

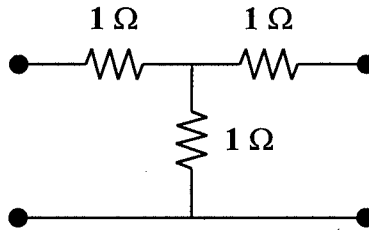
## CHAPTER 18 - TWO-PORT NETWORKS

List of topics for this chapter :

- Impedance Parameters
- Admittance Parameters
- Hybrid Parameters
- Transmission Parameters
- Relationship between Parameters
- Interconnection of Networks
- Computing Two-Port Parameters Using PSpice
- Applications

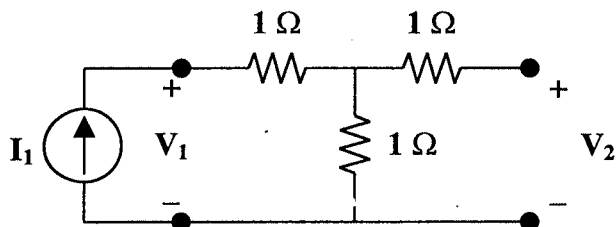
### IMPEDANCE PARAMETERS

**Problem 18.1** Determine the Z parameters for the following circuit.



The solution for this problem is straightforward and involves a series of exciting the circuit with a current source at both the input and the output and then measuring the voltage at the input and the output.

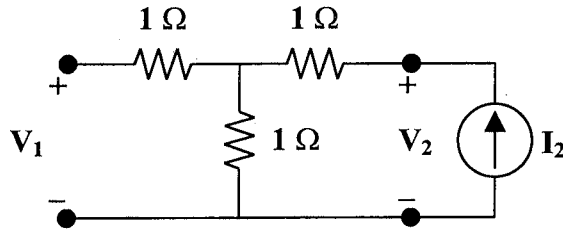
(a) To find  $z_{11}$  and  $z_{21}$ , consider the following circuit.



$$z_{11} = \frac{V_1}{I_1} = 1 + 1 = 2$$

By voltage division,  $V_2 = 1I_1$  and we then get  $z_{21} = \frac{V_2}{I_1} = 1$

(b) To find  $z_{22}$  and  $z_{12}$  we need to use the following circuit.



$$z_{22} = \frac{V_2}{I_2} = 1 + 1 = 2$$

By voltage division,  $V_1 = 1I_2$  and we then get  $z_{12} = \frac{V_1}{I_2} = 1$

$$[z] = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \Omega$$

**Problem 18.2** [18.3] Determine the  $z$  parameters of the two-port circuits in Figure 18.1.

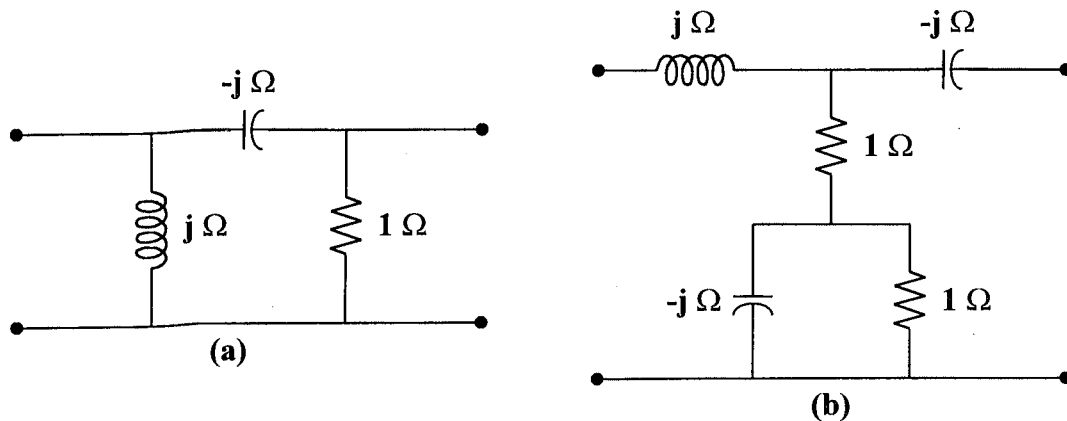
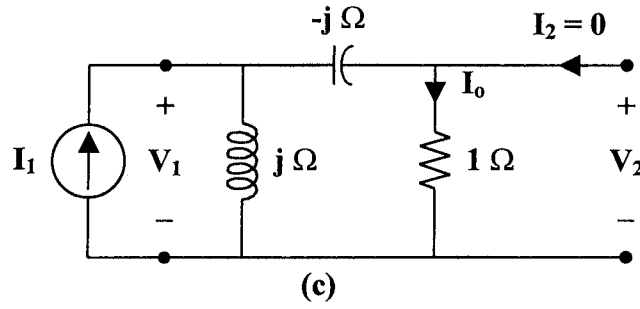


Figure 18.1

(a) To find  $z_{11}$  and  $z_{21}$ , consider the circuit in Fig. (c).



$$z_{11} = \frac{V_1}{I_1} = j \parallel (1-j) = \frac{j(1-j)}{j+1-j} = 1+j$$

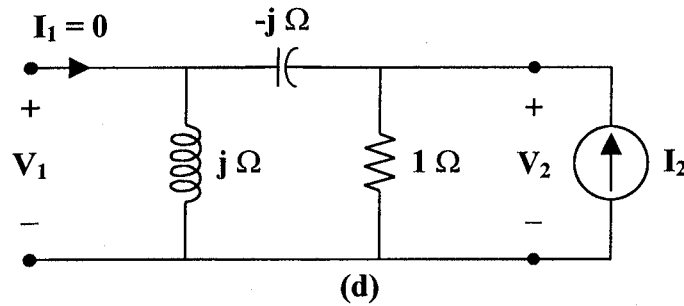
By current division,

$$I_0 = \frac{j}{j+1-j} I_1 = j I_1$$

$$V_2 = I_0 = j I_1$$

$$z_{21} = \frac{V_2}{I_1} = j$$

To get  $z_{22}$  and  $z_{12}$ , consider the circuit in Fig. (d).



$$z_{22} = \frac{V_2}{I_2} = 1 \parallel (j-j) = 0$$

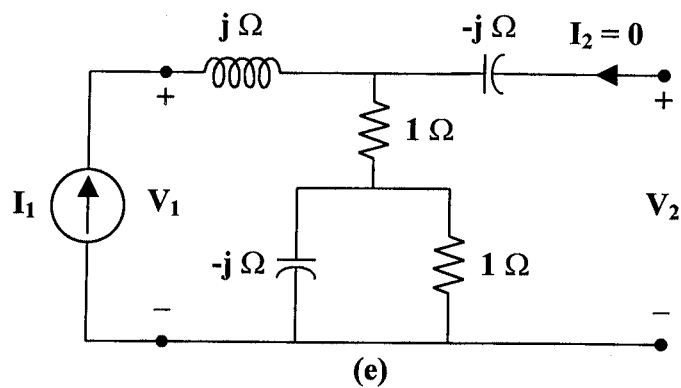
$$V_1 = j I_2$$

$$z_{12} = \frac{V_1}{I_2} = j$$

Thus,

$$[z] = \begin{bmatrix} 1+j & j \\ j & 0 \end{bmatrix} \Omega$$

(b) To find  $z_{11}$  and  $z_{21}$ , consider the circuit in Fig. (e).

