these points were not outside our original trial control limits in Figure 15.4 on page 15-16. However, the elimination of the assignable cause and the resulting reduction in $\bar{R}$ has narrowed the $\bar{x}$ chart control limits so that these points are now out of control. Because the R chart is in control, the points on the $\bar{x}$ chart that are out of control suggest that the process level has shifted when subgroups 1 and 12 were taken. Investigation reveals that these subgroups were observed immediately after start-up at the beginning of the day and immediately after start-up following the lunch break. We find that, if we allow a five-minute machine warm-up period, we can eliminate the process level problem.

Because we have again found and eliminated an assignable cause, we must compute newly revised center lines and control limits. Dropping subgroups 1 and 12 from the data set, we recompute

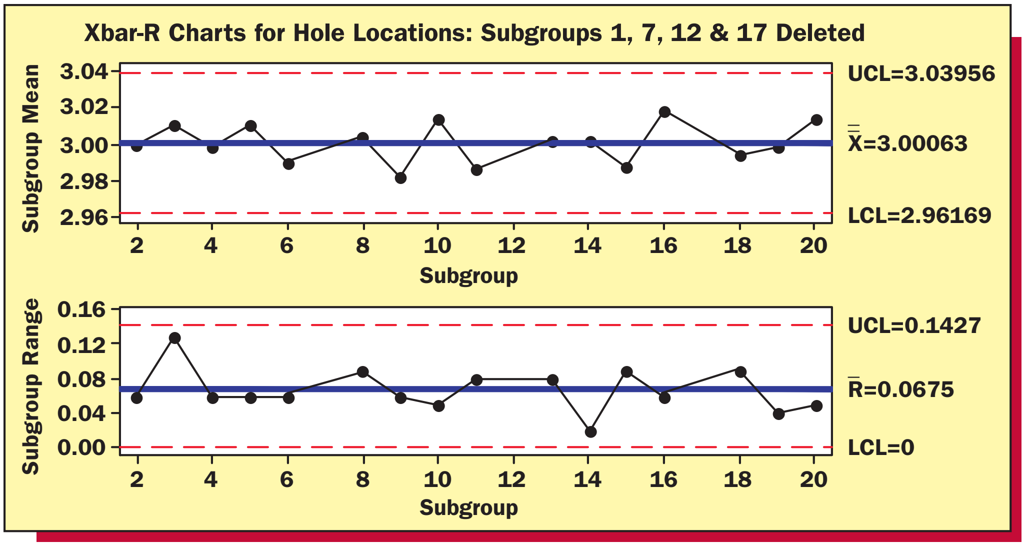
$$\bar{\bar{x}} = \frac{48.01}{16} = 3.0006 \qquad \text{and} \qquad \bar{R} = \frac{1.08}{16} = .0675$$

Using the newest $\bar{\bar{x}}$ and $\bar{R}$ values, we compute newly revised control limits as follows:

$$\text{UCL}_{\bar{x}} = \bar{\bar{x}} + A_2\bar{R} = 3.0006 + .577(.0675) = 3.0396$$
$$\text{LCL}_{\bar{x}} = \bar{\bar{x}} - A_2\bar{R} = 3.0006 - .577(.0675) = 2.9617$$
$$\text{UCL}_R = D_4\bar{R} = 2.114(.0675) = .1427$$

Again, the R chart does not have a LCL. We obtain the newly revised $\bar{x}$ and R charts that are shown in the MINITAB output of Figure 15.8 on the next page. We see that all the points on each chart are inside their respective control limits. This says that the actions taken to remove assignable causes have brought the process into statistical control. However, it is important to point out that, although the process is in statistical control, **this does not necessarily mean that the**

FIGURE 15.8 MINITAB Output of $\bar{x}$ and R Charts for the Hole Location Data: Subgroups 1, 7, 12, and 17 Omitted. The Charts Show Good Control.



process is capable of producing products that meet the customer's needs. That is, while the control charts tell us that no assignable causes of process variation remain, the charts do not (directly) tell us anything about how much common cause variation exists. If there is too much common cause variability, the process will not meet customer or manufacturer specifications. We will discuss this further in Section 15.5.

When both the $\bar{x}$ and R charts are in statistical control, we can use the control limits for ongoing process monitoring. New $\bar{x}$ and R values for subsequent subgroups are plotted with respect to these limits. We summarize analyzing $\bar{x}$ and R charts as follows:

Analyzing $\bar{x}$ and R Charts to Establish Process Control

1 Remember that it is important to use both the $\bar{x}$ chart and the R chart to study the process.

2 Begin by analyzing the R chart for statistical control.

- a Find and eliminate assignable causes that are indicated by the R chart.
- b Revise both the $\bar{x}$ and R chart control limits, dropping data for subgroups corresponding to assignable causes that have been found and eliminated in 2*a*.
- c Check the revised R chart for control.
- d Repeat 2*a*, *b*, and *c* as necessary until the R chart shows statistical control.

3 When the R chart is in statistical control, the $\bar{x}$ chart can be properly analyzed.

- a Find and eliminate assignable causes that are indicated by the $\bar{x}$ chart.
- b Revise both the $\bar{x}$ and R chart control limits, dropping data for subgroups corresponding to assignable causes that have been found and eliminated in 3*a*.
- c Check the revised $\bar{x}$ chart (and the revised R chart) for control.
- d Repeat 3*a*, *b*, and *c* (or, if necessary, 2*a*, *b*, and *c* and 3*a*, *b*, and *c*) as needed until both the $\bar{x}$ and R charts show statistical control.

4 When both the $\bar{x}$ and R charts are in control, use the control limits for process monitoring.

- a Plot $\bar{x}$ and R points for newly observed subgroups with respect to the established limits.
- b If either the $\bar{x}$ chart or the R chart indicates a lack of control, take corrective action on the process.

5 Periodically update the $\bar{x}$ and R chart control limits using all relevant data (data that describe the process as it now operates).

6 When a major process change is made, develop new control limits if necessary.

With respect to periodically updating the $\bar{x}$ and R chart control limits in ongoing process monitoring, note that employees often seem to be uncomfortable working with control limits that are frequently changing. Therefore, it is probably a good idea to update the control limits only when the new data would substantially change the limits. Of course, if an important process change is implemented, new data must be collected, and we may need to develop new control limits from scratch.

EXAMPLE 15.4 The Hole Location Case: Process Monitoring

C

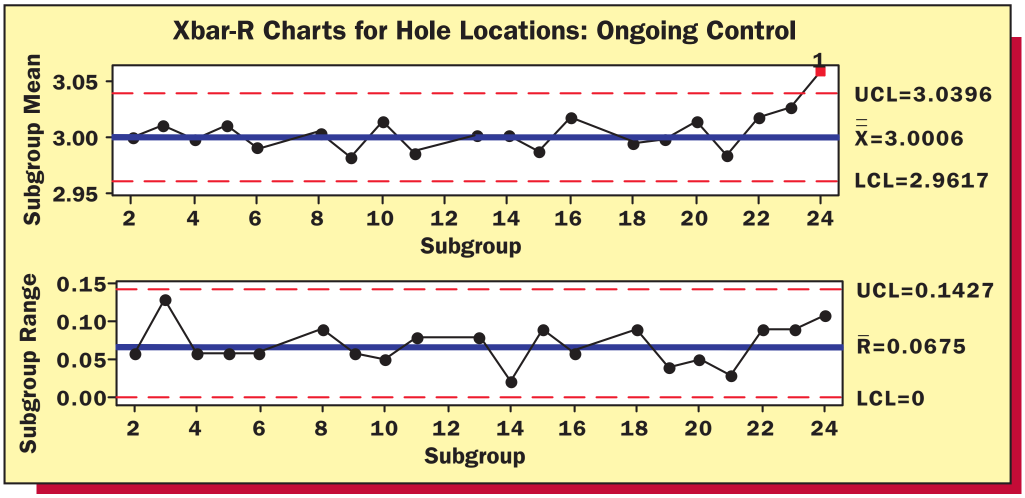
We consider the hole location problem and the revised $\bar{x}$ and R charts shown in Figure 15.8. Because the process has been brought into statistical control, we may use the control limits in Figure 15.8 to monitor the process. This would assume that we have used an appropriate subgrouping scheme and have observed enough subgroups to give potential assignable causes a chance to show up. In reality, we probably want to collect considerably more than 20 subgroups before setting control limits for ongoing control of the process.

We assume for this example that the control limits in Figure 15.8 are reasonable. Table 15.3 gives four subsequently observed subgroups of five hole location dimensions. The subgroup means and ranges for these data are plotted with respect to the ongoing control limits in the MINITAB output of Figure 15.9. We see that the R chart remains in control, while the mean for subgroup 24 is above the UCL on the $\bar{x}$ chart. This tells us that an assignable cause has increased the process mean. Therefore, action is needed to reduce the process mean.

TABLE 15.3 Four Subgroups of Five Hole Location Dimensions Observed after Developing Control Limits for Ongoing Process Monitoring

	Measurement (Inches)					Mean,	Range,
Subgroup	1	2	3	4	5	$\bar{x}$	R
21	2.98	3.00	2.97	2.99	2.98	2.984	.03
22	3.02	3.06	3.01	2.97	3.03	3.018	.09
23	3.03	3.08	3.01	2.99	3.02	3.026	.09
24	3.05	3.00	3.11	3.07	3.06	3.058	.11

FIGURE 15.9 MINITAB Output of $\bar{x}$ and R Charts for the Hole Location Data: Ongoing Control



Having seen how to interpret $\bar{x}$ and R charts, we are now better prepared to understand why we estimate the process standard deviation σ by $\bar{R}/d_2$. Recall that when μ and σ are known, the $\bar{x}$ chart control limits are $[\mu \pm 3(\sigma/\sqrt{n})]$. The standard deviation σ in these limits is the process standard deviation **when the process is in control.** When this standard deviation is unknown, **we estimate σ as if the process is in control, even though the process might not be in control.** The quantity $\bar{R}/d_2$ is an appropriate estimate of σ because **$\bar{R}$ is the average of individual ranges computed from rational subgroups—subgroups selected so that the chances that important process changes occur within a subgroup are minimized.** Thus each subgroup range, and therefore $\bar{R}/d_2$, estimates the process variation as if the process were in control. Of course, we could also compute the standard deviation of the measurements in each subgroup, and employ the average of the subgroup standard deviations to estimate σ. The key is not whether we use ranges or standard deviations to measure the variation within the subgroups. Rather, the key is that we must calculate a measure of variation for each subgroup and then must average the separate measures of subgroup variation in order to estimate the process variation as if the process is in control.

LO15-5 Detect the presence of assignable causes through pattern analysis.

Pattern analysis When we observe a plot point outside the control limits on a control chart, we have strong evidence that an assignable cause exists. In addition, several other data patterns indicate the presence of assignable causes. Precise description of these patterns is often made easier by **dividing the control band into zones—designated *A*, *B*, and *C*.** Zone boundaries are set at points that are one and two standard deviations (of the plotted statistic) on either side of the center line. We obtain six zones—each zone being one standard deviation wide—with three zones on each side of the center line. The zones that stretch one standard deviation above and below the center line are designated as ***C* zones.** The zones that extend from one to two standard deviations away from the center line are designated as ***B* zones.** The zones that extend from two to three standard deviations away from the center line are designated as ***A* zones.** Figure 15.10 illustrates a control chart with the six zones and shows how the zone boundaries for an $\bar{x}$ chart and an R chart are calculated. This figure also shows the values of the zone boundaries for the hole location $\bar{x}$ and R charts shown in Figure 15.8 (page 15-19). In calculating these boundaries, we use $\bar{\bar{x}} = 3.0006$ and $\bar{R} = .0675$, which we computed from subgroups 1 through 20 with subgroups 1, 7, 12, and 17 removed from the data set; that is, we are using $\bar{\bar{x}}$ and $\bar{R}$ when the process is in control. For example, the upper A–B boundary for the $\bar{x}$ chart has been calculated as follows:

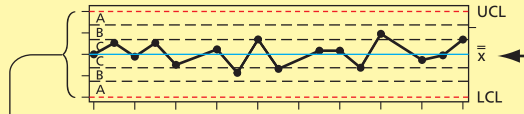
$$\bar{\bar{x}} + \frac{2}{3}(A_2\bar{R}) = 3.0006 + \frac{2}{3}(.577(.0675)) = 3.0266$$

Finally, Figure 15.10 shows (based on a normal distribution of plot points) the percentages of points that we would expect to observe in each zone when the process is in statistical control. For instance, we would expect to observe 34.13 percent of the plot points in the upper portion of zone C.

For an $\bar{x}$ chart, if the distribution of process measurements is reasonably normal, then the distribution of subgroup means will be approximately normal, and the percentages shown in Figure 15.10 apply. That is, the plotted subgroup means for an "in control" $\bar{x}$ chart should look as if they have been randomly selected from a normal distribution. Any distribution of plot points that looks very different from the expected percentages will suggest the existence of an assignable cause.

Several companies (for example, Western Electric [AT&T] and Ford Motor Company) have established sets of rules for identifying assignable causes; use of such rules is called **pattern analysis.** We summarize some commonly accepted rules on the next page. Note that many of these rules are illustrated in Figures 15.11, 15.12, and 15.13, which show several common out-of-control patterns. Also note that it is tempting to use many rules to decide when an assignable cause exists. However, if we use too many rules, we can end up with an unacceptably high chance of a **false out-of-control signal** (that is, an out-of-control signal when there is no assignable cause present). For most control charts, the use of the rules described on the next page will yield an overall probability of a false signal in the range of 1 to 2 percent.

FIGURE 15.10 Zone Boundaries



	Zone Boundaries	$\bar{x}$ Chart	R Chart
(.135%)	Upper Control Limit:	$\bar{\bar{x}} + A_2\bar{R} = 3.0396$	$D_4\bar{R} = .1427$
Zone *A* (2.145%)	Upper *A–B* Boundary:	$\bar{\bar{x}} + \frac{2}{3}(A_2\bar{R}) = 3.0266$	$\bar{R} + \frac{2}{3}(D_4\bar{R} - \bar{R}) = .1176$
Zone *B* (13.59%)	Upper *B–C* Boundary:	$\bar{\bar{x}} + \frac{1}{3}(A_2\bar{R}) = 3.0136$	$\bar{R} + \frac{1}{3}(D_4\bar{R} - \bar{R}) = .0926$
Zone *C* (34.13%)	Center Line:	$\bar{\bar{x}} = 3.0006$	$\bar{R} = .0675$
Zone *C* (34.13%)	Lower *B–C* Boundary:	$\bar{\bar{x}} - \frac{1}{3}(A_2\bar{R}) = 2.9876$	$\bar{R} - \frac{1}{3}(D_4\bar{R} - \bar{R}) = .0424$
Zone *B* (13.59%)	Lower *A–B* Boundary:	$\bar{\bar{x}} - \frac{2}{3}(A_2\bar{R}) = 2.9746$	$\bar{R} - \frac{2}{3}(D_4\bar{R} - \bar{R}) = .0174$
Zone *A* (2.145%)	Lower Control Limit:	$\bar{\bar{x}} - A_2\bar{R} = 2.9617$	$D_3\bar{R}$ = does not exist
(.135%)			

Note: Use the lower *B–C* and *A–B* bounderies if they are 0 or positive.

FIGURE 15.11
Two Out of Three Points in Zone *A*

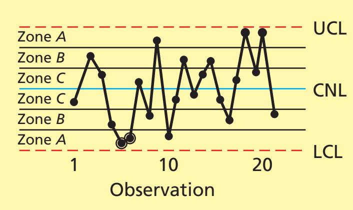
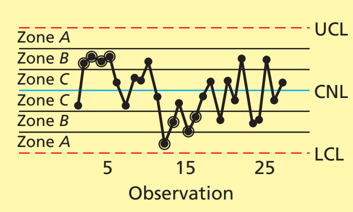


FIGURE 15.12
Four Out of Five Points in Zone *B*



Source of Figures 15.11 and 15.12: H. Gitlow, S. Gitlow, A. Oppenheim, and R. Oppenheim, *Tools and Methods for the Improvement of Quality*, pp. 191–93, 209–211. Copyright © 1989. Reprinted by permission of McGraw-Hill Companies, Inc.

Pattern Analysis for $\bar{x}$ and R Charts

If one or more of the following conditions exist, it is reasonable to conclude that one or more assignable causes are present:

1 One plot point beyond zone *A* (that is, beyond the three standard deviation control limits).

2 Two out of three consecutive plot points in zone *A* (or beyond) on one side of the center line of the control chart. Sometimes a zone boundary that separates zones *A* and *B* is called a **two standard deviation warning limit.** Figure 15.11 illustrates this pattern. Specifically, note that plot points 5 and 6 are two consecutive plot points in zone *A* and that plot points 19 and 21 are two out of three consecutive plot points in zone *A*.

3 Four out of five consecutive plot points in zone *B* (or beyond) on one side of the center line of the control chart. Figure 15.12 illustrates this pattern. Specifically, note that plot points 2, 3, 4, and 5 are four consecutive plot points in zone *B* and that plot points 12, 13, 15, and 16 are four out of five consecutive plot points in zone *B* (or beyond).

