CHAPTER NINE

Section 9.1E

1.a.	1	$(\exists x)Fx$ ✓	SM
	2	$(\exists x) \sim Fx$ ✓	SM
	3	Fa	1 ∃D
	4	$\sim Fb$	2 ∃D
		o	

The tree has a completed open branch.

c.	1	$(\exists x)(Fx \,\&\, \sim Gx)$ ✓		SM
	2	$(\forall x)(Fx \supset Gx)$		SM
	3	$Fa \,\&\, \sim Ga$ ✓		1 ∃D
	4	Fa		3 &D
	5	$\sim Ga$		3 &D
	6	$Fa \supset Ga$ ✓		2 ∀D
	7	$\sim Fa$	Ga	6 ⊃D
		×	×	

The tree is closed.

e.	1	$\sim (\forall x)(Fx \supset Gx)$ ✓	SM
	2	$\sim (\exists x)Fx$ ✓	SM
	3	$\sim (\exists x)Gx$ ✓	SM
	4	$(\exists x) \sim (Fx \supset Gx)$ ✓	1 ~ ∀D
	5	$(\forall x) \sim Fx$	2 ~ ∃D
	6	$(\forall x) \sim Gx$	3 ~ ∃D
	7	$\sim (Fa \supset Ga)$ ✓	4 ∃D
	8	Fa	7 ~ ⊃D
	9	$\sim Ga$	7 ~ ⊃D
	10	$\sim Fa$	5 ∀D
		×	

The tree is closed.

g.	1	$(\exists x)Fx$ ✓	SM
	2	$(\exists y)Gy$ ✓	SM
	3	$(\exists z)(Fz \,\&\, Gz)$ ✓	SM
	4	Fa	1 ∃D
	5	Gb	2 ∃D
	6	$Fc \,\&\, Gc$ ✓	3 ∃D
	7	Fc	6 &D
	8	Gc	6 &D
		o	

The tree has a completed open branch.

i.	1	(∀x) (∀y) (Fxy ⊃ Fyx)		SM
	2	(∃x) (∃y) (Fxy & ~ Fyx)✓		SM
	3	(∃y) (Fay & ~ Fya)✓		2 ∃D
	4	Fab & ~ Fba✓		3 ∃D
	5	Fab		4 &D
	6	~ Fba		4 &D
	7	(∀y) (Fay ⊃ Fya)		1 ∀D
	8	Fab ⊃ Fba✓		7 ∀D
	9	~ Fab	Fba	8 ⊃D
		×	×	

The tree is closed.

k.	1	(∃x)Fx ⊃ (∀x)Fx✓		SM
	2	~ (∀x) (Fx ⊃ (∀y)Fy)✓		SM
	3	(∃x) ~ (Fx ⊃ (∀y)Fy)✓		2 ~ ∀D
	4	~ (Fa ⊃ (∀y)Fy)✓		3 ∃D
	5	Fa		4 ~ ⊃D
	6	~ (∀y)Fy✓		4 ~ ⊃D
	7	(∃y) ~ Fy✓		6 ~ ∀D
	8	~ Fb		7 ∃D
	9	~ (∃x)Fx✓	(∀x)Fx	1 ⊃D
	10	(∀x) ~ Fx		9 ~ ∃D
	11	~ Fa		10 ∀D
	12	×	Fb	9 ∀D
			×	

The tree is closed.

m.	1	(∀x) (Fx ⊃ (∃y)Gyx)		SM
	2	~ (∀x) ~ Fx✓		SM
	3	(∀x) (∀y) ~ Gxy		SM
	4	(∃x) ~ ~ Fx✓		2 ~ ∀D
	5	~ ~ Fa✓		4 ∃D
	6	Fa		5 ~ ~ D
	7	Fa ⊃ (∃y)Gya✓		1 ∀D
	8	~ Fa	(∃y)Gya✓	7 ⊃D
	9	×	Gba	8 ∃D
	10		(∀y) ~ Gby	3 ∀D
	11		~ Gba	10 ∀D
			×	

The tree is closed.

o.	1	(∃x)Lxx✓		SM
	2	~ (∃x)(∃y)(Lxy & Lyx)✓		SM
	3	(∀x) ~ (∃y)(Lxy & Lyx)		2 ~ ∃D
	4	Laa		1 ∃D
	5	~ (∃y)(Lay & Lya)✓		3 ∀D
	6	(∀y) ~ (Lay & Lya)		5 ~ ∃D
	7	~ (Laa & Laa)✓		6 ∀D
	8	~ Laa	~ Laa	7 ~ &D
		×	×	

The tree is closed.

q.	1	(∃x)(Fx ∨ Gx)✓		SM
	2	(∀x)(Fx ⊃ ~ Gx)		SM
	3	(∀x)(Gx ⊃ ~ Fx)		SM
	4	~ (∃x)(~ Fx ∨ ~ Gx)✓		SM
	5	(∀x) ~ (~ Fx ∨ ~ Gx)		4 ~ ∃D
	6	Fa ∨ Ga		1 ∃D
	7	Fa ⊃ ~ Ga✓		2 ∀D
	8	Ga ⊃ ~ Fa		3 ∀D
	9	~ (~ Fa ∨ ~ Ga)✓		5 ∀D
	10	~ ~ Fa✓		9 ~ ∨D
	11	~ ~ Ga✓		9 ~ ∨D
	12	Ga		11 ~ ~ D
	13	Fa		10 ~ ~ D
	14	~ Fa	~ Ga	7 ⊃D
		×	×	

The tree is closed.