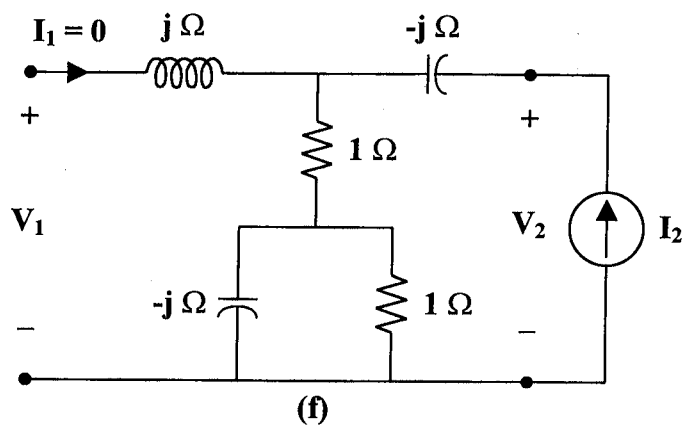


$$z_{11} = \frac{V_1}{I_1} = j + 1 + 1 \parallel (-j) = 1 + j + \frac{-j}{1-j} = 1.5 + j0.5$$

$$V_2 = (1.5 - j0.5)I_1$$

$$z_{21} = \frac{V_2}{I_1} = 1.5 - j0.5$$

To get  $z_{22}$  and  $z_{12}$ , consider the circuit in Fig. (f).



$$z_{22} = \frac{V_2}{I_2} = -j + 1 + 1 \parallel (-j) = 1.5 - j1.5$$

$$V_1 = (1.5 - j0.5)I_2$$

$$z_{12} = \frac{V_1}{I_2} = 1.5 - j0.5$$

Thus,

$$[z] = \begin{bmatrix} 1.5 + j0.5 & 1.5 - j0.5 \\ 1.5 - j0.5 & 1.5 - j1.5 \end{bmatrix} \Omega$$

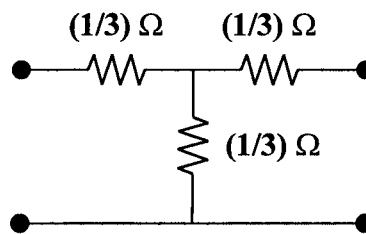
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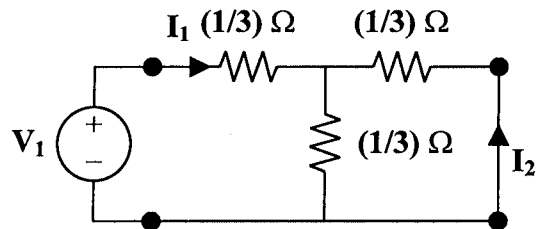
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### ADMITTANCE PARAMETERS

**Problem 18.3** Determine the Y parameters for the following circuit.



(a) To find  $y_{11}$  and  $y_{21}$  we consider the circuit below.



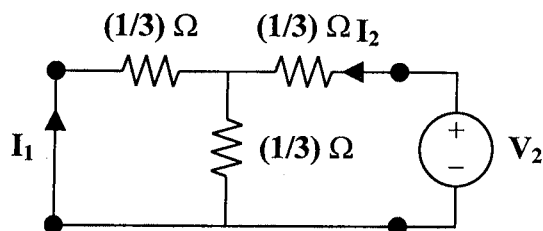
$$y_{11} = \frac{I_1}{V_1} = \frac{I_1}{I_1 \left( \frac{1}{3} + \left[ \frac{(1/3)(1/3)}{(1/3) + (1/3)} \right] \right)} = \frac{1}{\frac{1}{3} + \frac{1}{6}} = 2$$

From this we see that  $I_1 = 2V_1$ . In addition, from current division we get

$$I_2 = -\frac{(1/3)}{(1/3) + (1/3)} I_1 = -\frac{1}{2} I_1$$

$$y_{21} = \frac{I_2}{V_1} = \frac{-(1/2)I_1}{(1/2)I_1} = -1$$

(b) The following circuit will allow us to find  $y_{22}$  and  $y_{12}$ .



$$y_{22} = \frac{I_2}{V_2} = \frac{I_2}{I_2 \left( \frac{1}{3} + \left[ \frac{(1/3)(1/3)}{(1/3) + (1/3)} \right] \right)} = \frac{1}{\frac{1}{3} + \frac{1}{6}} = 2$$

$I_2 = 2V_2$ . In addition, we can use current division to get  $I_1$ .

$$I_1 = - \left( \frac{(1/3)}{(1/3) + (1/3)} \right) I_2 = -\frac{1}{2} I_2$$

$$y_{12} = \frac{I_1}{V_2} = \frac{-(1/2)I_2}{(1/2)I_2} = -1$$

$$[y] = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \text{S}$$

**Problem 18.4** [18.21] Find the  $y$  parameters for the circuit in Figure 18.2.

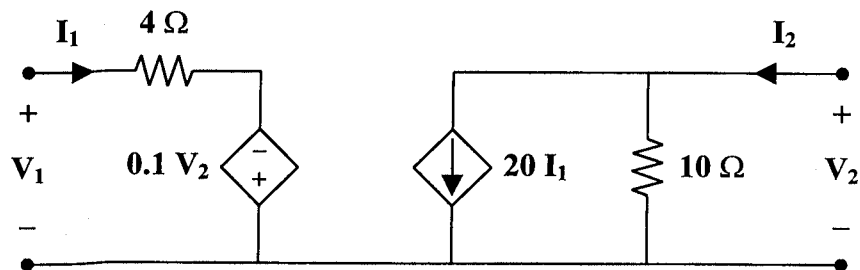
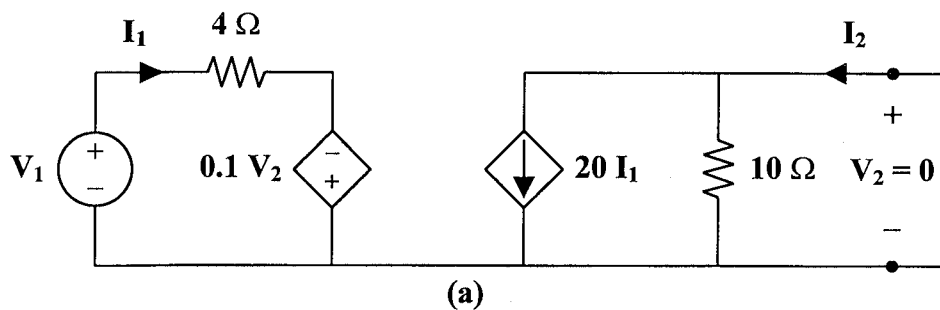


Figure 18.2

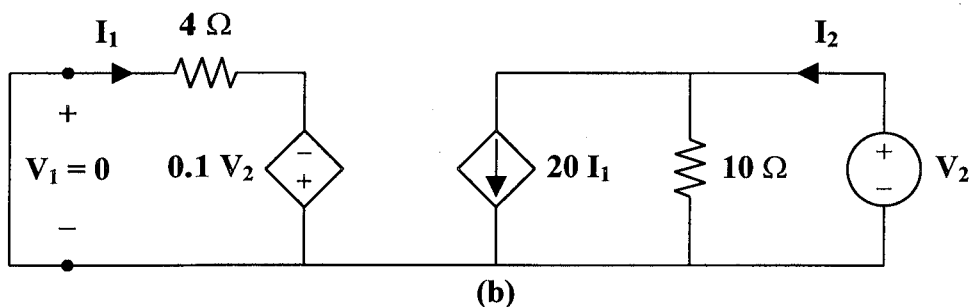
Consider the circuit in Fig. (a).