4 **A run of at least eight plot points.** Here we define a **run** to be **a sequence of plot points of the same type.** For example, we can have a run of points on one side of (above or below) the center line. Such a run is illustrated in part (a) of Figure 15.13 on the next page, which shows a run above the center line. We might also observe a run of steadily increasing plot points (a **run up**) or a run of steadily decreasing plot points (a **run down**). These patterns are illustrated in parts (b) and (c) of Figure 15.13. Any of the above types of runs consisting of at least eight points is an out-of-control signal.

5 A **nonrandom pattern** of plot points. Such a pattern might be an **increasing** or **decreasing trend, a fanning-out or funneling-in pattern, a cycle, an alternating pattern,** or any other pattern that is very inconsistent with the percentages given in Figure 15.10 (see parts (d) through (h) of Figure 15.13).

If none of the patterns or conditions in 1 through 5 exists, then the process is **in control.**

FIGURE 15.13 Other Out-of-Control Patterns

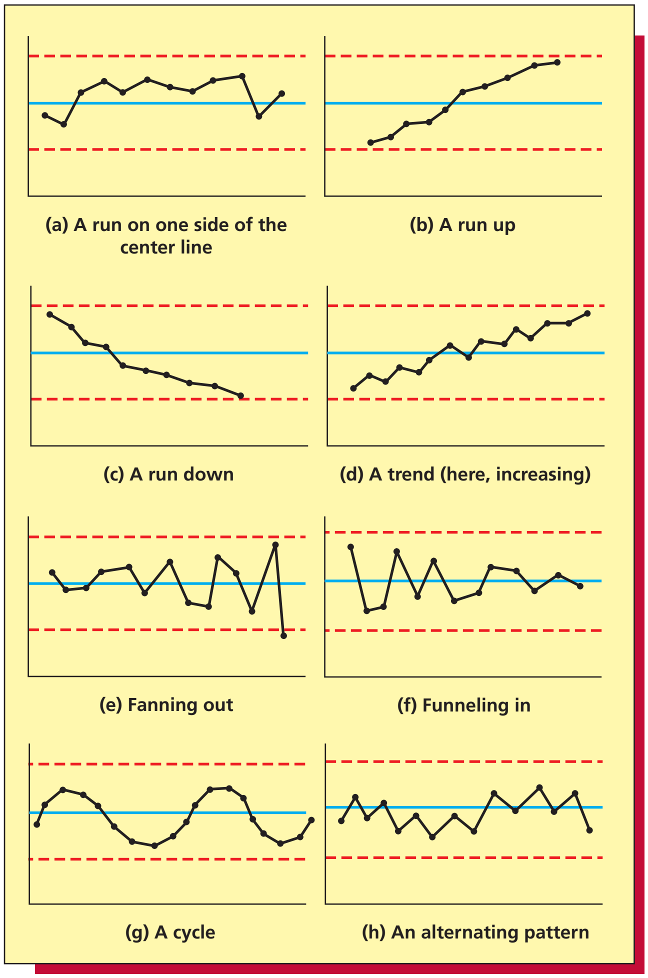
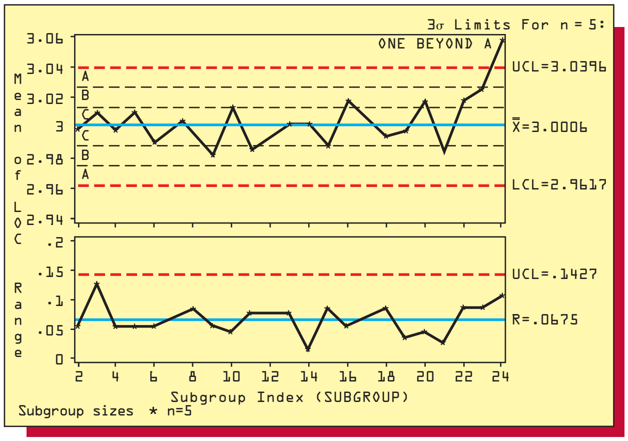


FIGURE 15.14 Ongoing $\bar{x}$ and *R* Charts for the Hole Location Data—Zones *A*, *B*, and *C* Included



As a specific example, Figure 15.14 shows ongoing $\bar{x}$ and *R* charts for the hole location problem. Here the $\bar{x}$ chart includes zone boundaries with zones *A*, *B*, and *C* labeled. Notice that the first out-of-control condition (one plot point beyond zone *A*) exists. Looking at the last five plot points on the $\bar{x}$ chart, we see that the third out-of-control condition (four out of five consecutive plot points in zone *B* or beyond) also exists.

Exercises for Sections 15.3 and 15.4

connect

CONCEPTS

15.5 Explain (1) the purpose of an $\bar{x}$ chart, (2) the purpose of an *R* chart, (3) why both charts are needed.

15.6 Explain why the initial control limits calculated for a set of subgrouped data are called "trial control limits."

15.7 Explain why a change in process variability shows up on both the $\bar{x}$ and *R* charts.

METHODS AND APPLICATIONS

15.8 A pizza restaurant monitors the size (measured by the diameter) of the 10-inch pizzas that it prepares. Pizza crusts are made from doughs that are prepared and prepackaged in boxes of 15 by a supplier. Doughs are thawed and pressed in a pressing machine. The toppings are added, and the pizzas are baked. The wetness of the doughs varies from box to box, and if the dough is too wet or greasy, it is difficult to press, resulting in a crust that is too small. The first shift of workers begins work at 4 P.M., and a new shift takes over at 9 P.M. and works until closing. The pressing machine

TABLE 15.4 10 Samples of Pizza Crust Diameters DS PizzaDiam

Subgroup	Time	Pizza Crust Diameter (Inches) 1	2	3	4	5	Mean, $\bar{x}$	Range, R
1	4 P.M.	9.8	9.0	9.0	9.2	9.2	9.24	0.8
2	5 P.M.	9.5	10.3	10.2	10.0	10.0	10.00	0.8
3	6 P.M.	10.5	10.3	9.8	10.0	10.3	10.18	0.7
4	7 P.M.	10.7	9.5	9.8	10.0	10.0	10.00	1.2
5	8 P.M.	10.0	10.5	10.0	10.5	10.5	10.30	0.5
6	9 P.M.	10.0	9.0	9.0	9.2	9.3	9.30	1.0
7	10 P.M.	11.0	10.0	10.3	10.3	10.0	10.32	1.0
8	11 P.M.	10.0	10.2	10.1	10.3	11.0	10.32	1.0
9	12 A.M.	10.0	10.4	10.4	10.5	10.0	10.26	0.5
10	1 A.M.	11.0	10.5	10.1	10.2	10.2	10.40	0.9

New shift at 9 P.M., pressing machine adjusted at the start of each shift (4 P.M. and 9 P.M.).

is readjusted at the beginning of each shift. The restaurant takes five consecutive pizzas prepared at the beginning of each hour from opening to closing on a particular day. The diameter of each baked pizza in the subgroups is measured, and the pizza crust diameters obtained are given in Table 15.4. Use the pizza crust diameter data to do the following: DS PizzaDiam

a Show that $\bar{\bar{x}} = 10.032$ and $\bar{R} = .84$.

b Find the center lines and control limits for the $\bar{x}$ and R charts for the pizza crust data.

c Set up the $\bar{x}$ and R charts for the pizza crust data.

d Is the R chart for the pizza crust data in statistical control? Explain.

e Is the $\bar{x}$ chart for the pizza crust data in statistical control? If not, use the $\bar{x}$ chart and the information given with the data to try to identify any assignable causes that might exist.

f Suppose that, based on the $\bar{x}$ chart, the manager of the restaurant decides that the employees do not know how to properly adjust the dough pressing machine. Because of this, the manager thoroughly trains the employees in the use of this equipment. Because an assignable cause (incorrect adjustment of the pressing machine) has been found and eliminated, we can remove the subgroups affected by this unusual process variation from the data set. We therefore drop subgroups 1 and 6 from the data. Use the remaining eight subgroups to show that we obtain revised center lines of $\bar{\bar{x}} = 10.2225$ and $\bar{R} = .825$.

g Use the revised values of $\bar{\bar{x}}$ and $\bar{R}$ to compute revised $\bar{x}$ and R chart control limits for the pizza crust diameter data. Set up $\bar{x}$ and R charts using these revised limits. Be sure to omit subgroup means and ranges for subgroups 1 and 6 when setting up these charts.

h Has removing the assignable cause brought the process into statistical control? Explain.

15.9 THE HOT CHOCOLATE TEMPERATURE CASE[5]

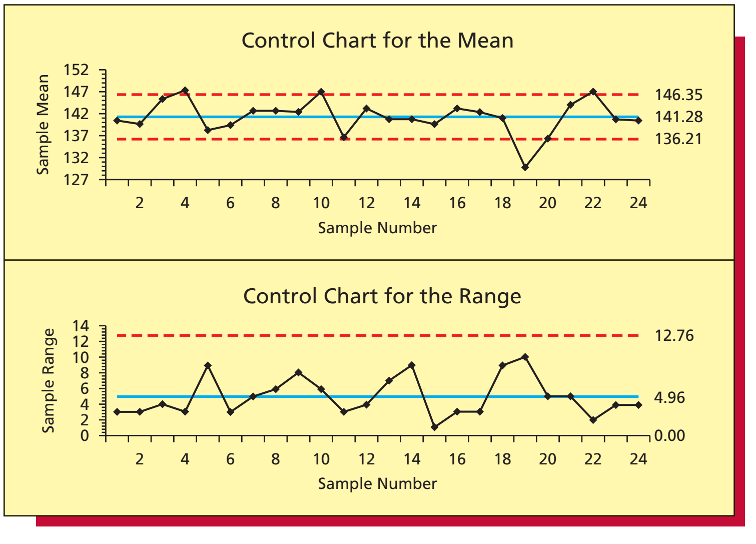
Since 1994 a number of consumers have filed and won large claims against national fast-food chains as a result of being scalded by excessively hot beverages such as coffee, tea, and hot chocolate. Because of such litigation, the food service staff at a university dining hall wishes to study the temperature of the hot chocolate dispensed by its hot chocolate machine. The dining hall staff believes that there might be substantial variations in hot chocolate temperatures from meal to meal. Therefore, it is decided that at least one subgroup of hot chocolate temperatures will be observed during each meal—breakfast (6:30 A.M. to 10 A.M.), lunch (11 A.M. to 1:30 P.M.), and dinner (5 P.M. to 7:30 P.M.). In addition, because the hot chocolate machine is heavily used during most meals, the dining hall staff also believes that hot chocolate temperatures might vary substantially from the beginning to the end of a single meal. It follows that the staff will obtain rational subgroups by selecting a subgroup a half hour after the beginning of each meal and by selecting another subgroup a half hour prior to the end of each meal. Specifically, each subgroup will be selected by pouring three cups of hot chocolate over a 10-minute time span using periodic sampling (the second cup will be poured 5 minutes after the first, and the third cup will be poured 5 minutes after the second). The temperature of the hot chocolate will be measured by a candy thermometer (to the nearest degree Fahrenheit) immediately after each cup is poured. Table 15.5 on the next page gives the results for 24 subgroups of three hot chocolate temperatures taken at each meal served at the dining hall over a four-day period. Here a subgroup consists of the three temperatures labeled 1 through 3 in a single

[5]The data for this case were collected for a student's term project with the cooperation of the Food Service at Miami University, Oxford, Ohio.

TABLE 15.5 24 Subgroups of Three Hot Chocolate Temperatures (Measurements to the Nearest Degree Fahrenheit) DS HotChoc

Day	Meal	Subgroup	Temperature 1	2	3	Subgroup Mean, $\bar{x}$	Subgroup Range, R
Monday	Breakfast	1	142°	140°	139°	140.33°	3°
		2	141	138	140	139.67	3
	Lunch	3	143	146	147	145.33	4
		4	146	149	147	147.33	3
	Dinner	5	133	142	140	138.33	9
		6	138	139	141	139.33	3
Tuesday	Breakfast	7	145	143	140	142.67	5
		8	139	144	145	142.67	6
	Lunch	9	139	141	147	142.33	8
		10	150	144	147	147.00	6
	Dinner	11	138	135	137	136.67	3
		12	145	141	144	143.33	4
Wednesday	Breakfast	13	138	145	139	140.67	7
		14	145	136	141	140.67	9
	Lunch	15	140	139	140	139.67	1
		16	142	143	145	143.33	3
	Dinner	17	144	142	141	142.33	3
		18	137	140	146	141.00	9
Thursday	Breakfast	19	125	129	135	129.67	10
		20	134	139	136	136.33	5
	Lunch	21	145	141	146	144.00	5
		22	147	146	148	147.00	2
	Dinner	23	140	143	139	140.67	4
		24	139	139	143	140.33	4

FIGURE 15.15 Excel add-in (MegaStat) Output of $\bar{x}$ and R Charts for the Hot Chocolate Temperature Data



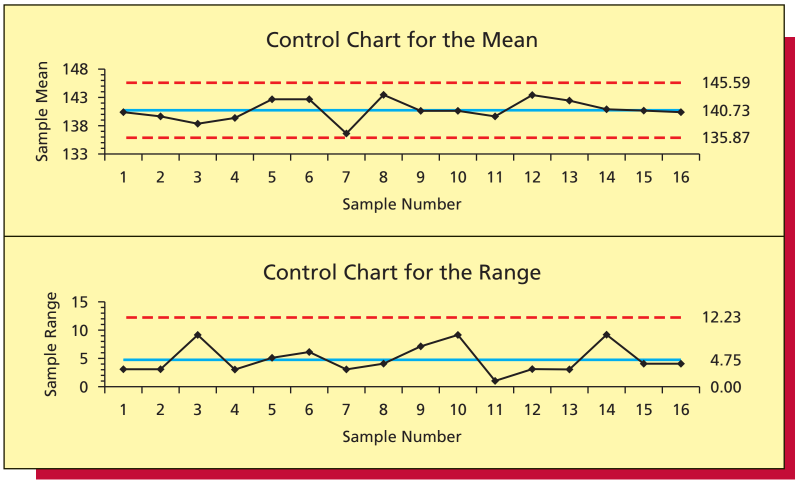
row in the table. The $\bar{x}$ and R charts for these data are given in the Excel add-in (MegaStat) output in Figure 15.15. DS HotChoc

a Using the fact that $\bar{\bar{x}} = 141.28$ and $\bar{R} = 4.96$, compute the control limits for the $\bar{x}$ and R charts and verify that they are as shown in Figure 15.15.

b Is the R chart in statistical control? Explain.

c Is the $\bar{x}$ chart in statistical control? If not, use the $\bar{x}$ chart to identify any out-of-control points.

FIGURE 15.16 **Excel Add-in (MegaStat) Output of Revised $\bar{x}$ and R Charts for the Hot Chocolate Temperature Data. The Process Is Now in Control.**



d Looking at the $\bar{x}$ chart, we see that the subgroup means that are above the UCL were observed during lunch (note subgroups 4, 10, and 22). Investigation and process documentation reveal that on these days the hot chocolate machine was not turned off between breakfast and lunch. Discussion among members of the dining hall staff further reveals that, because there is less time between breakfast and lunch than there is between lunch and dinner or dinner and breakfast, the staff often fails to turn off the hot chocolate machine between breakfast and lunch. Apparently, this is the reason behind the higher hot chocolate temperatures observed during lunch. Investigation also shows that the dining hall staff failed to turn on the hot chocolate machine before breakfast on Thursday (see subgroup 19)—in fact, a student had to ask that the machine be turned on. This caused the subgroup mean for subgroup 19 to be far below the $\bar{x}$ chart LCL. The dining hall staff concludes that the hot chocolate machine needs to be turned off after breakfast and then turned back on 15 minutes before lunch (prior experience suggests that it takes the machine 15 minutes to warm up). The staff also concludes that the machine should be turned on 15 minutes before each meal. In order to ensure that these actions are taken, an automatic timer is purchased to turn on the hot chocolate machine at the appropriate times. This brings the process into statistical control. Figure 15.16 shows $\bar{x}$ and R charts with revised control limits calculated using the subgroups that remain after the subgroups for the out-of-control lunches (subgroups 3, 4, 9, 10, 21, and 22) and the out-of-control breakfast (subgroups 19 and 20) are eliminated from the data set. Are these revised control charts in statistical control? Explain.

15.10 A company packages a bulk product in bags with a 50-pound label weight. During a typical day's operation of the fill process, 22 subgroups of five bag fills are observed. Using the observed data, $\bar{\bar{x}}$ and $\bar{R}$ are calculated to be 52.9364 pounds and 1.6818 pounds, respectively. When the 22 $\bar{x}$'s and 22 R's are plotted with respect to the appropriate control limits, the first 6 subgroups are found to be out of control. This is traced to a mechanical start-up problem, which is remedied. Using the remaining 16 subgroups, $\bar{\bar{x}}$ and $\bar{R}$ are calculated to be 52.5875 pounds and 1.2937 pounds, respectively.

a Calculate appropriate revised $\bar{x}$ and R chart control limits.

b When the remaining 16 $\bar{x}$'s and 16 R's are plotted with respect to the appropriate revised control limits, they are found to be within these limits. What does this imply?