$$V_1 = 4I_1 \longrightarrow y_{11} = \frac{I_1}{V_1} = \frac{I_1}{4I_1} = 0.25$$

$$I_2 = 20I_1 = 5V_1 \longrightarrow y_{21} = \frac{I_2}{V_1} = 5$$

Consider the circuit in Fig. (b).



$$4I_1 = 0.1V_2 \longrightarrow y_{12} = \frac{I_1}{V_2} = \frac{0.1}{4} = 0.025$$

$$I_2 = 20I_1 + \frac{V_2}{10} = 0.5V_2 + 0.1V_2 = 0.6V_2 \longrightarrow y_{22} = \frac{I_2}{V_2} = 0.6$$

Thus,

$$[y] = \begin{bmatrix} 0.25 & 0.025 \\ 5 & 0.6 \end{bmatrix} \text{S}$$

Alternatively, from the given circuit,

$$V_1 = 4I_1 - 0.1V_2$$

$$I_2 = 20I_1 + 0.1V_2$$

Comparing these with the equations for the h parameters, show that

$$h_{11} = 4, \quad h_{12} = -0.1, \quad h_{21} = 20, \quad h_{22} = 0.1$$

Using Table 18.1,

$$y_{11} = \frac{1}{h_{11}} = \frac{1}{4} = 0.25$$

$$y_{12} = \frac{-h_{12}}{h_{11}} = \frac{0.1}{4} = 0.025$$

$$y_{21} = \frac{h_{21}}{h_{11}} = \frac{20}{4} = 5$$

$$y_{22} = \frac{\Delta_h}{h_{11}} = \frac{0.4 + 2}{4} = 0.6$$

as above.

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## HYBRID PARAMETERS

**Problem 18.5** [18.27] Obtain the  $h$  and  $g$  parameters of the two-port network shown in Figure 18.3.

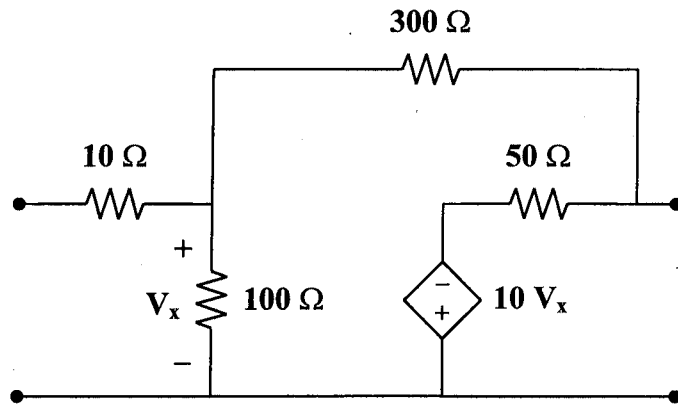
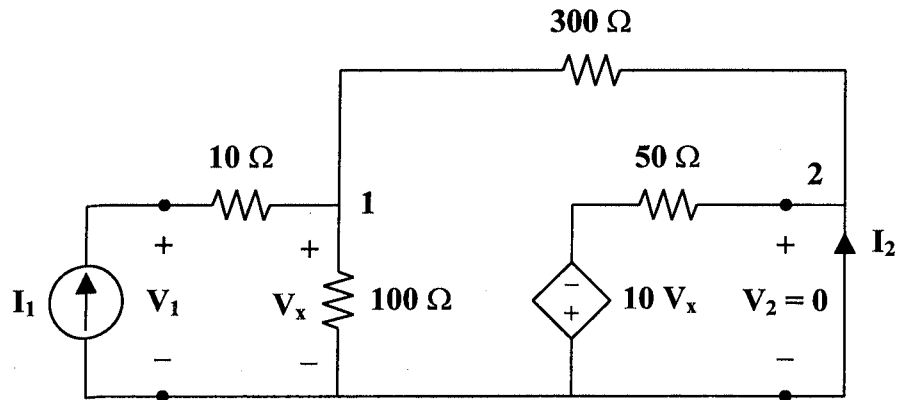


Figure 18.3

Refer to Fig. (a) to get  $h_{11}$  and  $h_{21}$ .



(a)



At node 1,

$$I_1 = \frac{V_x}{100} + \frac{V_x - 0}{300} \longrightarrow 300I_1 = 4V_x \quad (1)$$

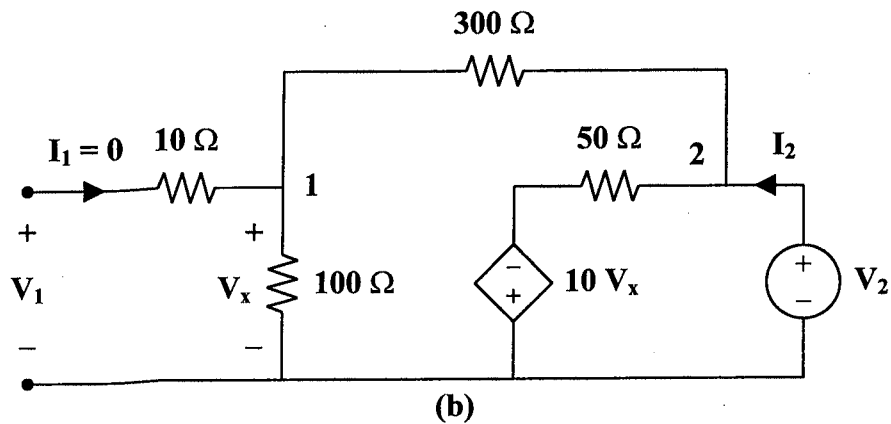
$$V_x = \frac{300}{4}I_1 = 75I_1$$

But  $V_1 = 10I_1 + V_x = 85I_1 \longrightarrow h_{11} = \frac{V_1}{I_1} = 85 \Omega$

$$I_2 = \frac{0 + 10V_x}{50} - \frac{V_x}{300} = \frac{V_x}{5} - \frac{V_x}{300} = \frac{75}{5}I_1 - \frac{75}{300}I_1 = 14.75I_1$$

$$h_{21} = \frac{I_2}{I_1} = 14.75$$

To get  $h_{22}$  and  $h_{12}$ , refer to Fig. (b).



At node 2,

$$I_2 = \frac{V_2}{400} + \frac{V_2 + 10V_x}{50} \longrightarrow 400I_2 = 9V_2 + 80V_x$$

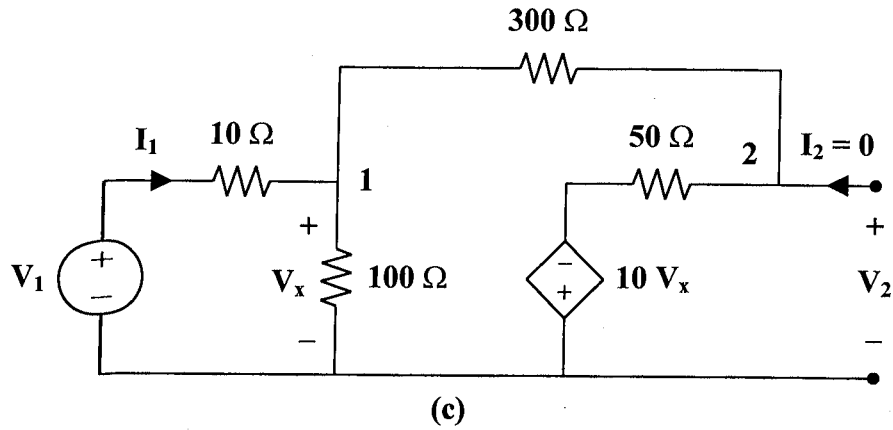
But  $V_x = \frac{100}{400}V_2 = \frac{V_2}{4}$

Hence,  $400I_2 = 9V_2 + 20V_2 = 29V_2$

$$h_{22} = \frac{I_2}{V_2} = \frac{29}{400} = 0.0725 \text{ S}$$

$$V_1 = V_x = \frac{V_2}{4} \longrightarrow h_{12} = \frac{V_1}{V_2} = \frac{1}{4} = 0.25$$

$$[h] = \begin{bmatrix} 85 \Omega & 0.25 \\ 14.75 & 0.0725 \text{ S} \end{bmatrix}$$



To get  $g_{11}$  and  $g_{21}$ , refer to Fig. (c).

$$\text{At node 1, } I_1 = \frac{V_x}{100} + \frac{V_x + 10V_x}{350} \longrightarrow 350I_1 = 14.5V_x \quad (2)$$

$$\text{But } I_1 = \frac{V_1 - V_x}{10} \longrightarrow 10I_1 = V_1 - V_x$$

$$\text{or } V_x = V_1 - 10I_1 \quad (3)$$

Substituting (3) into (2) gives

$$350I_1 = 14.5V_1 - 145I_1 \longrightarrow 495I_1 = 14.5V_1$$

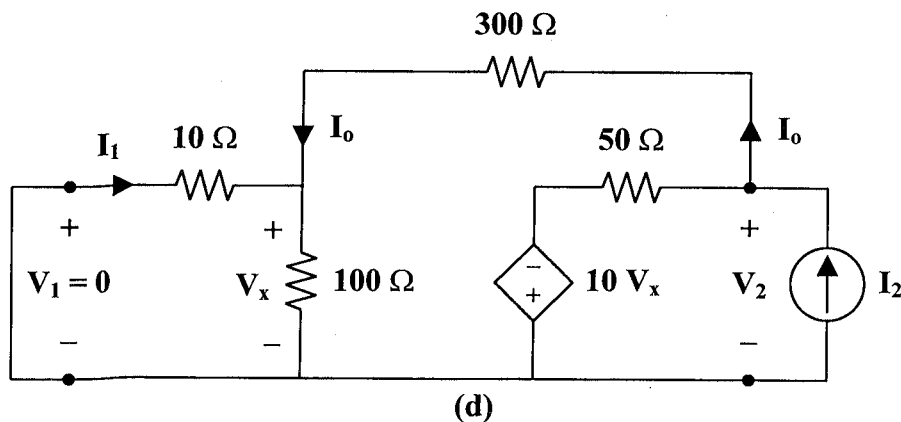
$$g_{11} = \frac{I_1}{V_1} = \frac{14.5}{495} = 0.02929 \text{ S}$$

$$V_2 = (50) \left( \frac{11}{350} V_x \right) - 10V_x = -8.4286V_x$$

$$V_2 = -8.4286V_1 + 84.286I_1 = -8.4286V_1 + (84.286) \left( \frac{14.5}{495} \right) V_1$$

$$V_2 = -5.96V_1 \longrightarrow g_{21} = \frac{V_2}{V_1} = -5.96$$

To get  $g_{22}$  and  $g_{12}$ , refer to Fig. (d).



$$10 \parallel 100 = 9.091$$

$$I_2 = \frac{V_2 + 10V_x}{50} + \frac{V_2}{300 + 9.091}$$

$$309.091I_2 = 7.1818V_2 + 61.818V_x \quad (4)$$

But 
$$V_x = \frac{9.091}{309.091}V_2 = 0.02941V_2 \quad (5)$$

Substituting (5) into (4) gives

$$309.091I_2 = 9V_2$$

$$g_{22} = \frac{V_2}{I_2} = 34.34 \Omega$$

$$I_o = \frac{V_2}{309.091} = \frac{34.34I_2}{309.091}$$

$$I_1 = \frac{-100}{110}I_o = \frac{-34.34I_2}{(1.1)(309.091)}$$

$$g_{12} = \frac{I_1}{I_2} = -0.101$$

Thus,

$$[g] = \begin{bmatrix} 0.02929 \text{ S} & -0.101 \\ -5.96 & 34.34 \Omega \end{bmatrix}$$

## TRANSMISSION PARAMETERS

**Problem 18.6** [18.41] Obtain the  $t$  parameters for the network in Figure 18.4.

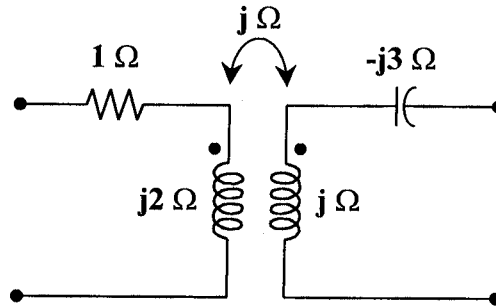
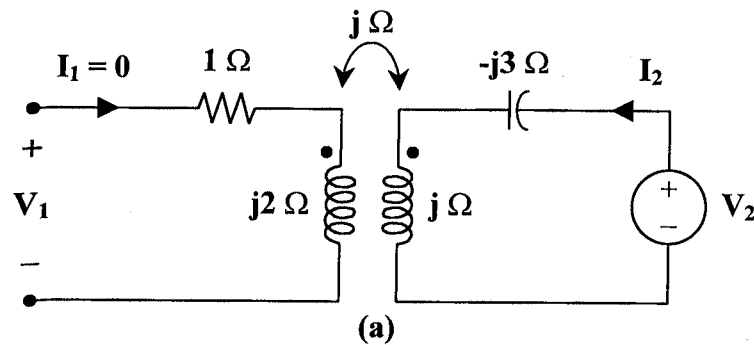


Figure 18.4

To get  $a$  and  $c$ , consider the circuit in Fig. (a).



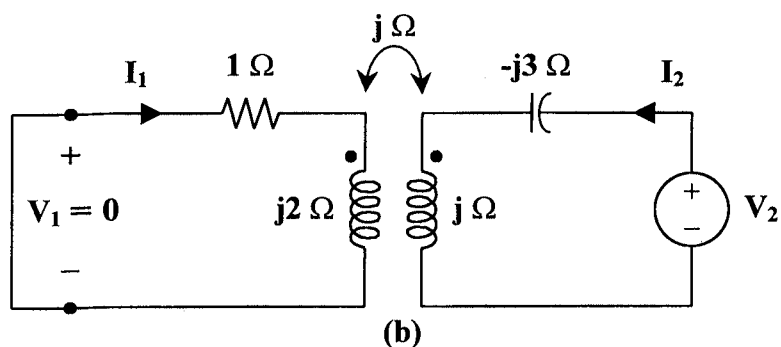
$$V_2 = I_2(j - j3) = -j2I_2$$

$$V_1 = -jI_2$$

$$a = \frac{V_2}{V_1} = \frac{-j2I_2}{-jI_2} = 2$$

$$c = \frac{I_2}{V_1} = \frac{1}{-j} = j$$

To get **b** and **d**, consider the circuit in Fig. (b).