15.11 In the book *Tools and Methods for the Improvement of Quality,* Gitlow, Gitlow, Oppenheim, and Oppenheim discuss an example of using $\bar{x}$ and R charts to study tuning knob diameters. In their problem description the authors say this:

> A manufacturer of high-end audio components buys metal tuning knobs to be used in the assembly of its products. The knobs are produced automatically by a subcontractor using a single machine that is supposed to produce them with a constant diameter. Nevertheless, because of persistent final assembly problems with the knobs, management has decided to examine this process output by requesting that the subcontractor keep an $\bar{x}$ and R chart for knob diameter.

TABLE 15.6 25 Subgroups of Tuning Knob Diameters DS KnobDiam

Time	Subgroup Number	Diameter Measurement 1	2	3	4	Average, $\bar{x}$	Range, R
8:30 A.M.	1	836	846	840	839	840.25	10
9:00	2	842	836	839	837	838.50	6
9:30	3	839	841	839	844	840.75	5
10:00	4	840	836	837	839	838.00	4
10:30	5	838	844	838	842	840.50	6
11:00	6	838	842	837	843	840.00	6
11:30	7	842	839	840	842	840.75	3
12:00	8	840	842	844	836	840.50	8
12:30 P.M.	9	842	841	837	837	839.25	5
1:00	10	846	846	846	845	845.75	1
1:30	11	849	846	848	844	846.75	5
2:00	12	845	844	848	846	845.75	4
2:30	13	847	845	846	846	846.00	2
3:00	14	839	840	841	838	839.50	3
3:30	15	840	839	839	840	839.50	1
4:00	16	842	839	841	837	839.75	5
4:30	17	841	845	839	839	841.00	6
5:00	18	841	841	836	843	840.25	7
5:30	19	845	842	837	840	841.00	8
6:00	20	839	841	842	840	840.50	3
6:30	21	840	840	842	836	839.50	6
7:00	22	844	845	841	843	843.25	4
7:30	23	848	843	844	836	842.75	12
8:00	24	840	844	841	845	842.50	5
8:30	25	843	845	846	842	844.00	4

Source: H. Gitlow, S. Gitlow, A. Oppenheim, and R. Oppenheim, *Tools and Methods for the Improvement of Quality*, p. 301. Copyright © 1989. Reprinted by permission of McGraw-Hill Companies, Inc.

On a particular day the subcontractor selects four knobs every half hour and carefully measures their diameters. Twenty-five subgroups are obtained, and these subgroups (along with their subgroup means and ranges) are given in Table 15.6. DS KnobDiam

a For these data, show that $\bar{\bar{x}} = 841.45$ and $\bar{R} = 5.16$. Then use these values to calculate control limits and to set up $\bar{x}$ and R charts for the 25 subgroups of tuning knob diameters. Do these $\bar{x}$ and R charts indicate the existence of any assignable causes? Explain.

b An investigation is carried out to find out what caused the large range for subgroup 23. The investigation reveals that a water pipe burst at 7:25 P.M. and that the mishap resulted in water leaking under the machinery used in the tuning knob production process. The resulting disruption is the apparent cause for the out-of-control range for subgroup 23. The water pipe is mended, and because this fix is reasonably permanent, we are justified in removing subgroup 23 from the data set. Using the remaining 24 subgroups, show that revised center lines are $\bar{\bar{x}} = 841.40$ and $\bar{R} = 4.88$.

c Use the revised values of $\bar{\bar{x}}$ and $\bar{R}$ to set up revised $\bar{x}$ and R charts for the remaining 24 subgroups of diameters. Be sure to omit the mean and range for subgroup 23.

d Looking at the revised R chart, is this chart now in statistical control? What does your answer say about whether we can use the $\bar{x}$ chart to decide if the process mean is changing?

e Looking at the revised $\bar{x}$ chart, is this chart in statistical control? What does your answer tell us about the process mean?

f An investigation is now undertaken to find the cause of the very high $\bar{x}$ values for subgroups 10, 11, 12, and 13. We again quote Gitlow, Gitlow, Oppenheim, and Oppenheim:

> The investigation leads to the discovery that . . . a keyway wedge had cracked and needed to be replaced on the machine. The mechanic who normally makes this repair was out to lunch, so the machine operator made the repair. This individual had not been properly trained for the repair; for this reason, the wedge was not properly aligned in the keyway, and the subsequent points were out of control. Both the operator and the mechanic agree that the need for this repair was not unusual. To correct this problem it is decided to train the machine operator and provide the appropriate tools for making

this repair in the mechanic's absence. Furthermore, the maintenance and engineering staffs agree to search for a replacement part for the wedge that will not be so prone to cracking.

Because the assignable causes responsible for the very high $\bar{x}$ values for subgroups 10, 11, 12, and 13 have been found and eliminated, we remove these subgroups from the data set. Show that removing subgroups 10, 11, 12, and 13 (in addition to the previously removed subgroup 23) results in the revised center lines $\bar{\bar{x}} = 840.46$ and $\bar{R} = 5.25$. Then use these revised values to set up revised $\bar{x}$ and R charts for the remaining 20 subgroups.

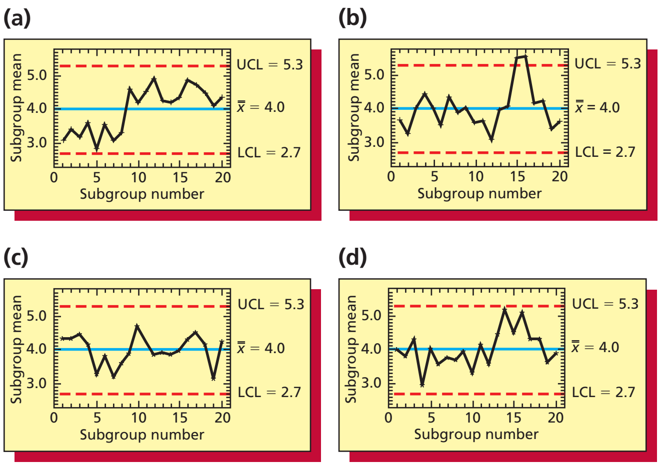
g Are all of the subgroup means and ranges for these newly revised $\bar{x}$ and R charts inside their respective control limits? Note: See Exercise 15.14 for more on this situation.

15.12 In an issue of *Quality Progress,* Gunter presents several control charts. Four of these charts are reproduced in Figure 15.17. For each chart, find any evidence of a lack of statistical control (that is, for each chart identify any evidence of the existence of one or more assignable causes). In each case, if such evidence exists, clearly explain why the plot points indicate that the process is not in control.

15.13 In the book *Tools and Methods for the Improvement of Quality,* Gitlow, Gitlow, Oppenheim, and Oppenheim present several control charts in a discussion and exercises dealing with pattern analysis. These control charts, which include appropriate A, B, and C zones, are reproduced in Figure 15.18. For each chart, identify any evidence of a lack of statistical control (that is, for each chart identify any evidence suggesting the existence of one or more assignable causes). In each case, if such evidence exists, clearly explain why the plot points indicate that the process is not in control.

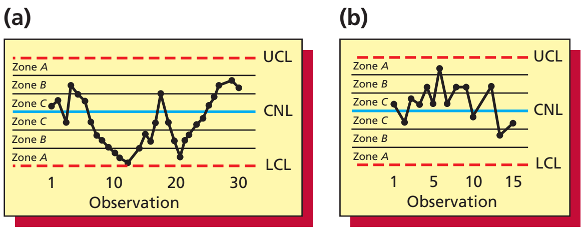
15.14 Consider Exercise 15.11. After removing subgroups 10, 11, 12, 13, and 23 from Table 15.6, we found that $\bar{\bar{x}} = 840.46$ and $\bar{R} = 5.25$. Using the formula $\bar{\bar{x}} + (2/3)\,A_2\bar{R}$, calculate the upper $A-B$ boundary for performing pattern analysis. Then, using the fact that the last three $\bar{x}$ values are 843.25, 842.50, and 844.0 (see Table 15.6 on page 15-27, and recall that we removed subgroup 23), identify a condition that indicates a lack of statistical control.

FIGURE 15.17 Charts for Exercise 15.12



Source: B. Gunter, "Process Capability Studies Part 3: The Tale of the Charts," *Quality Progress* (June 1991), pp. 77–82. Copyright © 1991. American Society for Quality. Used with permission.

FIGURE 15.18 Charts for Exercise 15.13



Source: H. Gitlow, S. Gitlow, A. Oppenheim, and R. Oppenheim, *Tools and Methods for the Improvement of Quality*, pp. 191–93, 209–11. Copyright © 1989. Reprinted by permission of McGraw-Hill Companies, Inc.

15.5 Comparison of a Process with Specifications: Capability Studies

LO15-6 Decide whether a process is capable of meeting specifications.

If we have a process in **statistical control,** we have found and **eliminated** the **assignable causes of process variation.** Therefore, the individual process measurements fluctuate over time with a **constant standard deviation σ** around a **constant mean μ.** It follows that we can use the individual process measurements to estimate μ and σ. Doing this lets us determine if the process is capable of producing output that meets specifications. Specifications are based on fitness for use criteria—that is, the specifications are established by design engineers or customers. Even if a process is in statistical control, it may exhibit too much **common cause variation** (represented by σ) to meet specifications.

As will be shown in Example 15.5, one way to study the capability of a process that is in statistical control is to construct a **histogram** from a set of individual process measurements. The histogram can then be compared with the product specification limits. In addition, we know that if all possible individual process measurements are normally distributed with mean μ and standard deviation σ, then 99.73 percent of these measurements will be in the interval $[\mu - 3\sigma, \mu + 3\sigma]$. Estimating μ and σ by $\bar{\bar{x}}$ and $\bar{R}/d_2$, we obtain the **natural tolerance limits**[6] for the process.

Natural Tolerance Limits

The **natural tolerance limits** for a normally distributed process that is in statistical control are

$$\left[\bar{\bar{x}} \pm 3\left(\frac{\bar{R}}{d_2}\right)\right] = \left[\bar{\bar{x}} - 3\left(\frac{\bar{R}}{d_2}\right),\quad \bar{\bar{x}} + 3\left(\frac{\bar{R}}{d_2}\right)\right]$$

where d_2 is a constant that depends on the subgroup size n. Values of d_2 are given in Table 15.2 (page 15-15) for subgroup sizes $n = 2$ to $n = 25$. These limits contain approximately 99.73 percent of the individual process measurements.

If the natural tolerance limits are inside the specification limits, then almost all (99.73 percent) of the individual process measurements are produced within the specification limits. In this case we say that the process is **capable** of meeting specifications. Furthermore, if we use $\bar{x}$ and R charts to monitor the process, then as long as the process remains in statistical control, the process will continue to meet the specifications. If the natural tolerance limits are wider than the specification limits, we say that the process is **not capable.** Here some individual process measurements are outside the specification limits.

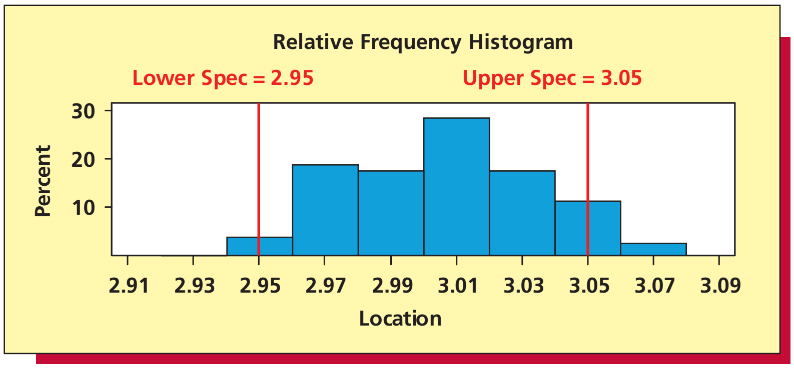
EXAMPLE 15.5 The Hole Location Case: Meeting Customer Requirements

C

Again consider the hole punching process for air conditioner compressor shells. Recall that we were able to get this process into a state of statistical control with $\bar{\bar{x}} = 3.0006$ and $\bar{R} = .0675$ by removing several assignable causes of process variation.

Figure 15.19 gives a relative frequency histogram of the 80 individual hole location measurements used to construct the $\bar{x}$ and R charts of Figure 15.8 (page 15-19). This histogram

FIGURE 15.19 **A Relative Frequency Histogram of the Hole Location Data (Based on the Data with Subgroups 1, 7, 12, and 17 Omitted)**



[6]There are a number of alternative formulas for the natural tolerance limits. Here we give the version that is the most clearly related to using $\bar{x}$ and R charts. At the end of this section we present an alternative formula.

suggests that the population of all individual hole location dimensions is approximately normally distributed.

Because the process is in statistical control, $\bar{\bar{x}} = 3.0006$ is an estimate of the process mean, and $\bar{R}/d_2 = .0675/2.326 = .0290198$ is an estimate of the process standard deviation. Here $d_2 = 2.326$ is obtained from Table 15.2 (page 15-15) corresponding to the subgroup size $n = 5$. Furthermore, the natural tolerance limits

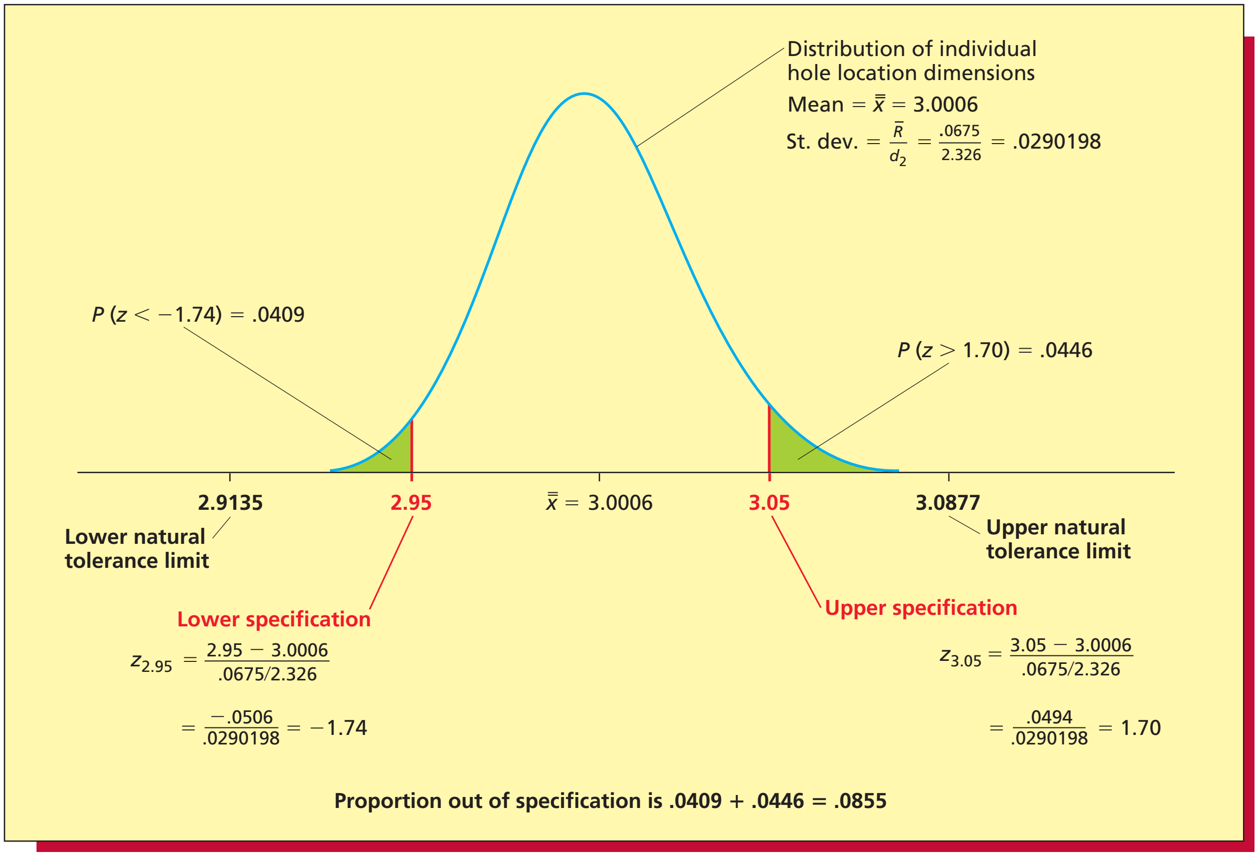
$$\left[\bar{\bar{x}} \pm 3\left(\frac{\bar{R}}{d_2}\right)\right] = \left[3.0006 \pm 3\left(\frac{.0675}{2.326}\right)\right]$$
$$= [3.0006 \pm .0871]$$
$$= [2.9135, 3.0877]$$

tell us that almost all (approximately 99.73 percent) of the individual hole location dimensions produced by the hole punching process are between 2.9135 inches and 3.0877 inches.