For mesh 1,

$$0 = (1 + j2)I_1 - jI_2$$

or

$$\frac{I_2}{I_1} = \frac{1 + j2}{j} = 2 - j$$

$$\mathbf{d} = \frac{-I_2}{I_1} = -2 + j$$

For mesh 2,

$$V_2 = I_2(j - j3) - jI_1$$

$$V_2 = I_1(2 - j)(-j2) - jI_1 = (-2 - j5)I_1$$

$$\mathbf{b} = \frac{-V_2}{I_1} = 2 + j5$$

Thus,

$$[\mathbf{t}] = \begin{bmatrix} 2 & 2 + j5 \\ j & -2 + j \end{bmatrix}$$

## RELATIONSHIP BETWEEN PARAMETERS

**Problem 18.7** [18.45] Prove that the *g* parameters can be obtained from the *z* parameters as

$$\mathbf{g}_{11} = \frac{1}{\mathbf{z}_{11}} \quad \mathbf{g}_{12} = \frac{-\mathbf{z}_{12}}{\mathbf{z}_{11}} \quad \mathbf{g}_{21} = \frac{\mathbf{z}_{21}}{\mathbf{z}_{11}} \quad \mathbf{g}_{22} = \frac{\Delta_z}{\mathbf{z}_{11}}$$

For the z parameters

$$\mathbf{V}_1 = \mathbf{z}_{11} \mathbf{I}_1 + \mathbf{z}_{12} \mathbf{I}_2 \quad (1)$$

$$\mathbf{V}_2 = \mathbf{z}_{21} \mathbf{I}_1 + \mathbf{z}_{22} \mathbf{I}_2 \quad (2)$$

From (1),

$$\mathbf{I}_1 = \frac{1}{\mathbf{z}_{11}} \mathbf{V}_1 - \frac{\mathbf{z}_{12}}{\mathbf{z}_{11}} \mathbf{I}_2 \quad (3)$$

Substituting (3) into (2) gives

$$\mathbf{V}_2 = \frac{\mathbf{z}_{21}}{\mathbf{z}_{11}} \mathbf{V}_1 + \left( \mathbf{z}_{22} - \frac{\mathbf{z}_{21} \mathbf{z}_{12}}{\mathbf{z}_{11}} \right) \mathbf{I}_2$$

or

$$\mathbf{V}_2 = \frac{\mathbf{z}_{21}}{\mathbf{z}_{11}} \mathbf{V}_1 + \frac{\Delta_z}{\mathbf{z}_{11}} \mathbf{I}_2 \quad (4)$$

Comparing (3) and (4) with the following equations

$$\mathbf{I}_1 = \mathbf{g}_{11} \mathbf{V}_1 + \mathbf{g}_{12} \mathbf{I}_2$$

$$\mathbf{V}_2 = \mathbf{g}_{21} \mathbf{V}_1 + \mathbf{g}_{22} \mathbf{I}_2$$

indicates that

$$\underline{\mathbf{g}_{11} = \frac{1}{\mathbf{z}_{11}}}, \quad \underline{\mathbf{g}_{12} = \frac{-\mathbf{z}_{12}}{\mathbf{z}_{11}}}, \quad \underline{\mathbf{g}_{21} = \frac{\mathbf{z}_{21}}{\mathbf{z}_{11}}}, \quad \underline{\mathbf{g}_{22} = \frac{\Delta_z}{\mathbf{z}_{11}}}$$

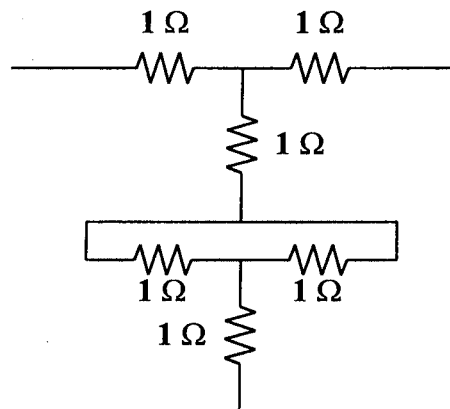
as required.

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## INTERCONNECTION OF NETWORKS

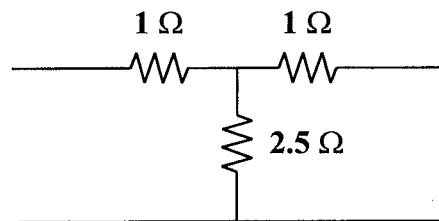
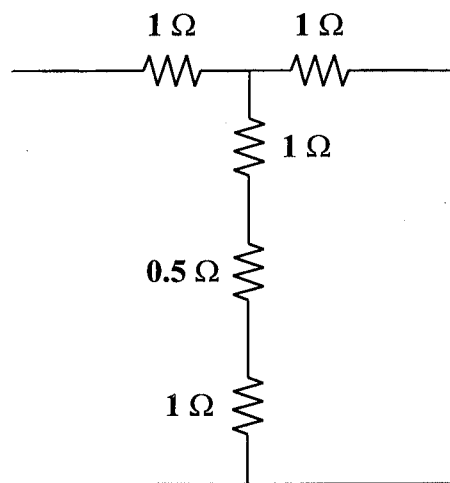
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**Problem 18.8**      Connect two of the circuits from Problem 18.1 in series as shown below. Determine the z parameters.

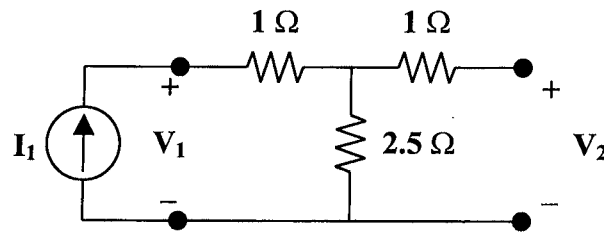


- **Carefully DEFINE the problem.**  
Each component is labeled completely. The problem is clear if all that is being asked is to find the z parameters.
- **PRESENT everything you know about the problem.**  
This is basically a linear circuit with resistors.
- **Establish a set of ALTERNATIVE solutions and determine the one that promises the greatest likelihood of success.**  
It is clear that all we need to do is to excite the circuit with a current source and make open circuit measurements as we have done earlier in this chapter.
- **ATTEMPT a problem solution.**

The first thing we need to do is to use circuit reductions to make our calculations easier. The following circuits are reductions of the above. Please note that the two 1-ohm resistors are in parallel.

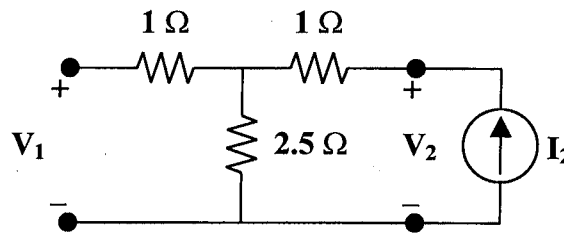


(a) To find  $z_{11}$  and  $z_{21}$  requires the following circuit



$$z_{11} = \frac{V_1}{I_1} = \frac{I_1(1+2.5)}{I_1} = 3.5 \quad \text{and} \quad z_{21} = \frac{V_2}{I_1} = \frac{2.5I_1}{I_1} = 2.5$$

(b) The following circuit can be used to generate  $z_{22}$  and  $z_{12}$ .



$$z_{22} = \frac{V_2}{I_2} = \frac{I_2(1+2.5)}{I_2} = 3.5$$

$$z_{12} = \frac{V_1}{I_2} = \frac{I_2 \cdot 2.5}{I_2} = 2.5$$

$$[z] = \begin{bmatrix} 3.5 & 2.5 \\ 2.5 & 3.5 \end{bmatrix} \Omega$$

➤ **EVALUATE** the solution and check for accuracy.