Suppose a major customer requires that the hole location dimension must meet specifications of 3.00 ± .05 inches. That is, the customer requires that every individual hole location dimension must be between 2.95 inches and 3.05 inches. The natural tolerance limits, [2.9135, 3.0877], which contain almost all individual hole location dimensions, are wider than the specification limits [2.95, 3.05]. This says that some of the hole location dimensions are outside the specification limits. Therefore, the process is not capable of meeting the specifications. Note that the histogram in Figure 15.19 also shows that some of the hole location dimensions are outside the specification limits.

Figure 15.20 illustrates the situation, assuming that the individual hole location dimensions are normally distributed. The figure shows that the natural tolerance limits are wider than the

FIGURE 15.20 **Calculating the Fraction out of Specification for the Hole Location Data. Specifications Are 3.00 ± .05.**



specification limits. The shaded areas under the normal curve make up the fraction of product that is outside the specification limits. Figure 15.20 also shows the calculation of the estimated fraction of hole location dimensions that are out of specification. We estimate that 8.55 percent of the dimensions do not meet the specifications.

Because the process is not capable of meeting specifications, it must be improved by removing common cause variation. This is management's responsibility. Suppose engineering and management conclude that the excessive variation in the hole locations can be reduced by redesigning the machine that punches the holes in the compressor shells. Also suppose that after a research and development program is carried out to do this, the process is run using the new machine and 20 new subgroups of $n = 5$ hole location measurements are obtained. The resulting $\bar{x}$ and R charts (not given here) indicate that the process is in control with $\bar{\bar{x}} = 3.0002$ and $\bar{R} = .0348$. Furthermore, a histogram of the 100 hole location dimensions used to construct the $\bar{x}$ and R charts indicates that all possible hole location measurements are approximately normally distributed. It follows that we estimate that almost all individual hole location dimensions are contained within the new natural tolerance limits

$$\left[\bar{\bar{x}} \pm 3\left(\frac{\bar{R}}{d_2}\right)\right] = \left[3.0002 \pm 3\left(\frac{.0348}{2.326}\right)\right]$$
$$= [3.0002 \pm .0449]$$
$$= [2.9553, 3.0451]$$

BI

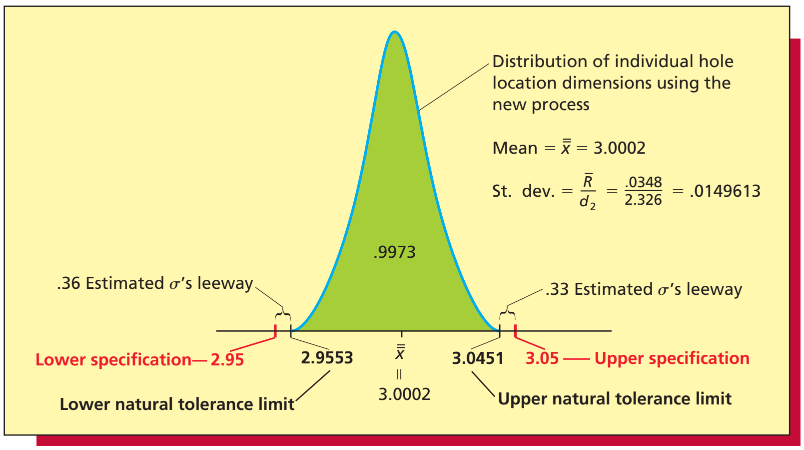
As illustrated in Figure 15.21, these tolerance limits are within the specification limits 3.00 ± .05. Therefore, the new process is now capable of producing almost all hole location dimensions inside the specifications. The new process is capable because the estimated process standard deviation has been substantially reduced (from $\bar{R}/d_2 = .0675/2.326 = .0290$ for the old process to $\bar{R}/d_2 = .0348/2.326 = .0149613$ for the redesigned process).

Next, note that (for the improved process) the z value corresponding to the lower specification limit (2.95) is

$$z_{2.95} = \frac{2.95 - 3.0002}{.0149613} = -3.36$$

This says that the lower specification limit is 3.36 estimated process standard deviations below $\bar{\bar{x}}$. Because the lower natural tolerance limit is 3 estimated process standard deviations below $\bar{\bar{x}}$, there is a **leeway** of .36 estimated process standard deviations between the lower natural tolerance limit and the lower specification limit (see Figure 15.21). Also, note that the z value corresponding to

FIGURE 15.21 **A Capable Process: The Natural Tolerance Limits Are within the Specification Limits**



the upper specification limit (3.05) is

$$z_{3.05} = \frac{3.05 - 3.0002}{.0149613} = 3.33$$

This says that the upper specification limit is 3.33 estimated process standard deviations above $\bar{\bar{x}}$. Because the upper natural tolerance limit is 3 estimated process standard deviations above $\bar{\bar{x}}$, there is a **leeway** of .33 estimated process standard deviations between the upper natural tolerance limit and the upper specification limit (see Figure 15.21). Because some leeway exists between the natural tolerance limits and the specification limits, the distribution of process measurements (that is, the curve in Figure 15.21) can shift slightly to the right or left (or can become slightly more spread out) without violating the specifications. Obviously, the more leeway, the better.

To understand why process leeway is important, recall that a process must be in statistical control before we can assess the capability of the process. In fact:

In order to demonstrate that a company's product meets customer requirements, the company must present

1. $\bar{x}$ and R charts that are in statistical control.
2. Natural tolerance limits that are within the specification limits.

However, even if a capable process shows good statistical control, the process mean and/or the process variation will occasionally change (due to new assignable causes or unexpected recurring problems). If the process mean shifts and/or the process variation increases, a process will need some leeway between the natural tolerance limits and the specification limits in order to avoid producing out-of-specification product. We can determine the amount of process leeway (if any exists) by defining what we call the **sigma level capability** of the process.

Sigma Level Capability

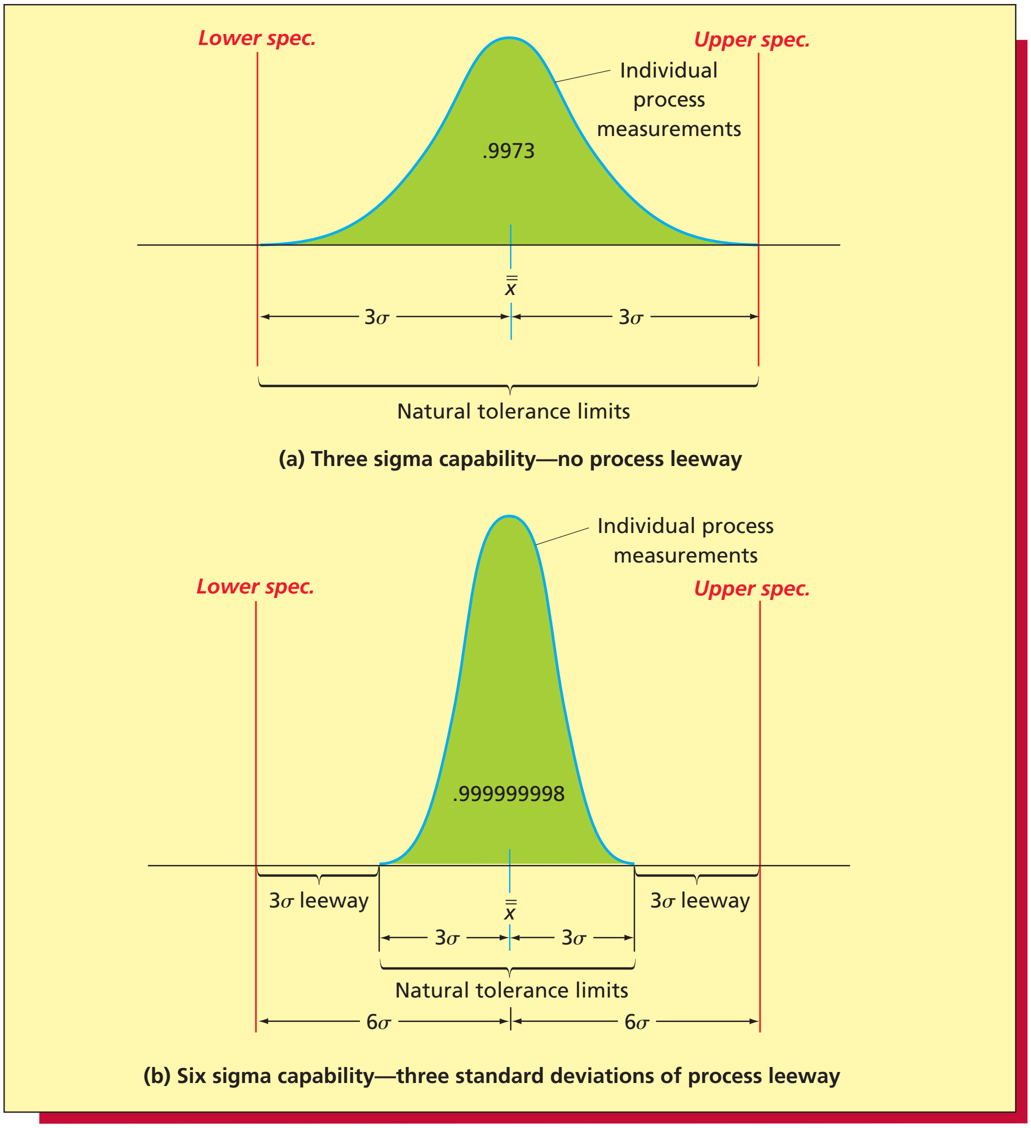
The **sigma level capability** of a process is the number of estimated process standard deviations between the estimated process mean, $\bar{\bar{x}}$, and the specification limit that is closest to $\bar{\bar{x}}$.

For instance, in the previous example the lower specification limit (2.95) is 3.36 estimated standard deviations below the estimated process mean, $\bar{\bar{x}}$, and the upper specification limit (3.05) is 3.33 estimated process standard deviations above $\bar{\bar{x}}$. It follows that the upper specification limit is closest to the estimated process mean $\bar{\bar{x}}$, and because this specification limit is 3.33 estimated process standard deviations from $\bar{\bar{x}}$, we say that the hole punching process has 3.33 sigma capability.

If a process has a sigma level capability of three or more, then there are at least three estimated process standard deviations between $\bar{\bar{x}}$ and the specification limit that is closest to $\bar{\bar{x}}$. It follows that, if the distribution of process measurements is normally distributed, then the process is capable of meeting the specifications. For instance, Figure 15.22(a) on the next page illustrates a process with three sigma capability. This process is just barely capable—that is, there is no process leeway. Figure 15.22(b) on the next page illustrates a process with six sigma capability. This process has three standard deviations of leeway. In general, we see that if a process is capable, the sigma level capability expresses the amount of process leeway. The higher the sigma level capability, the more process leeway. More specifically, for a capable process, the sigma level capability minus three gives the number of estimated standard deviations of process leeway. For example, because the hole punching process has 3.33 sigma capability, this process has $3.33 - 3 = .33$ estimated standard deviations of leeway.

The difference between three sigma and six sigma capability is dramatic. To illustrate this, look at Figure 15.22(a), which shows that a normally distributed process with three sigma capability produces 99.73 percent good quality (the area under the distribution curve between

FIGURE 15.22 Sigma Level Capability and Process Leeway



the specification limits is .9973). On the other hand, Figure 15.22(b) shows that a normally distributed process with six sigma capability produces 99.9999998 percent good quality. Said another way, if the process mean is centered between the specifications, and if we produce large quantities of product, then a normally distributed process with three sigma capability will produce an average of 2,700 defective products per million, while a normally distributed process with six sigma capability will produce an average of only .002 defective products per million.

In the long run, however, process shifts due to assignable causes are likely to occur. It can be shown that, if we monitor the process by using an $\bar{x}$ chart that employs a typical subgroup size of 4 to 6, the largest sustained shift of the process mean that might remain undetected by the $\bar{x}$ chart is a shift of 1.5 process standard deviations. In this worst case, it can be shown that a normally distributed three sigma capable process will produce an average of 66,800 defective products per million (clearly unacceptable), while a normally distributed six sigma capable process will produce an average of only 3.4 defective products per million. Therefore, if a six sigma capable process is monitored by $\bar{x}$ and R charts, then, when a process shift occurs, we can detect the shift (by using the control charts), and we can take immediate corrective action before a substantial number of defective products are produced.

This is, in fact, how control charts are supposed to be used to prevent the production of defective product. That is, our strategy is

Prevention Using Control Charts

1. Reduce common cause variation in order to create leeway between the natural tolerance limits and the specification limits.
2. Use control charts to establish statistical control and to monitor the process.
3. When the control charts give out-of-control signals, take immediate action on the process to reestablish control before out-of-specification product is produced.

A number of U.S. companies have adopted what they call a **six sigma philosophy.** In fact, these companies refer to themselves as **six sigma companies.** It is the goal of these companies to achieve six sigma capability for all processes in the entire organization. For instance, Motorola, Inc., the first company to adopt a six sigma philosophy, began a five-year quality improvement program in 1987. The goal of Motorola's companywide defect reduction program is to achieve six sigma capability for all processes—for instance, manufacturing processes, delivery, information systems, order completeness, accuracy of transactions records, and so forth. As a result of its six sigma plan, Motorola claims to have saved more than $1.5 billion. The corporation won the Malcolm Baldrige National Quality Award in 1988, and Motorola's six sigma plan has become a model for firms that are committed to quality improvement. Other companies that have adopted the six sigma philosophy include IBM, Digital Equipment Corporation, and General Electric.

To conclude this section, we make two comments. First, it has been traditional to measure process capability by using what is called the $\boldsymbol{C_{p_k}}$ **index.** This index is calculated by dividing the sigma level capability by three. For example, because the hole punching process illustrated in Figure 15.21 (page 15-31) has a sigma level capability of 3.33, the C_{p_k} for this process is 1.11. In general, if C_{p_k} is at least 1, then the sigma level capability of the process is at least 3 and thus the process is capable. Historically, C_{p_k} has been used because its value relative to the number 1 describes the process capability. We prefer using sigma level capability to characterize process capability because we believe that it is more intuitive.

Second, when a process is in control, then the estimates $\bar{R}/d_2$ and s of the process standard deviation will be very similar. This implies that we can compute the natural tolerance limits by using the alternative formula $[\bar{\bar{x}} \pm 3s]$. For example, because the mean and standard deviation of the 80 observations used to construct the $\bar{x}$ and R charts in Figure 15.8 (page 15-19) are $\bar{\bar{x}} = 3.0006$ and $s = .028875$, we obtain the natural tolerance limits

$$
\begin{aligned}
[\bar{\bar{x}} \pm 3s] &= [3.0006 \pm 3(.028875)] \\
&= [2.9140,\ 3.0872]
\end{aligned}
$$

These limits are very close to those obtained in Example 15.5 on pages 15-29 and 15-30, [2.9135, 3.0877], which were computed by using the estimate $\bar{R}/d_2 = .0290198$ of the process standard deviation. Use of the alternative formula $[\bar{\bar{x}} \pm 3s]$ is particularly appropriate when there are long-run process variations that are not measured by the subgroup ranges (in which case $\bar{R}/d_2$ underestimates the process standard deviation). Because statistical control in any real application of SPC will not be perfect, some people believe that this version of the natural tolerance limits is the most appropriate.

Exercises for Section 15.5

CONCEPTS

15.15 Write a short paragraph explaining why a process that is in statistical control is not necessarily capable of meeting customer requirements (specifications).

connect

15.16 Explain the interpretation of the natural tolerance limits for a process. What assumptions must be made in order to properly make this interpretation? How do we check these assumptions?

15.17 Explain how the natural tolerance limits compare to the specification limits when

a A process is capable of meeting specifications.

b A process is not capable of meeting specifications.

15.18 For each of the following, explain

a Why it is important to have leeway between the natural tolerance limits and the specification limits.

b What is meant by the sigma level capability for a process.

c Two reasons why it is important to achieve six sigma capability.

METHODS AND APPLICATIONS

15.19 THE HOT CHOCOLATE TEMPERATURE CASE

Consider the hot chocolate temperature situation in Exercise 15.9 (page 15-24). We found that $\bar{x}$ and R charts based on subgroups of size 3 for these data are in statistical control with $\bar{\bar{x}} = 140.73$ and $\bar{R} = 4.75$. DS HotChoc

a Assuming that the hot chocolate temperatures are approximately normally distributed, calculate a range of values that contains almost all (approximately 99.73 percent) of the hot chocolate temperatures.

b Find reasonable estimates of the maximum and minimum hot chocolate temperatures that would be served at the dining hall.

c Suppose the dining hall staff has determined that all of the hot chocolate it serves should have a temperature between 130°F and 150°F. Is the process capable of meeting these specifications? Why or why not?

d Find the sigma level capability of the process.

15.20 Suppose that $\bar{x}$ and R charts based on subgroups of size 3 are used to monitor the moisture content of a type of paper. The $\bar{x}$ and R charts are found to be in statistical control with $\bar{\bar{x}} = 6.0$ percent and $\bar{R} = .4$ percent. Further, a histogram of the individual moisture content readings suggests that these measurements are approximately normally distributed.

a Compute the natural tolerance limits (limits that contain almost all the individual moisture content readings) for this process.

b If moisture content specifications are 6.0 percent $\pm .5$ percent, is this process capable of meeting the specifications? Why or why not?

c Estimate the fraction of paper that is out of specification.

d Find the sigma level capability of the process.

15.21 A grocer has a contract with a produce wholesaler that specifies that the wholesaler will supply the grocer with grapefruit that weigh at least .75 pounds each. In order to monitor the grapefruit weights, the grocer randomly selects three grapefruit from each of 25 different crates of grapefruit received from the wholesaler. Each grapefruit's weight is determined and, therefore, 25 subgroups of three grapefruit weights are obtained. When $\bar{x}$ and R charts based on these subgroups are constructed, we find that these charts are in statistical control with $\bar{\bar{x}} = .8467$ and $\bar{R} = .11$. Further, a histogram of the individual grapefruit weights indicates that these measurements are approximately normally distributed.

a Calculate a range of values that contains almost all (approximately 99.73 percent) of the individual grapefruit weights.

b Find a reasonable estimate of the maximum weight of a grapefruit that the grocer is likely to sell.

c Suppose that the grocer's contract with its produce supplier specifies that grapefruits are to weigh a minimum of .75 lb. Is this lower specification being met? Explain. Note here that there is no upper specification because we would like grapefruits to be as large as possible.

d If the lower specification of .75 lb. is not being met, estimate the fraction of grapefruits that weigh less than .75 lb. Hint: Find an estimate of the standard deviation of the individual grapefruit weights.

15.22 Consider the pizza crust diameters for 10-inch pizzas given in Exercise 15.8 (pages 15-23 to 15-24). We found that, by removing an assignable cause, we were able to bring the process into statistical control with $\bar{\bar{x}} = 10.2225$ and $\bar{R} = .825$. DS PizzaDiam

a Recalling that the subgroup size for the pizza crust $\bar{x}$ and R charts is 5, and assuming that the pizza crust diameters are approximately normally distributed, calculate the natural tolerance limits for the diameters.

b Using the natural tolerance limits, estimate the largest diameter likely to be sold by the restaurant as a 10-inch pizza.

c Using the natural tolerance limits, estimate the smallest diameter likely to be sold by the restaurant as a 10-inch pizza.

d Are all 10-inch pizzas sold by this restaurant really at least 10 inches in diameter? If not, estimate the fraction of pizzas that are not at least 10 inches in diameter.

15.23 Consider the bag fill situation in Exercise 15.10 (page 15-26). We found that the elimination of a start-up problem brought the filling process into statistical control with $\bar{\bar{x}} = 52.5875$ and $\bar{R} = 1.2937$.

a Recalling that the fill weight $\bar{x}$ and R charts are based on subgroups of size 5, and assuming that the fill weights are approximately normally distributed, calculate the natural tolerance limits for the process.

b Suppose that management wishes to reduce the mean fill weight in order to save money by "giving away" less product. However, because customers expect each bag to contain at least 50 pounds of product, management wishes to leave some process leeway. Therefore, after the mean fill weight is reduced, the lower natural tolerance limit is to be no less than 50.5 lb. Based on the natural tolerance limits, how much can the mean fill weight be reduced? If the product costs \$2 per pound, and if 1 million bags are sold per year, what is the yearly cost reduction achieved by lowering the mean fill weight?

15.24 Suppose that a normally distributed process (centered at target) has three sigma capability. If the process shifts 1.5 sigmas to the right, show that the process will produce defective products at a rate of 66,800 per million.

15.25 Suppose that a product is assembled using 10 different components, each of which must meet specifications for five different quality characteristics. Therefore, we have 50 different specifications that potentially could be violated. Further suppose that each component possesses three sigma capability (process centered at target) for each quality characteristic. Then, if we assume normality and independence, find the probability that all 50 specifications will be met.

15.6 Charts for Fraction Nonconforming

LO15-7 Use p charts to monitor process quality.

Sometimes, rather than collecting measurement data, we inspect items and simply decide whether each item conforms to some desired criterion (or set of criteria). For example, a fuel tank does or does not leak, an order is correctly or incorrectly processed, a batch of chemical product is acceptable or must be reprocessed, or plastic wrap appears clear or too hazy. When an inspected unit does not meet the desired criteria, it is said to be **nonconforming** (or **defective**). When an inspected unit meets the desired criteria, it is said to be **conforming** (or **nondefective**). Traditionally, the terms *defective* and *nondefective* have been employed. Lately, the terms *nonconforming* and *conforming* have become popular.

The control chart that we set up for this type of data is called a ***p* chart.** To construct this chart, we observe subgroups of n units over time. We inspect or test the n units in each subgroup and determine the number d of these units that are nonconforming. We then calculate for each subgroup

$$\hat{p} = d/n = \text{the fraction of nonconforming units in the subgroup}$$

and we plot the $\hat{p}$ values versus time on the p chart. If the process being studied is in statistical control and producing a fraction p of nonconforming units, and if the units inspected are independent, then the number of nonconforming units d in a subgroup of n units inspected can be described by a binomial distribution. If, in addition, n is large enough so that np is greater than 2,[7] then both d and the fraction $\hat{p}$ of nonconforming units are approximately described by normal distributions. Furthermore, the population of all possible $\hat{p}$ values has mean $\mu_{\hat{p}} = p$ and standard deviation

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

Therefore, if p is known we can compute three standard deviation control limits for values of $\hat{p}$ by setting

$$\text{UCL} = p + 3\sqrt{\frac{p(1-p)}{n}} \qquad \text{and} \qquad \text{LCL} = p - 3\sqrt{\frac{p(1-p)}{n}}$$

However, because it is unlikely that p will be known, we usually must estimate p from process data. The estimate of p is

$$\bar{p} = \frac{\text{Total number of nonconforming units in all subgroups}}{\text{Total number inspected in all subgroups}}$$

[7]Some statisticians believe that this condition should be $np > 5$. However, for p charts many think $np > 2$ is sufficient.

Substituting $\bar{p}$ for p, we obtain the following:

Center Line and Control Limits for a *p* Chart

$$\text{Center line} = \bar{p} \qquad \text{UCL} = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} \qquad \text{LCL} = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

Note that if the LCL calculates negative, there is no lower control limit for the p chart.

The control limits calculated using these formulas are considered to be **trial control limits.** Plot points above the upper control limit suggest that one or more assignable causes have increased the process fraction nonconforming. Plot points below the lower control limit may suggest that an improvement in the process performance has been observed. However, plot points below the lower control limit may also tell us that an inspection problem exists. Perhaps defective items are still being produced, but for some reason the inspection procedure is not finding them. If the chart shows a lack of control, assignable causes must be found and eliminated and the trial control limits must be revised. Here data for subgroups associated with assignable causes that have been eliminated will be dropped, and data for newly observed subgroups will be added when calculating the revised limits. This procedure is carried out until the process is in statistical control. When control is achieved, the limits can be used to monitor process performance. **The process capability for a process that is in statistical control is expressed using $\bar{p}$, the estimated process fraction nonconforming.** When the process is in control and $\bar{p}$ is too high to meet internal or customer requirements, common causes of process variation must be removed in order to reduce $\bar{p}$. This is a management responsibility.

EXAMPLE 15.6 The Sales Invoice Case: Improving Customer Service

To improve customer service, a corporation wishes to study the fraction of incorrect sales invoices that are sent to its customers. Every week a random sample of 100 sales invoices sent during the week is selected, and the number of sales invoices containing at least one error is determined. The data for the last 30 weeks are given in Table 15.7. To construct a p chart for these data, we plot the fraction of incorrect invoices versus time. Because the true overall fraction p of incorrect invoices is unknown, we estimate p by (see Table 15.7)

$$\bar{p} = \frac{1 + 5 + 4 + \cdots + 0}{3{,}000} = \frac{69}{3{,}000} = .023$$

Because $n\bar{p} = 100(.023) = 2.3$ is greater than 2, the population of all possible $\hat{p}$ values has an approximate normal distribution if the process is in statistical control. Therefore, we calculate the center line and control limits for the p chart as follows:

$$\text{Center line} = \bar{p} = .023$$

$$\begin{aligned}\text{UCL} &= \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = .023 + 3\sqrt{\frac{.023(1-.023)}{100}}\\ &= .023 + .04497\\ &= .06797\end{aligned}$$

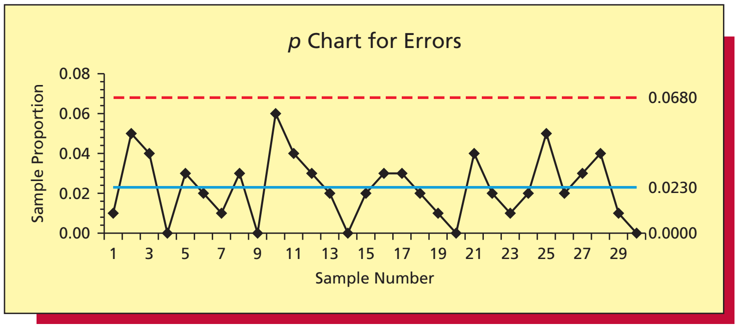
$$\begin{aligned}\text{LCL} &= \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = .023 - .04497\\ &= -.02197\end{aligned}$$

Because the LCL calculates negative, there is no lower control limit for this p chart. The Excel add-in (MegaStat) output of the p chart for these data is shown in Figure 15.23. We note that no plot points are outside the control limits, and we fail to see any nonrandom patterns of points.

TABLE 15.7 Sales Invoice Data—100 Invoices Sampled Weekly DS Invoice

Week	Number of Incorrect Sales Invoices (d)	Fraction of Incorrect Sales Invoices ($\hat{p} = d/100$)	Week	Number of Incorrect Sales Invoices (d)	Fraction of Incorrect Sales Invoices ($\hat{p} = d/100$)
1	1	.01	16	3	.03
2	5	.05	17	3	.03
3	4	.04	18	2	.02
4	0	.00	19	1	.01
5	3	.03	20	0	.00
6	2	.02	21	4	.04
7	1	.01	22	2	.02
8	3	.03	23	1	.01
9	0	.00	24	2	.02
10	6	.06	25	5	.05
11	4	.04	26	2	.02
12	3	.03	27	3	.03
13	2	.02	28	4	.04
14	0	.00	29	1	.01
15	2	.02	30	0	.00

FIGURE 15.23 Excel Add-in (MegaStat) Output of a *p* Chart for the Sales Invoice Data



We conclude that the process is in statistical control with a relatively constant process fraction nonconforming of $\bar{p} = .023$. That is, the process is stable with an average of approximately 2.3 incorrect invoices per each 100 invoices processed. Because no assignable causes are present, there is no reason to believe that any of the plot points have been affected by unusual process variations. That is, it will not be worthwhile to look for unusual circumstances that have changed the average number of incorrect invoices per 100 invoices processed. If an average of 2.3 incorrect invoices per each 100 invoices is not acceptable, then management must act to remove common causes of process variation. For example, perhaps sales personnel need additional training or perhaps the invoice itself needs to be redesigned.

BI

In the previous example, subgroups of 100 invoices were randomly selected each week for 30 weeks. In general, subgroups must be taken often enough to detect possible sources of variation in the process fraction nonconforming. For example, if we believe that shift changes may significantly influence the process performance, then we must observe at least one subgroup per shift in order to study the shift-to-shift variation. Subgroups must also be taken long enough to allow the major sources of process variation to show up. As a general rule, at least 25 subgroups will be needed to estimate the process performance and to test for process control.

We have said that the size n of each subgroup should be large enough so that np (which is usually estimated by $n\bar{p}$) is greater than 2 (some practitioners prefer np to be greater than 5). Because we often monitor a p that is quite small (.05 or .01 or less), n must often be quite large. Subgroup sizes of 50 to 200 or more are common. Another suggestion is to choose a subgroup size that is large enough to give a positive lower control limit (often, when employing a p chart, smaller subgroup sizes give a calculated lower control limit that is negative). A positive LCL is desirable because it allows us to detect opportunities for process improvement. Such an opportunity exists when we observe a plot point below the LCL. If there is no LCL, it would obviously be impossible to obtain a plot point below the LCL. It can be shown that

A condition that guarantees that the subgroup size is large enough to yield a **positive lower control limit for a *p* chart is**

$$n > \frac{9(1 - p_0)}{p_0}$$

where **p_0 is an initial estimate of the fraction nonconforming** produced by the process. This condition is appropriate when **three standard deviation control limits** are employed.

For instance, suppose experience suggests that a process produces 2 percent nonconforming items. Then, in order to construct a p chart with a positive lower control limit, the subgroup size employed must be greater than

$$\frac{9(1 - p_0)}{p_0} = \frac{9(1 - .02)}{.02} = 441$$

As can be seen from this example, for small values of p_0, the above condition may require very large subgroup sizes. For this reason, it is not crucial that the lower control limit be positive.

We have thus far discussed how often—that is, over what specified periods of time (each hour, shift, day, week, or the like)—we should select subgroups. We have also discussed how large each subgroup should be. We next consider how we actually choose the items in a subgroup. One common procedure—which often yields large subgroup sizes—is to include in a subgroup **all (that is, 100 percent) of the units produced in a specified period of time.** For instance, a subgroup might consist of all the units produced during a particular hour. When employing this kind of scheme, we must carefully consider the independence assumption. The binomial distribution assumes that successive units are produced independently. It follows that a p chart would not be appropriate if the likelihood of a unit being nonconforming depends on whether other units produced in close proximity are nonconforming. Another procedure is to **randomly select the units in a subgroup from all the units produced in a specified period of time.** This was the procedure used in Example 15.6 to obtain the subgroups of sales invoices. As long as the subgroup size is small relative to the total number of units produced in the specified period, the units in the randomly selected subgroup should probably be **independent.** However, if the rate of production is low, it could be difficult to obtain a large enough subgroup when using this method. In fact, even if we inspect 100 percent of the process output over a specified period, and even if the production rate is quite high, it might still be difficult to obtain a large enough subgroup. This is because (as previously discussed) we must select subgroups often enough to detect possible assignable causes of variation. If we must select subgroups fairly often, the production rate may not be high enough to yield the needed subgroup size in the time in which the subgroup must be selected.

In general, the large subgroup sizes that are required can make it difficult to set up useful p charts. For this reason, we sometimes (especially when we are monitoring a very small p) relax the requirement that np be greater than 2. Practice shows that even if np is somewhat smaller than 2, we can still use the three standard deviation p chart control limits. In such a case, we detect assignable causes by looking for points outside the control limits and by looking for runs of points on the same side of the center line. In order for the distribution of all possible $\hat{p}$ values to be sufficiently normal to use the pattern analysis rules we presented for $\bar{x}$ charts, $n\bar{p}$ must be greater than 2. In this case we carry out pattern analysis for a p chart as we do for an $\bar{x}$ chart (see page 15-22), and we use the following zone boundaries:

Upper A–B boundary: $\bar{p} + 2\sqrt{\dfrac{\bar{p}(1 - \bar{p})}{n}}$ Lower B–C boundary: $\bar{p} - \sqrt{\dfrac{\bar{p}(1 - \bar{p})}{n}}$

Upper B–C boundary: $\bar{p} + \sqrt{\dfrac{\bar{p}(1 - \bar{p})}{n}}$ Lower A–B boundary: $\bar{p} - 2\sqrt{\dfrac{\bar{p}(1 - \bar{p})}{n}}$

Here, when the LCL calculates negative, it should not be placed on the control chart. Zone boundaries, however, can still be placed on the control chart as long as they are not negative.

Exercises for Section 15.6

CONCEPTS

15.26 In your own words, define a *nonconforming unit.*

15.27 Describe two situations in your personal life in which you might wish to plot a control chart for fraction nonconforming.

15.28 Explain why it can sometimes be difficult to obtain rational subgroups when using a control chart for fraction nonconforming.

METHODS AND APPLICATIONS

15.29 Suppose that $\bar{p} = .1$ and $n = 100$. Calculate the upper and lower control limits, UCL and LCL, of the corresponding p chart.

15.30 Suppose that $\bar{p} = .04$ and $n = 400$. Calculate the upper and lower control limits, UCL and LCL, of the corresponding p chart.

15.31 In an issue of *Quality Progress,* William J. McCabe discusses using a p chart to study a company's order entry system. The company was experiencing problems meeting the promised 60-day delivery schedule. An investigation found that the order entry system frequently lacked all the information needed to correctly process orders. Figure 15.24 gives a p chart analysis of the percentage of orders having missing information.

a From Figure 15.24 we see that $\bar{p} = .527$. If the subgroup size for this p chart is $n = 250$, calculate the upper and lower control limits, UCL and LCL.

b Is the p chart of Figure 15.24 in statistical control? That is, are there any assignable causes affecting the fraction of orders having missing information?

c On the basis of the p chart in Figure 15.24, McCabe says,

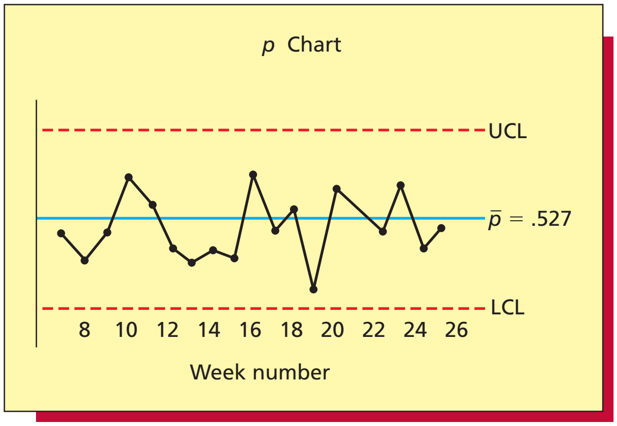
> The process was stable and one could conclude that the cause of the problem was built into the system. The major cause of missing information was salespeople not paying attention to detail, combined with management not paying attention to this problem. Having sold the product, entering the order into the system was generally left to clerical people while the salespeople continued selling.