Our check for accuracy was successful.

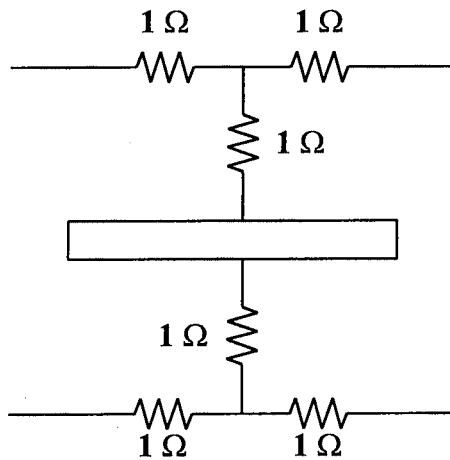
➤ **Has the problem been solved SATISFACTORILY?** If so, present the solution; if not, then return to “ALTERNATIVE solutions” and continue through the process again.



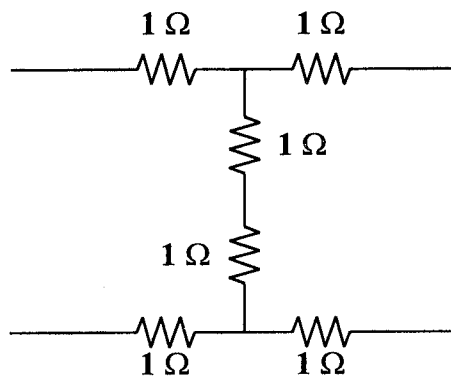
This problem has been solved satisfactorily. However, a concern is raised as to why we would want a circuit where some of the resistance is lost. So when you report the solution, you should point out that this looks like something that needs further investigating.

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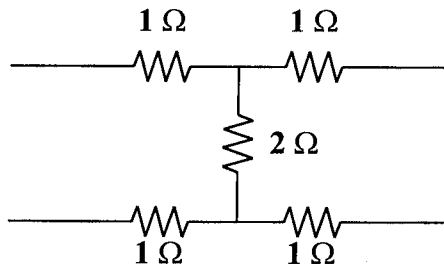
**Problem 18.9** Your boss now comes back and asks you to connect the two circuits together in series as shown in the following circuit.



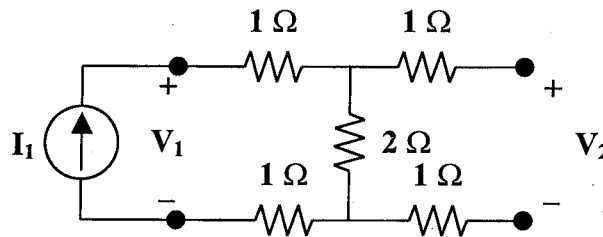
It is clear that the two circuits are connected by a short, which produces the following circuit.



The two  $1\text{-ohm}$  resistors can be combined into a  $2\text{-ohm}$  resistor and we have this circuit



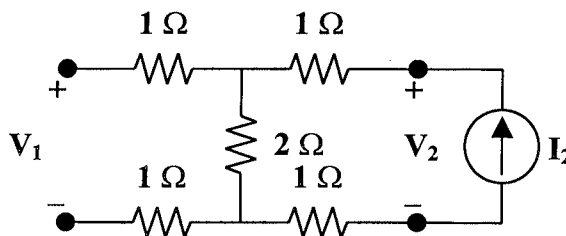
(a) To solve for  $z_{11}$  and  $z_{21}$  we use the following circuit.



$$z_{11} = \frac{V_1}{I_1} = \frac{I_1(1+2+1)}{I_1} = 4$$

$$z_{21} = \frac{V_2}{I_1} = \frac{I_1 \cdot 2}{I_1} = 2$$

(b) We now use the following circuit to determine  $z_{22}$  and  $z_{12}$ .



$$z_{22} = \frac{V_2}{I_2} = \frac{I_2(1+2+1)}{I_2} = 4$$

$$z_{12} = \frac{V_1}{I_2} = \frac{I_2 2}{I_2} = 2$$

$$[z] = \begin{bmatrix} 4 & 2 \\ 4 & 2 \end{bmatrix} \Omega$$

Clearly this is a different answer. We again take our results to the boss.

**Problem 18.10** Our boss now asks us to determine how to successfully cascade, in series.

- **Carefully DEFINE the problem.**  
Now, after trying two different configurations, which, if either, is correct.
- **PRESENT everything you know about the problem.**

Most of what we know is represented in the two previous problems and the fact that the two circuits need to function in series. We look at some textbooks and find that when two circuits are connected in series, the z parameters add.

- **Establish a set of ALTERNATIVE solutions and determine the one that promises the greatest likelihood of success.**

The two previous problems represent two alternatives for connecting the circuits together. Clearly one of these is not correct. There is a third way of connecting the circuits in series and that is to connect the two tops together in the center. However, that results in a dead short from the input to the output. This then is not the answer. We could add the z parameters together and see what we get.

- **ATTEMPT a problem solution.**

$$[z] = [z_1] + [z_1] = 2[z_1] = 2 \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix} \Omega$$

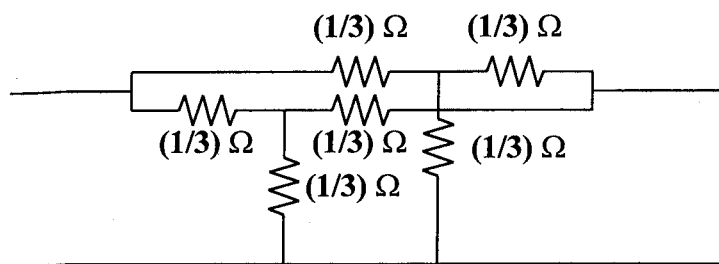
- **EVALUATE the solution and check for accuracy.**

Since this agrees with the solution from the second form of the circuit, it is clear that it is very important how circuits are cascaded. It can be concluded from this that when two circuits are cascaded in series, they each must have a common reference. Furthermore, when the circuits are interconnected, their commons need to be connected together.

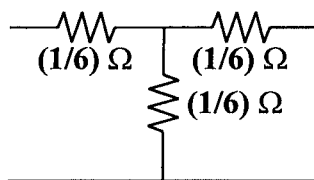
- **Has the problem been solved SATISFACTORILY? If so, present the solution; if not, then return to “ALTERNATIVE solutions” and continue through the process again.**

This problem has been solved satisfactorily and the results will be presented to the boss..

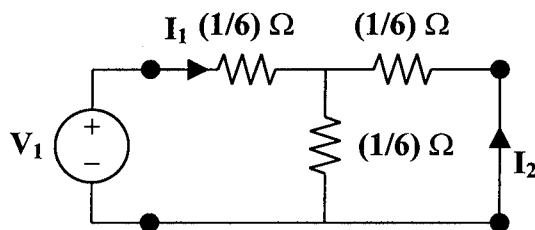
**Problem 18.11** Using two circuits identical to the one in problem 18.3, connect them in parallel so that the outputs and inputs are connected as shown in the following figure.



Now, calculate the  $y$  parameters for this circuit. Our work is made easier if we reduce the circuit through circuit reduction techniques. Since all of the resistors have the same value, we can treat them as if they are in parallel with each other. This gives the following equivalent circuit.



- (a) The following circuit is used to calculate  $y_{11}$  and  $y_{21}$ .

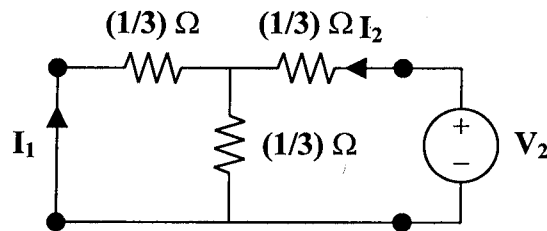


$$y_{11} = \frac{I_1}{V_1} = \frac{I_1}{I_1 \left( \frac{1}{6} + \frac{(1/6)(1/6)}{(1/6) + (1/6)} \right)} = \frac{1}{\left( \frac{1}{6} + \frac{1}{12} \right)} = 4$$

Since,  $V_1 = (1/4)I_1$  and  $I_2 = -\left( \frac{(1/6)}{(1/6) + (1/6)} \right) I_1 = -\frac{1}{2}I_1$

$$y_{21} = \frac{I_2}{V_1} = \frac{-(1/2)I_1}{(1/4)I_1} = -2$$

(b) The following circuit will help us solve for  $y_{22}$  and  $y_{12}$ .



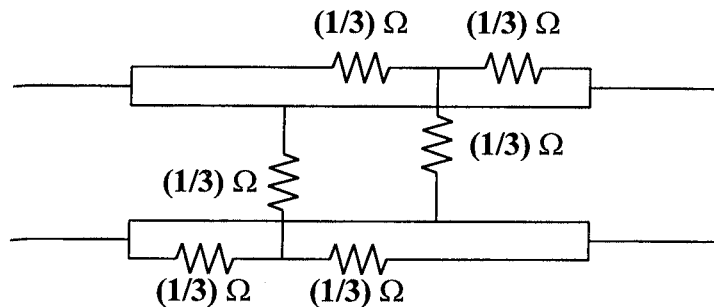
$$y_{22} = \frac{I_2}{V_2} = \frac{I_2}{I_2 \left( \frac{1}{6} + \frac{(1/6)(1/6)}{(1/6) + (1/6)} \right)} = \frac{1}{\left( \frac{1}{6} + \frac{1}{12} \right)} = 4$$

Since,  $V_2 = I_2/4$  and  $I_1 = -\frac{(1/6)}{(1/6) + (1/6)} I_2 = -\frac{1}{2}I_2$

$$y_{12} = \frac{I_1}{V_2} = \frac{-(1/2)I_2}{(1/4)I_2} = -2$$

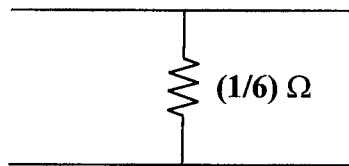
$$[y] = \begin{bmatrix} 4 & -2 \\ -2 & 4 \end{bmatrix} \text{S}$$

**Problem 18.12** Using the circuit in Problem 18.3, connect two of them in parallel so that the top of one is directly connected to the bottom of the other as shown in the following circuit.



Calculate the y parameters for this circuit.

Our job will be easier if we first reduce the circuit using circuit reduction techniques. It is immediately clear that the input is directly connected to the output by two shorts. This produces the circuit below.



This produces a set of y parameters equal to  $[y] = \begin{bmatrix} \infty & -\infty \\ -\infty & \infty \end{bmatrix} \mathbf{S}$ . This makes sense since the short circuit conditions with ideal voltage sources creates infinite currents.

**Problem 18.13** Using the circuit from Problem 18.3, determine the proper manner of connecting two of these circuits in parallel. Our textbooks tell us that if we connect circuits together in parallel, their y matrices add. Thus,

$$[y] = [y_1] + [y_1] = 2[y_1] = 2 \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 4 & -2 \\ -2 & 4 \end{bmatrix}$$

Clearly the first attempt at combining two circuits in parallel (Problem 18.11) produces the above results. Again, it does make a difference how circuits are cascaded. In this case, both circuits must have a common and those commons are to be joined together in order for the cascaded circuit to work.

## COMPUTING TWO-PORT PARAMETERS USING PSPICE

**Problem 18.14** [18.69] Using PSpice, find the transmission parameters for the network in Figure 18.5.

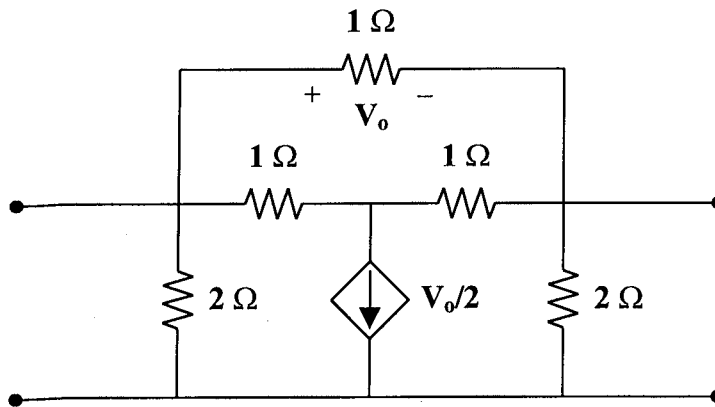
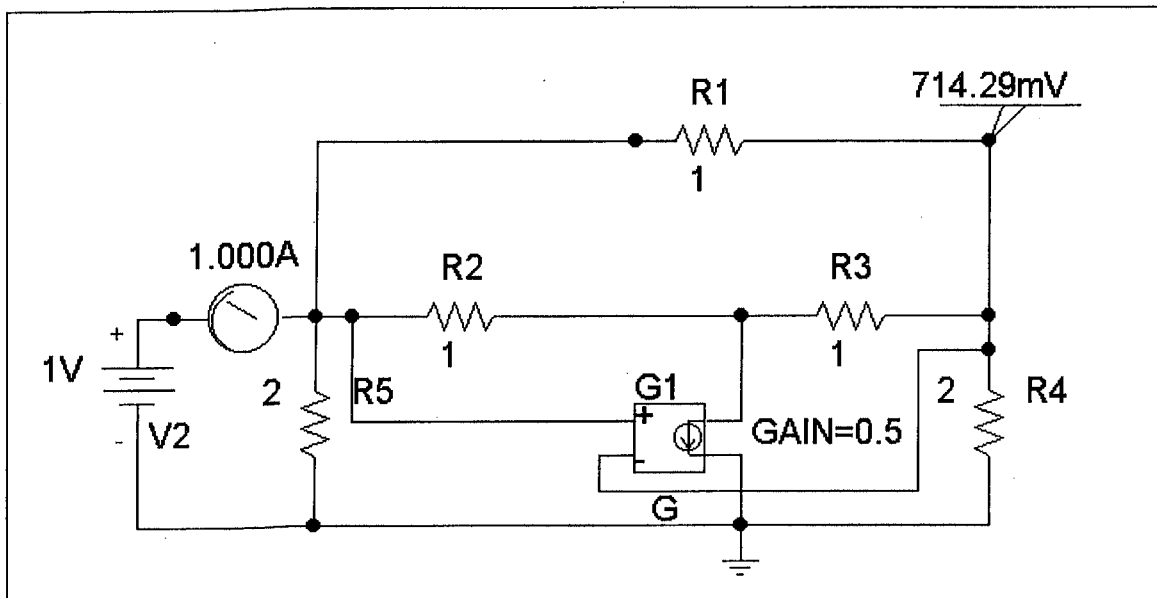