Can you suggest possible improvements to the order entry system?

15.32 In the book *Tools and Methods for the Improvement of Quality,* Gitlow, Gitlow, Oppenheim, and Oppenheim discuss a data entry operation that makes a large number of entries every day. Over a 24-day period, daily samples of 200 data entries are inspected. Table 15.8 gives the number of erroneous entries per 200 that were inspected each day. DS DataErr

a Use the data in Table 15.8 to compute $\bar{p}$. Then use this value of $\bar{p}$ to calculate the control limits for a p chart of the data entry operation, and set up the p chart. Include zone boundaries on the chart.

FIGURE 15.24 A p Chart for the Fraction of Orders with Missing Information



Source: W. J. McCabe, "Examining Processes Improves Operations," *Quality Progress* (July 1989), pp. 26–32. Copyright © 1989 American Society for Quality. Used with permission.

TABLE 15.8 The Number of Erroneous Entries for 24 Daily Samples of 200 Data Entries DS DataErr

Day	Number of Erroneous Entries	Day	Number of Erroneous Entries
1	6	13	2
2	6	14	4
3	6	15	7
4	5	16	1
5	0	17	3
6	0	18	1
7	6	19	4
8	14	20	0
9	4	21	4
10	0	22	15
11	1	23	4
12	8	24	1

Source: H. Gitlow, S. Gitlow, A. Oppenheim, and R. Oppenheim, *Tools and Methods for the Improvement of Quality*, pp. 168–172. Copyright © 1989. Reprinted by permission of McGraw-Hill Companies, Inc.

b Is the data entry process in statistical control, or are assignable causes affecting the process? Explain.

c Investigation of the data entry process is described by Gitlow, Gitlow, Oppenheim, and Oppenheim as follows:

> In our example, to bring the process under control, management investigated the observations which were out of control (days 8 and 22) in an effort to discover and remove the special causes of variation in the process. In this case, management found that on day 8 a new operator had been added to the workforce without any training. The logical conclusion was that the new environment probably caused the unusually high number of errors. To ensure that this special cause would not recur, the company added a one-day training program in which data entry operators would be acclimated to the work environment.
>
> A team of managers and workers conducted an investigation of the circumstances occurring on day 22. Their work revealed that on the previous night one of the data entry consoles malfunctioned and was replaced with a standby unit. The standby unit was older and slightly different from the ones currently used in the department. The repairs on the regular console were not expected to be completed until the morning of day 23. To correct this special source of variation, the team recommended purchasing a spare console that would match the existing equipment and disposing of the outdated model presently being used as the backup. Management then implemented the suggestion.

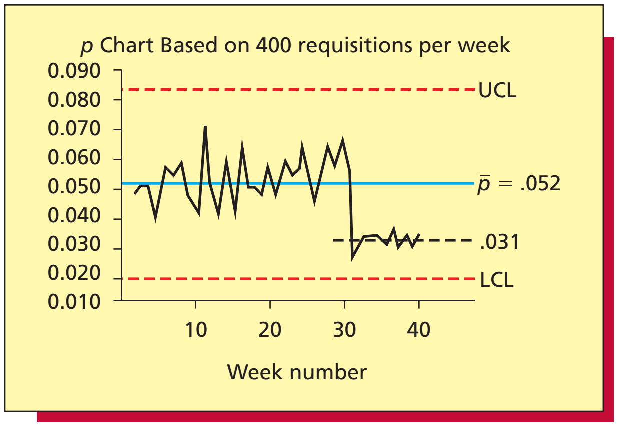
Because the assignable causes on days 8 and 22 have been found and eliminated, we can remove the data for these days from the data set. Remove the data and calculate the new value of $\bar{p}$. Then set up a revised p chart for the remaining 22 subgroups.

d Did the actions taken bring the process into statistical control? Explain.

15.33 In an issue of *Quality Progress,* William J. McCabe discusses using a p chart to study the percentage of errors made by 21 buyers processing purchase requisitions. The p chart presented by McCabe is shown in Figure 15.25. In his explanation of this chart, McCabe says,

> The causes of the errors . . . could include out-of-date procedures, unreliable office equipment, or the perceived level of management concern with errors. These causes are all associated with the system and are all under management control.
>
> Focusing on the 21 buyers, weekly error rates were calculated for a 30-week period (the data existed, but weren't being used). A p-chart was set up for the weekly department error rate. It showed a 5.2 percent average rate for the department. In week 31, the manager called the buyers together and made two statements: "I care about errors because they affect our costs and delivery schedules," and "I am going to start to count errors by individual buyers so I can understand the causes." The p-chart . . . shows an almost immediate drop from 5.2 percent to 3.1 percent.
>
> The explanation is that the common cause system (supervision, in this case) had changed; the improvement resulted from eliminating buyer sloppiness in the execution of orders. The p-chart indicates that buyer errors are now stable at 3.1 percent. The error rate will stay there until the common cause system is changed again.

FIGURE 15.25 ***p* Chart for the Weekly Department Error Rate for 21 Buyers Processing Purchase Requisitions**



Source: W. J. McCabe, "Examining Processes Improves Operations," *Quality Progress* (July 1989), pp. 26–32. Copyright © 1989 American Society for Quality. Used with permission.

a The p chart in Figure 15.25 shows that $\bar{p} = .052$ for weeks 1 through 30. Noting that the subgroup size for this chart is $n = 400$, calculate the control limits UCL and LCL for the p chart during weeks 1 through 30.

b The p chart in Figure 15.25 shows that after week 30 the value of $\bar{p}$ is reduced to .031. Assuming that the process has been permanently changed after week 30, calculate new control limits based on $\bar{p} = .031$. If we use these new control limits after week 30, is the improved process in statistical control? Explain.

15.34 The customer service manager of a discount store monitors customer complaints. Each day a random sample of 100 customer transactions is selected. These transactions are monitored, and the number of complaints received concerning these transactions during the next 30 days is recorded. The numbers of complaints received for 20 consecutive daily samples of 100 transactions are, respectively, 2, 5, 10, 1, 5, 6, 9, 4, 1, 7, 1, 5, 7, 4, 5, 4, 6, 3, 10, and 5.
DS Complaints

a Use the data to compute $\bar{p}$. Then use this value of $\bar{p}$ to calculate the control limits for a p chart of the complaints data. Set up the p chart.

b Are the customer complaints for this 20-day period in statistical control? That is, have any unusual problems caused an excessive number of complaints during this period? Explain why or why not.

c Suppose the discount store receives 13 complaints in the next 30 days for the 100 transactions that have been randomly selected on day 21. Should the situation be investigated? Explain why or why not.

15.7 Cause-and-Effect and Defect Concentration Diagrams (Optional)

LO15-8 Use diagrams to discern the causes of quality problems (Optional).

We saw in Chapter 2 that Pareto charts are often used to identify quality problems that require attention. When an opportunity for improvement has been identified, it is necessary to examine potential causes of the problem or defect (the undesirable **effect**). Because many processes are complex, there are often a very large number of possible **causes,** and it may be difficult to focus on the important ones. In this section we discuss two diagrams that can be employed to help uncover potential causes of process variation that are resulting in the undesirable effect.

The **cause-and-effect diagram** was initially developed by Japanese quality expert **Professor Kaoru Ishikawa.** In fact, these diagrams are often called **Ishikawa diagrams;** they are also called **fishbone charts** for reasons that will become obvious when we look at an example. Cause-and-effect diagrams are usually constructed by a quality team. For example, the team might consist of product designers and engineers, production workers, inspectors, supervisors and foremen, quality engineers, managers, sales representatives, and maintenance personnel. The team will set up the cause-and-effect diagram during a **brainstorming session.** After the problem (effect) is clearly stated, the team attempts to identify as many potential causes (sources of process variation) as possible. None of the potential causes suggested by team members should be criticized or rejected. The goal is to identify as many potential causes as possible. No attempt is made to actually develop solutions to the problem at this point. After beginning to brainstorm potential causes, it may be useful to observe the process in operation for a period of time before finishing the diagram. It is helpful to focus on finding sources of process variation rather than discussing reasons why these causes cannot be eliminated.

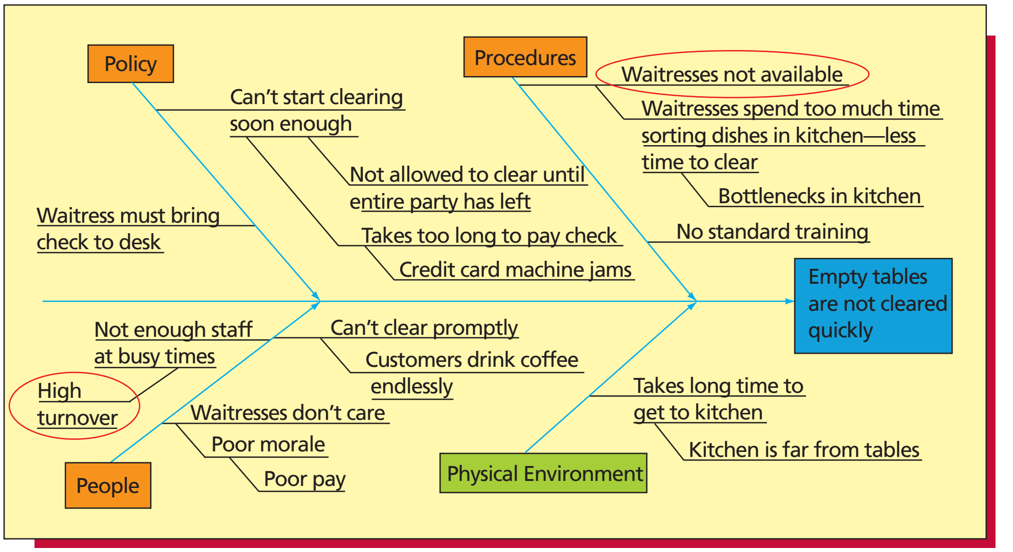
The causes identified by the team are organized into a cause-and-effect diagram as follows:

1. After clearly stating the problem, write it in an effect box at the far right of the diagram. Draw a horizontal (center) line connected to the effect box.
2. Identify major potential cause categories. Write them in boxes that are connected to the center line. Various approaches can be employed in setting up these categories. For example, Figure 15.26 on the next page is a cause-and-effect diagram for "why tables are not cleared quickly" in a restaurant. This diagram employs the categories:

   Policy   Procedures   People   Physical Environment

3. Identify subcauses and classify these according to the major potential cause categories identified in step 2. Identify new major categories if necessary. Place subcauses on the diagram as branches. See Figure 15.26.

FIGURE 15.26 A Cause-and-Effect Diagram for "Why Tables Are Not Cleared Quickly" in a Restaurant



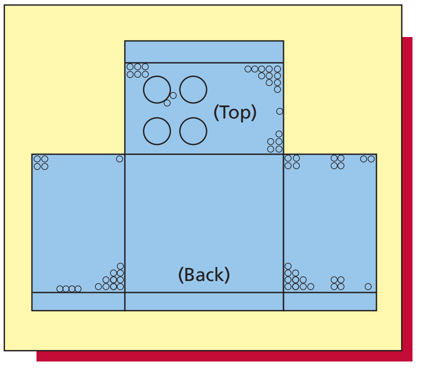
Source: M. Gaudard, R. Coates, and L. Freeman, "Accelerating Improvement," *Quality Progress* (October 1991), pp. 81–88. Copyright © 1991. American Society for Quality. Used with permission.

4 Try to decide which causes are most likely causing the problem or defect. Circle the most likely causes. See Figure 15.26.

After the cause-and-effect diagram has been constructed, the most likely causes of the problem or defect need to be studied. It is usually necessary to collect and analyze data in order to find out if there is a relationship between likely causes and the effect. We have studied various statistical methods (for instance, control charts, scatter plots, ANOVA, and regression) that help in this determination.

A **defect concentration diagram** is a picture of the product. It depicts all views—for example, front, back, sides, bottom, top, and so on. The various kinds of defects are then illustrated on the diagram. Often, by examining the locations of the defects, we can discern information concerning the causes of the defects. For example, in the October 1990 issue of *Quality Progress,* The Juran Institute presents a defect concentration diagram that plots the locations of chips in the enamel finish of a kitchen range. This diagram is shown in Figure 15.27. If the manufacturer of this range plans to use protective packaging to prevent chipping, it appears that the protective packaging should be placed on the corners, edges, and burners of the range.

FIGURE 15.27 Defect Concentration Diagram Showing the Locations of Enamel Chips on Kitchen Ranges



Source: "The Tools of Quality Part V: Check Sheets," from *QI Tools: Data Collection Workbook,* p. 11. Copyright © 1989. Juran Institute, Inc. Reprinted with permission from Juran Institute, Inc.

Exercises for Section 15.7

CONCEPTS

15.35 Explain the purpose behind constructing (a) a cause-and-effect diagram and (b) a defect concentration diagram.

15.36 Explain how to construct (a) a cause-and-effect diagram and (b) a defect concentration diagram.

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METHODS AND APPLICATIONS

15.37 In an issue of *Quality Progress,* Hoexter and Julien discuss the quality of the services delivered by law firms. One aspect of such service is the quality of attorney–client communication. Hoexter and Julien present a cause-and-effect diagram for "poor client–attorney telephone communications." This diagram is shown in Figure 15.28.

a Using this diagram, what (in your opinion) are the most important causes of poor client–attorney telephone communications?

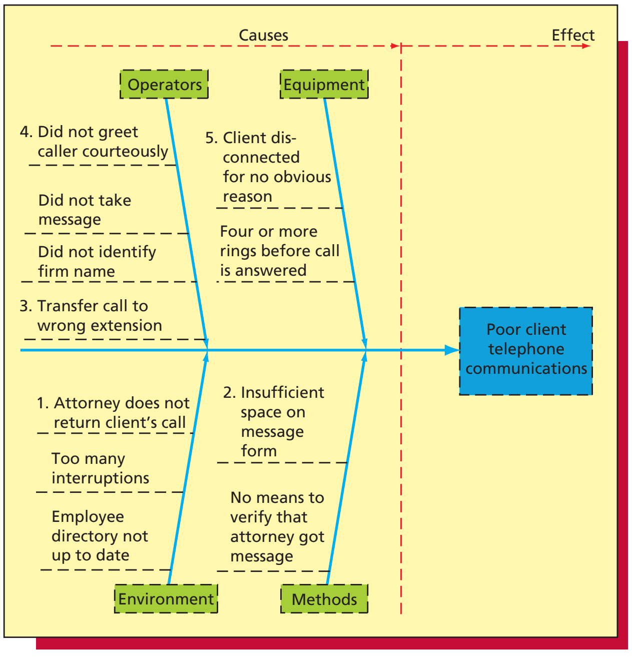
b Try to improve the diagram. That is, try to add causes to the diagram.

15.38 In an issue of *Quality Progress,* The Juran Institute presents an example that deals with the production of integrated circuits. The article describes the situation as follows:

> The manufacture of integrated circuits begins with silicon slices that, after a sequence of complex operations, will contain hundreds or thousands of chips on their surfaces. Each chip must be tested to establish whether it functions properly. During slice testing, some chips are found to be defective and are rejected. To reduce the number of rejects, it is necessary to know not only the percentage but also the locations and the types of defects. There are normally two major types of defects: functional and parametric. A functional reject occurs when a chip does not perform one of its functions. A parametric reject occurs when the circuit functions properly, but a parameter of the chip, such as speed or power consumption, is not correct.

Figure 15.29 gives a defect concentration diagram showing the locations of rejected chips within the integrated circuit. Only those chips that had five or more defects during the testing of 1,000 integrated circuits are shaded. Describe where parametric rejects tend to be, and describe where functional rejects tend to be.

FIGURE 15.28 A Cause-and-Effect Diagram for "Poor Client–Attorney Telephone Communications"



Source: R. Hoexter and M. Julien, "Legal Eagles Become Quality Hawks," *Quality Progress* (January 1994), pp. 31–33. Copyright © 1994 American Society for Quality. Used with permission.

FIGURE 15.29 Defect Concentration Diagram Showing the Locations of Rejected Chips on Integrated Circuits

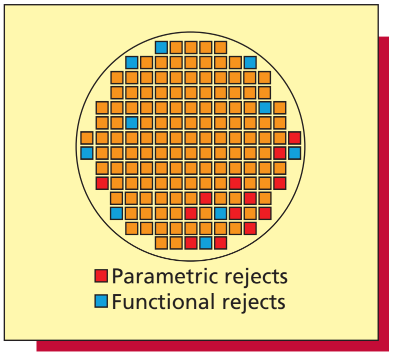


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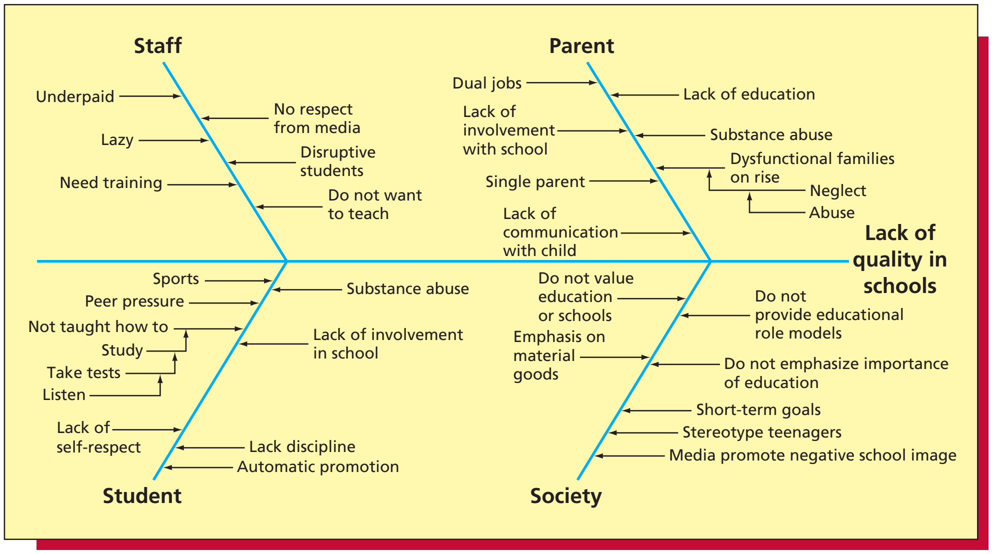
FIGURE 15.30 A Cause-and-Effect Diagram on the "Lack of Quality in Schools"



Source: F. P. Schargel, "Teaching TQM in an Inner City High School," *Quality Progress* (September 1994), pp. 87–90.
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15.39 In an issue of *Quality Progress,* Franklin P. Schargel presents a cause-and-effect diagram for the "lack of quality in schools." We present this diagram in Figure 15.30.

a Identify and circle the causes that you feel contribute the most to the "lack of quality in schools."

b Try to improve the diagram. That is, see if you can add causes to the diagram.

Chapter Summary

In this chapter we studied how to improve business processes by using **control charts.** We began by considering several meanings of quality, and we discussed the history of the quality movement in the United States. We saw that Walter Shewhart introduced statistical quality control while working at Bell Telephone Laboratories during the 1920s and 30s, and we also saw that W. Edwards Deming taught the Japanese how to use statistical methods to improve product quality following World War II. When the quality of Japanese products surpassed that of American-made goods, and when, as a result, U.S. manufacturers lost substantial shares of their markets, Dr. Deming consulted and lectured extensively in the United States. This sparked an American reemphasis on quality that continues to this day. We also briefly presented **Deming's 14 Points,** a set of management principles that, if followed, Deming believed would enable a company to improve quality and productivity, reduce costs, and gain competitive advantage.

We next learned that processes are influenced by **common cause variation** (inherent variation) and by **assignable cause variation** (unusual variation), and we saw that a control chart signals when assignable causes exist. Then we discussed how to sample a process. In particular, we explained that effective control charting requires **rational subgrouping.** Such subgroups minimize the chances that important process variations will occur within subgroups, and they maximize the chances that such variations will occur between subgroups.

Next we studied **$\bar{x}$ and R charts** in detail. We saw that $\bar{x}$ charts are used to monitor and stabilize the process mean (level), and that R charts are used to monitor and stabilize the process variability. In particular, we studied how to construct $\bar{x}$ and R charts by using **control chart constants,** how to recognize out-of-control conditions by employing **zone boundaries** and **pattern analysis,** and how to use $\bar{x}$ and R charts to get a process into statistical control.

While it is important to bring a process into statistical control, we learned that it is also necessary to meet the **customer's** or **manufacturer's requirements** (or **specifications**). Because statistical control does not guarantee that the process output meets specifications, we must carry out a **capability study** after the process has been brought into control. We studied how this is done by computing **natural tolerance limits,** which are limits that contain almost all the individual process measurements. We saw that, if the natural tolerance limits are inside the specification limits, then the process is **capable** of meeting the specifications. We also saw that we can measure how capable a process is by using **sigma level capability,** and we learned that a number of major businesses now orient their management philosophy around the concept of **six sigma capability.** In particular, we learned that, if a process is in statistical control and if the process has six sigma or better capability, then the defective rate will be very low (3.4 per million or less).

We continued by studying ***p* charts,** which are charts for fraction nonconforming. Such charts are useful when it is not possible (or when it is very expensive) to measure the quality characteristic of interest.

We concluded this chapter with an optional section on how to construct **cause-and-effect diagrams** and **defect concentration diagrams.** These diagrams are used to identify opportunities for process improvement and to discover sources of process variation.

It should be noted that two useful types of control charts not discussed in this chapter are **individuals charts** and ***c* charts.** These charts are discussed in Appendix L in the Online Learning Center www.mhhe.com/bowerman7e.

Glossary of Terms

acceptance sampling: A statistical sampling technique that enables us to accept or reject a quantity of goods (the lot) without inspecting the entire lot. (page 15-3)
assignable causes (of process variation): Unusual sources of process variation. Also called **special causes** or **specific causes** of process variation. (page 15-7)
capable process: A process that has the ability to produce products or services that meet customer or manufacturer requirements (specifications). (page 15-29)
cause-and-effect diagram: A diagram that enumerates (lists) the potential causes of an undesirable effect. (page 15-42)
common causes (of process variation): Sources of process variation that are inherent to the process design—that is, sources of usual process variation. (page 15-6)
conforming unit (nondefective): An inspected unit that meets a set of desired criteria. (page 15-36)
control chart: A graph of process performance that includes a center line and two control limits—an upper control limit, UCL, and a lower control limit, LCL. Its purpose is to detect assignable causes. (page 15-12)
C_{Pk} index: A process's sigma level capability divided by 3. (page 15-34)
defect concentration diagram: An illustration of a product that depicts the locations of defects that have been observed. (page 15-43)
ISO 9000: A series of international standards for quality assurance management systems. (page 15-5)
natural tolerance limits: Assuming a process is in statistical control and assuming process measurements are normally distributed, limits that contain almost all (approximately 99.73 percent) of the individual process measurements. (page 15-29)
nonconforming unit (defective): An inspected unit that does not meet a set of desired criteria. (page 15-36)
pattern analysis: Looking for patterns of plot points on a control chart in order to find evidence of assignable causes. (page 15-21)
***p* chart:** A control chart on which the proportion nonconforming (in subgroups of size n) is plotted versus time. (page 15-36)
quality of conformance: How well a process is able to meet the requirements (specifications) set forth by the process design. (page 15-2)
quality of design: How well the design of a product or service meets and exceeds the needs and expectations of the customer. (page 15-2)
quality of performance: How well a product or service performs in the marketplace. (page 15-2)
rational subgroups: Subgroups of process observations that are selected so that the chances that process changes will occur between subgroups is maximized. (page 15-9)
***R* chart:** A control chart on which subgroup ranges are plotted versus time. It is used to monitor the process variability (or spread). (pages 15-12 and 15-15)
run: A sequence of plot points on a control chart that are of the same type—for instance, a sequence of plot points above the center line. (page 15-22)
sigma level capability: The number of estimated process standard deviations between the estimated process mean, $\bar{\bar{x}}$, and the specification limit that is closest to $\bar{\bar{x}}$. (page 15-32)
statistical control: A state in which process measurements display a constant amount of variation around a constant mean (or level). (page 15-6)
statistical process control (SPC): A systematic method for analyzing process data in which we monitor and study the process variation. The goal is continuous process improvement. (page 15-6)
subgroup: A set of process observations that are grouped together for purposes of control charting. (page 15-9)
total quality management (TQM): Applying quality principles to all company activities. (page 15-4)
variables control charts: Control charts constructed by using measurement data. (page 15-12)
$\bar{x}$ chart (*x*-bar chart): A control chart on which subgroup means are plotted versus time. It is used to monitor the process mean (or level). (pages 15-12 and 15-15)

Important Formulas

Center line and control limits for an $\bar{x}$ chart: page 15-15
Center line and control limits for an R chart: page 15-15
Zone boundaries for an $\bar{x}$ chart: page 15-22
Zone boundaries for an R chart: page 15-22
Natural tolerance limits for normally distributed process measurements: page 15-29
Sigma level capability: page 15-32
C_{p_k} index: page 15-34
Center line and control limits for a p chart: page 15-37
Zone boundaries for a p chart: page 15-39

Supplementary Exercises

Exercises 15.40 through 15.43 are based on a case study adapted from an example presented in the paper "Managing with Statistical Models" by James C. Seigel (1982). Seigel's example concerned a problem encountered by Ford Motor Company.

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THE CAMSHAFT CASE DS Camshaft

An automobile manufacturer produces the parts for its vehicles in many different locations and transports them to assembly plants. In order to keep the assembly operations running efficiently, it is vital that all parts be within specification limits. One important part used in the assembly of V6 engines is the engine camshaft, and one important quality characteristic of this camshaft is the case hardness depth of its eccentrics. A camshaft eccentric is a metal disk positioned on the camshaft so that as the camshaft turns, the eccentric drives a lifter that opens and closes an engine valve. The V6 engine camshaft and its

FIGURE 15.31 A Camshaft and Related Parts

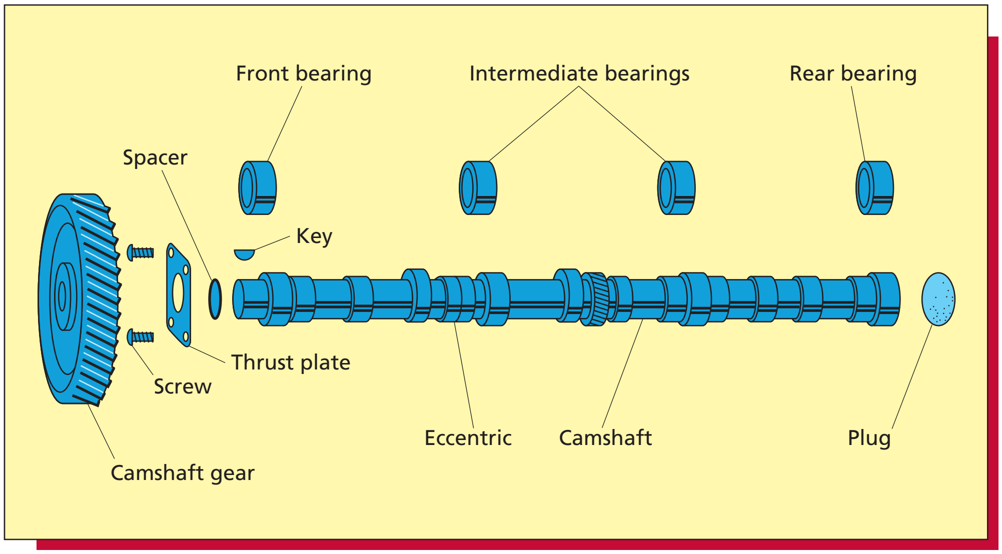


TABLE 15.9 Hardness Depth Data for Camshafts (Coil #1) DS Camshaft

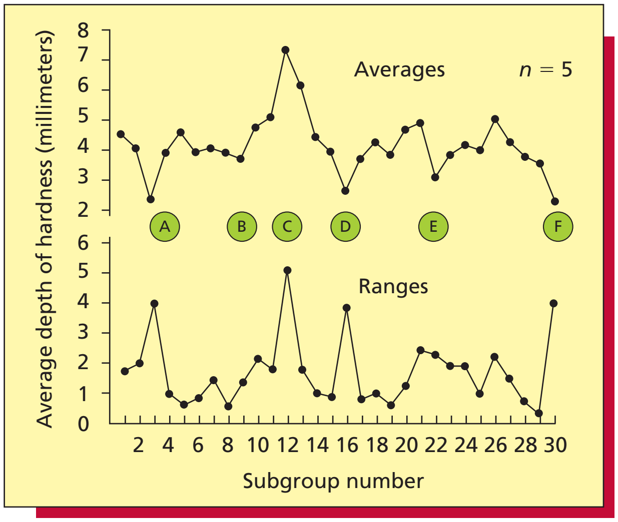
	Date	6/7	8	9	10	11	14	15	16	17	18	21	22	23	24	25
READINGS	1	3.7	5.5	4.0	4.5	4.7	4.3	5.1	4.3	4.0	3.7	4.4	5.0	7.2	4.9	4.7
	2	4.3	4.0	3.8	4.1	4.7	4.5	4.4	4.1	4.5	4.2	4.6	5.9	6.9	5.1	4.0
	3	5.5	4.3	3.0	3.5	5.0	3.6	4.0	3.7	4.1	4.9	5.4	6.5	6.0	4.5	3.9
	4	4.6	3.5	1.7	4.2	4.3	3.8	3.6	3.9	3.5	5.5	5.5	9.4	5.4	4.0	4.2
	5	4.9	3.6	0	3.9	4.4	4.1	3.7	4.0	3.0	5.9	6.3	10.1	5.5	4.2	3.7
Subgroup Mean $\bar{x}$		4.6	4.2	2.5	4.0	4.6	4.1	4.2	4	3.8	4.8	5.2	7.4	6.2	4.5	4.1
Subgroup Range R		1.8	2.0	4.0	1.0	0.7	0.9	1.5	0.6	1.5	2.2	1.9	5.1	1.8	1.1	1.0

	Date	28	29	30	7/1	2	5	6	7	8	9	12	13	14	15	16
READINGS	1	3.7	3.5	4.7	4.0	5.0	5.8	3.6	4.0	3.5	4.1	6.2	5.5	4.4	4.0	3.9
	2	3.9	3.8	5.0	3.7	4.1	6.3	3.9	3.6	5.5	4.8	5.1	5.0	4.0	3.6	3.5
	3	3.4	3.6	4.1	3.9	4.2	3.8	4.1	3.5	5.0	3.8	5.4	3.9	3.7	3.7	3.3
	4	3.0	4.1	3.9	4.4	5.2	5.2	3.0	5.5	4.0	3.9	3.9	4.2	3.9	3.5	1.7
	5	0	4.4	4.3	4.2	5.5	3.9	1.7	3.5	3.5	4.4	4.7	4.4	3.6	3.7	0
Subgroup Mean $\bar{x}$		2.8	3.9	4.4	4	4.8	5	3.3	4	4.3	4.2	5.1	4.6	3.9	3.7	2.5
Subgroup Range R		3.9	0.9	1.1	0.7	1.4	2.5	2.4	2.0	2.0	1.0	2.3	1.6	0.8	0.5	3.9

eccentrics are illustrated in Figure 15.31. These eccentrics are hardened by a process that passes the camshaft through an electrical coil that "cooks" or "bakes" the camshaft. Studies indicate that the hardness depth of the eccentric labeled in Figure 15.31 is representative of the hardness depth of all the eccentrics on the camshaft. Therefore, the hardness depth of this representative eccentric is measured at a specific location and is regarded to be the **hardness depth of the camshaft.** The optimal or target hardness depth for a camshaft is 4.5 mm. In addition, specifications state that, in order for the camshaft to wear properly, the hardness depth of a camshaft must be between 3.0 mm and 6.0 mm.

The automobile manufacturer was having serious problems with the process used to harden the camshaft. This problem was resulting in 12 percent rework and 9 percent scrap, or a total of 21 percent out-of-specification camshafts. The hardening process was automated. However, adjustments could be made to the electrical coil employed in the process. To begin study of the process, a problem-solving team selected 30 daily subgroups of $n = 5$ hardened camshafts and measured the hardness depth of each camshaft. For each subgroup, the team calculated the mean $\bar{x}$ and range R of the $n = 5$ hardness depth readings. The 30 subgroups are given in Table 15.9. The subgroup means and ranges are plotted in Figure 15.32. These means and ranges seem to exhibit substantial variability, which suggests that the hardening process was not in statistical control; we will compute control limits shortly.

FIGURE 15.32 Graphs of Performance ($\bar{x}$ and R) for Hardness Depth Data (Using Coil #1)



Although control limits had not yet been established, the problem-solving team took several actions to try to stabilize the process while the 30 subgroups were being collected:

1 At point *A*, which corresponds to a low average and a high range, the power on the coil was increased from 8.2 to 9.2.

2 At point *B* the problem-solving team found a bent coil. The coil was straightened, although at point *B* the subgroup mean and range do not suggest that any problem exists.

3 At point *C*, which corresponds to a high average and a high range, the power on the coil was decreased to 8.8.

4 At point *D*, which corresponds to a low average and a high range, the coil shorted out. The coil was straightened, and the team designed a gauge that could be used to check the coil spacing to the camshaft.

5 At point *E*, which corresponds to a low average, the spacing between the coil and the camshaft was decreased.

6 At point *F*, which corresponds to a low average and a high range, the first coil (Coil #1) was replaced. Its replacement (Coil #2) was a coil of the same type.