Figure 18.5

(a) Since  $A = \left. \frac{V_1}{V_2} \right|_{I_2=0}$  and  $C = \left. \frac{I_1}{V_2} \right|_{I_2=0}$ , we open-circuit the output port and let  $V_1 = 1$  V. The schematic is as shown below. After simulation, we obtain

$$A = 1/V_2 = 1/0.7143 = 1.4$$

$$C = I_2/V_2 = 1.0/0.7143 = 1.4$$



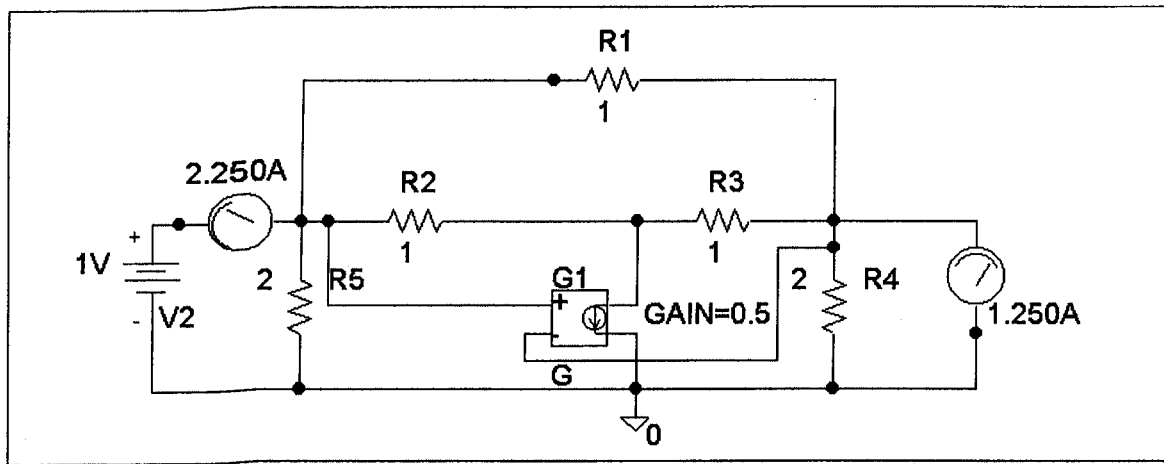
(b) To get B and D, we short-circuit the output port and let  $V_1 = 1$ . The schematic is shown below. After simulating the circuit, we obtain

$$B = -V_1/I_2 = -1/1.25 = -0.8$$

$$D = -I_1/I_2 = -2.25/1.25 = -1.8$$

Thus

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1.4 & -0.8 \\ 1.4 & -1.8 \end{bmatrix}$$

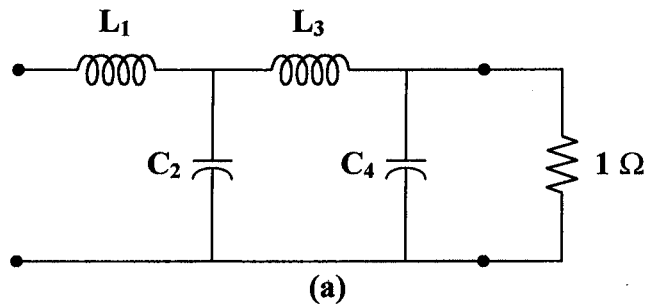


## APPLICATIONS

**Problem 18.15** [18.81] Design an LC ladder network to realize a lowpass filter with transfer function

$$H(s) = \frac{1}{s^4 + 2.613s^3 + 3.414s^2 + 2.613s + 1}$$

This is a fourth-order network which can be realized with the network shown in Fig. (a).





$$\Delta(s) = (s^4 + 3.414s^2 + 1) + (2.613s^3 + 2.613s)$$

$$H(s) = \frac{1}{1 + \frac{2.613s^3 + 2.613s}{s^4 + 3.414s^2 + 1}}$$

which indicates that

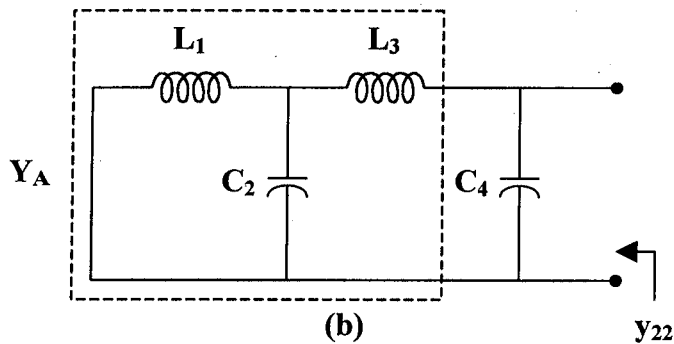
$$y_{21} = \frac{-1}{2.613s^3 + 2.613s}$$

$$y_{22} = \frac{s^4 + 3.414s + 1}{2.613s^3 + 2.613s}$$

We seek to realize  $y_{22}$ . By long division,

$$y_{22} = 0.383s + \frac{2.414s^2 + 1}{2.613s^3 + 2.613s} = sC_4 + Y_A$$

i.e.,  $C_4 = 0.383 \text{ F}$  and  $Y_A = \frac{2.414s^2 + 1}{2.613s^3 + 2.613s}$   
as shown in Fig. (b).



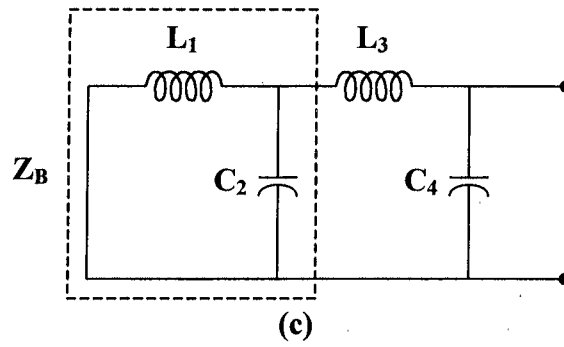
$$Z_A = \frac{1}{Y_A} = \frac{2.613s^3 + 2.613s}{2.414s^2 + 1}$$

By long division,

$$Z_A = 1.082s + \frac{1.531s}{2.414s^2 + 1} = sL_3 + Z_B$$

i.e.,  $L_3 = 1.082 \text{ H}$  and  $Z_B = \frac{1.531s}{2.414s^2 + 1}$

as shown in Fig. (c).



$$Y_B = \frac{1}{Z_B} = 1.577s + \frac{1}{1.531s} = sC_2 + \frac{1}{sL_1}$$

i.e.,  $C_2 = 1.577 \text{ F}$  and  $L_1 = 1.531 \text{ H}$

Thus, the network is shown in Fig. (d).