15.40 Using the data in Table 15.9: DS Camshaft

a Calculate $\bar{\bar{x}}$ and $\bar{R}$ and then find the center lines and control limits for $\bar{x}$ and R charts for the camshaft hardness depths.

b Set up the $\bar{x}$ and R charts for the camshaft hardness depth data.

c Are the $\bar{x}$ and R charts in statistical control? Explain.

Examining the actions taken at points *A* through *E* (in Figure 15.32), the problem-solving team learned that the power on the coil should be roughly 8.8 and that it is important to monitor the spacing between the camshaft and the coil. It also learned that it may be important to check for bent coils. The problem-solving team then (after replacing Coil #1 with Coil #2) attempted to control the hardening process by using this knowledge. Thirty new daily subgroups of $n = 5$ hardness depths were collected. The $\bar{x}$ and R charts for these subgroups are given in Figure 15.33 on the next page.

15.41 Using the values of $\bar{\bar{x}}$ and $\bar{R}$ in Figure 15.33: DS Camshaft

a Calculate the control limits for the $\bar{x}$ chart in Figure 15.33.

b Calculate the upper control limit for the R chart in Figure 15.33.

c Are the $\bar{x}$ and R charts for the 30 new subgroups using Coil #2 (which we recall was of the same type as Coil #1) in statistical control? Explain.

15.42 Consider the $\bar{x}$ and R charts in Figure 15.33. DS Camshaft

a Calculate the natural tolerance limits for the improved process.

b Recalling that specifications state that the hardness depth of each camshaft must be between 3.0 mm. and 6.0 mm., is the improved process capable of meeting these specifications? Explain.

c Use $\bar{\bar{x}}$ and $\bar{R}$ to estimate the fraction of hardness depths that are out of specification for the improved process.

FIGURE 15.33 $\bar{x}$ and R Charts for Hardness Depth Data Using Coil #2 (Same Type as Coil #1)

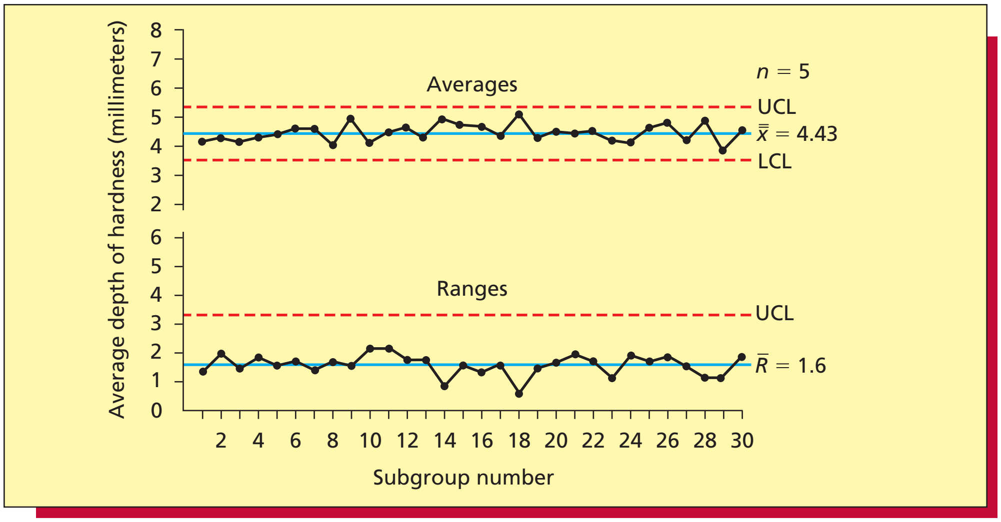
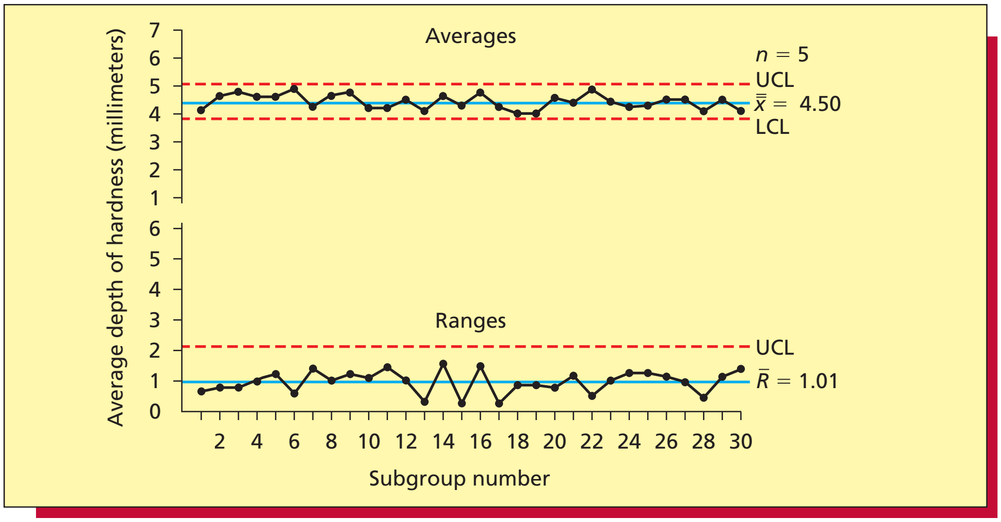


FIGURE 15.34 $\bar{x}$ and R Charts for Hardness Depth Data Using a Redesigned Coil



Because the hardening process shown in Figure 15.33 was not capable, the problem-solving team redesigned the coil to reduce the common cause variability of the process. Thirty new daily subgroups of $n = 5$ hardness depths were collected using the redesigned coil, and the resulting $\bar{x}$ and R charts are given in Figure 15.34.

15.43 Using the values of $\bar{\bar{x}}$ and $\bar{R}$ given in Figure 15.34: DS Camshaft

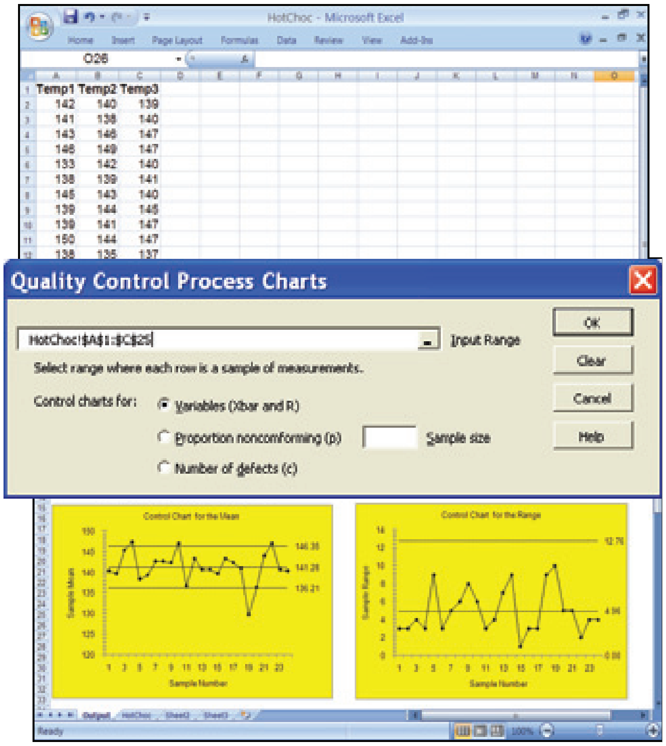
- **a** Calculate the control limits for the $\bar{x}$ and R charts in Figure 15.34.
- **b** Is the process (using the redesigned coil) in statistical control? Explain.
- **c** Calculate the natural tolerance limits for the process (using the redesigned coil).
- **d** **(1)** Is the process (using the redesigned coil) capable of meeting specifications of 3.0 mm. to 6.0 mm.? Explain. **(2)** Find and interpret the sigma level capability.

15.44 A bank officer wishes to study how many credit cardholders attempt to exceed their established credit limits. To accomplish this, the officer randomly selects a weekly sample of 100 of the cardholders who have been issued credit cards by the bank, and the number of cardholders who have attempted to exceed their credit limit during the week is recorded. The numbers of cardholders who exceeded their credit limit in 20 consecutive weekly samples of 100 cardholders are, respectively, 1, 4, 9, 0, 4, 6, 0, 3, 8, 5, 3, 5, 2, 9, 4, 4, 3, 6, 4, and 0. **(1)** Construct a control chart for the data and determine if the data are in statistical control. **(2)** If 12 cardholders in next week's sample of 100 cardholders attempt to exceed their credit limit, should the bank regard this as unusual variation in the process? DS CredLim

Appendix 15.1 ■ Control Charts Using MegaStat

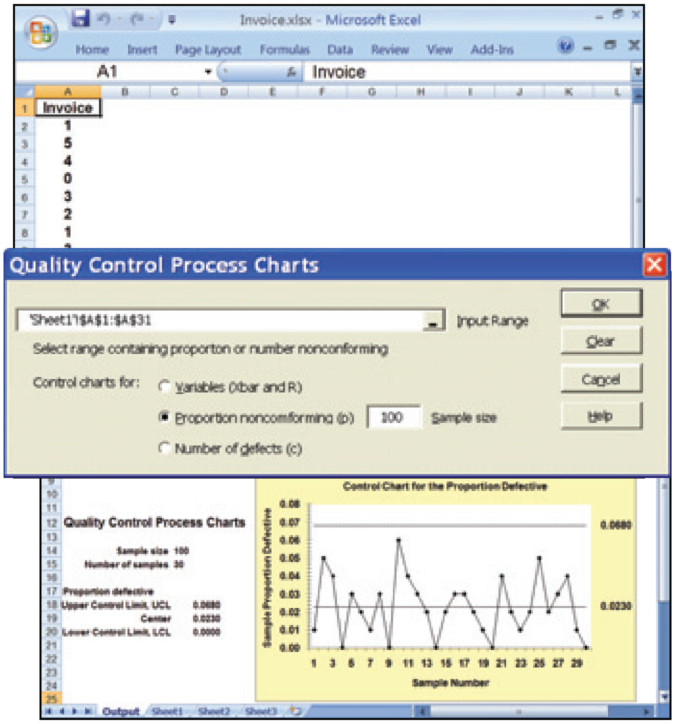
***X*-bar and *R* charts** in Figure 15.15 on page 15-25 (data file: HotChoc.xlsx):

- In cells A1, A2, and A3, enter the column labels Temp1, Temp2, and Temp3.
- In columns A, B, and C, enter the hot chocolate temperature data as 24 rows of 3 measurements, as laid out in the columns headed 1, 2, and 3 in Table 15.5 on page 15-25. When entered in this way, each row is a subgroup (sample) of three temperatures. Calculated means and ranges (as in Table 15.1 on page 15-10) need not be entered—only the raw data are needed.
- Select **Add-Ins : MegaStat : Quality Control Process Charts.**
- In the "Quality Control Process Charts" dialog box, click on "Variables (Xbar and R)."
- Use the AutoExpand feature to select the range A1 : C25 into the Input Range window. Here each row in the selected range is a subgroup (sample) of measurements.
- Click OK in the "Quality Control Process Charts" dialog box.
- The requested control charts are placed in an output file and may be edited using standard Excel editing features. See Appendix 1.1 (page 18) for additional information about editing Excel graphics.



***p* control chart** in Figure 15.23 on page 15-38 (data file: Invoice.xlsx):

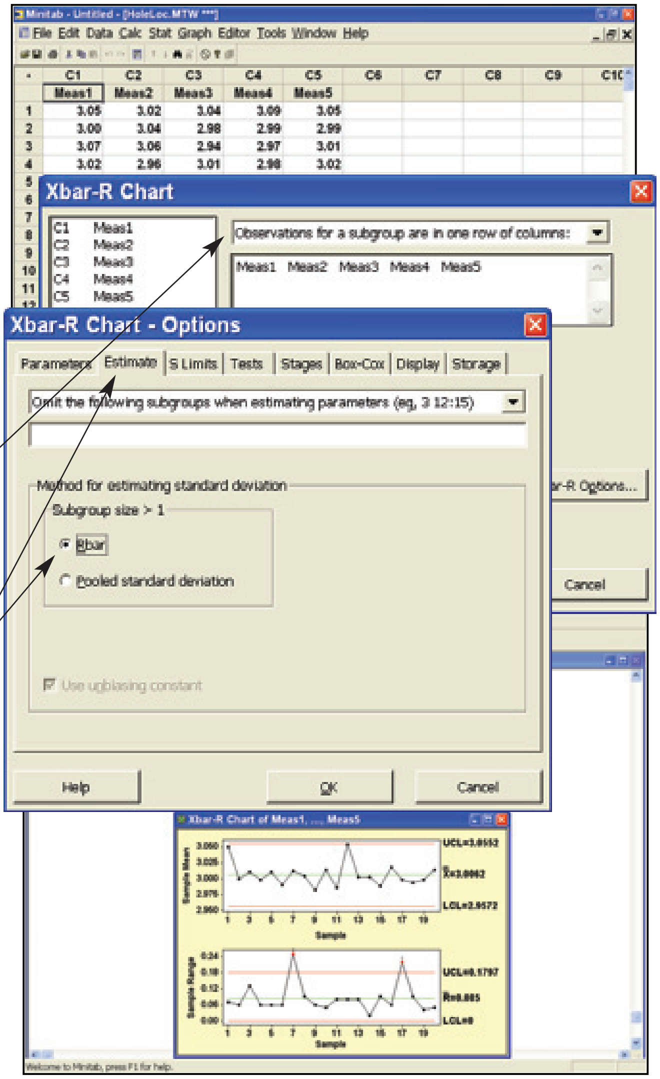
- Enter the 30 weekly error counts from Table 15.7 (page 15-38) into Column A with the label Invoice in cell A1.
- Select **Add-Ins : MegaStat : Quality Control Process Charts.**
- In the "Quality Control Process Charts" dialog box, select "Proportion nonconforming (p)."
- Use the AutoExpand feature to enter the range A1: A31 into the Input Range window.
- Enter the subgroup (sample) size (here equal to 100) into the Sample size box.
- Click OK in the "Quality Control Process Charts" dialog box.



Appendix 15.2 ■ Control Charts Using MINITAB

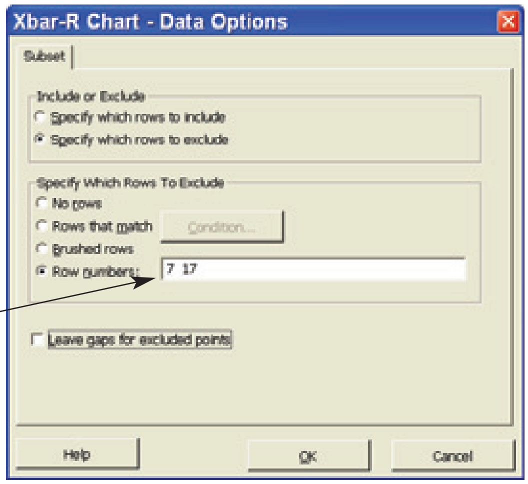
Combined *X*-bar and *R* control charts for the hole location data in Figure 15.4 on page 15-16 (data file: HoleLoc.MTW):

- In the Data window, enter the hole location measurements from Figure 15.1 (page 15-10) into columns C1 through C5 as shown in the screen with the measurements for each subgroup in a single row of columns C1 through C5—columns C1 through C5 have variable names Meas1, Meas2, Meas3, Meas4, and Meas5, which correspond to the five measurements in a single subgroup.
- Select **Stat : Control Charts : Variables Charts for Subgroups : Xbar-R.**
- In the Xbar-R Chart dialog box, select the "Observations for a subgroup are in one row of columns" option from the pull-down menu.
- Select Meas1–Meas5 into the variables window below the pull-down menu.
- Click on the "Xbar-R Chart - Options…" button.
- In the "Xbar-R Chart—Options" dialog box, click on the Estimate tab and select the Rbar option for "Method for estimating standard deviation."
- Click OK in the "Xbar-R Chart - Options" dialog box.
- Click OK in the Xbar-R Chart dialog box.
- The combined *X*-bar and *R* charts are displayed in a graphics window and can be edited using the usual MINITAB editing features.



To delete subgroups of data from the control chart (as in Figure 15.7 on page 15-18):

- In the Xbar-R Chart dialog box, click on the Data Options… button.
- Select the "Specify which rows to exclude" option under "Include or Exclude."
- Under "Specify Which Rows To Exclude," select the "Row numbers" option.
- In the Row numbers window, enter the subgroups that are to be deleted—subgroups 7 and 17 in the case of Figure 15.7.
- Follow the previously given steps to construct the *X*-bar and *R* charts.



***p* control chart** similar to Figure 15.23 on page 15-38 (data file: Invoice.MTW):

- In the Data window, enter the 30 weekly error counts from Table 15.7 (page 15-38) into column C1 with variable name Invoice.
- Select **Stat : Control Charts : Attributes Charts : p.**
- In the P Chart dialog box, enter Invoice into the Variables window.
- Enter 100 in the "Subgroup sizes" window to indicate that each error count is based on a sample of 100 invoices.
- Click OK in the P Chart dialog box.
- The *p* control chart will be displayed in a graphics window.