Section 9.2E

Note: In these answers, whenever a tree is open we give a complete tree. This is because the strategies we have suggested do not uniquely determine the order of decomposition, and so the first open branch to be completed on your tree may not be the first such branch completed on our tree. In accordance with strategy 5, you should stop when your tree has one completed open branch.

1.a.	1	$(\forall x)Fx \vee (\exists y)Gy \checkmark$		SM
	2	$(\exists x)(Fx \mathbin{\&} Gb) \checkmark$		SM
	3	$Fa \mathbin{\&} Gb \checkmark$		2 ∃D
	4	Fa		3 &D
	5	Gb		3 &D
	6	$(\forall x)Fx$	$(\exists y)Gy \checkmark$	1 ∨D
	7	Fa		6 ∀D
	8	Fb		6 ∀D
	9	o	Gc	6 ∃D
			o	

The tree has two completed open branches. The set is quantificationally consistent.

The literals ‘Fa’, ‘Fb’, and ‘Gb’ on the left completed open branch will all be true on any interpretation that makes the following assignments:

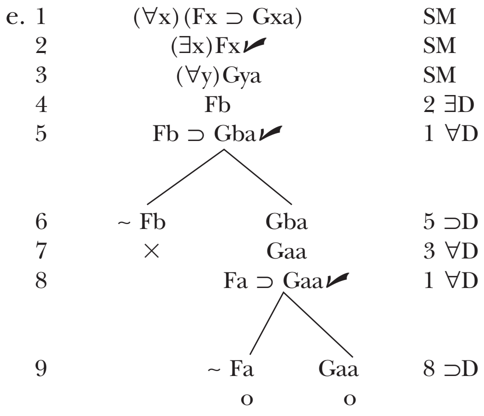
UD: The set {1, 2}
F: {<1>, <2>}
G: {<2>}
a: 1
b: 2

The literals ‘Fa’, ‘Gb’, and ‘Gc’ on the right completed open branch will all be true on any interpretation that makes the following assignments:

UD: The set {1, 2, 3}
F: {<1>}
G: {<2>, <3>}
a: 1
b: 2
c: 3

c.	1	$(\forall x)(Fx \supset Gxa)$		SM
	2	$(\exists x)Fx \checkmark$		SM
	3	$(\forall y) \sim Gya$		SM
	4	Fb		2 ∃D
	5	$Fb \supset Gba \checkmark$		1 ∀D
	6	$\sim Fb$	Gba	5 ⊃D
	7	×	$\sim Gba$	3 ∀D
			×	

The tree is closed. The set is quantificationally inconsistent.

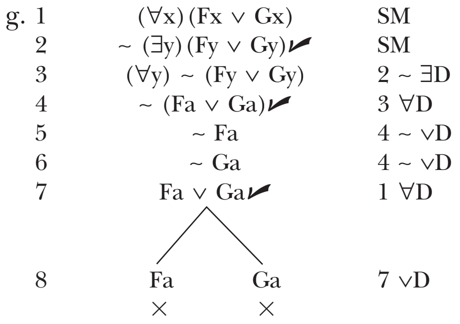


The tree has two completed open branches. The set is quantificationally consistent.

The literals ‘Fb’, ‘Gba’, ‘Gaa’, and ‘~ Fa’ on the left completed open branch will all be true on any interpretation that makes the following assignments:

UD: The set {1, 2}
a: 1
b: 2
F: {<2>}
G: {<1, 1>, <2, 1>}

The literals ‘Fb’, ‘Gba’, ‘Gaa’ on the right completed open branch will also be true on any interpretation that makes these assignments.



The tree is closed. The set is quantificationally inconsistent.

i. 1	(∀z)Hz		SM
2	(∃x)Hx ⊃ (∀y)Fy✓		SM
3	Ha		1 ∀D
4	~ (∃x)Hx✓	(∀y)Fy	2 ⊃D
5	(∀x) ~ Hx		4 ~ ∃D
6	~ Ha		5 ∀D
7	×	Fa	4 ∀D
		o	

The tree has one completed open branch. The set is quantificationally consistent.

The literals ‘Ha’ and ‘Fa’ on the completed open branch will both be true on any interpretation that makes the following assignments:

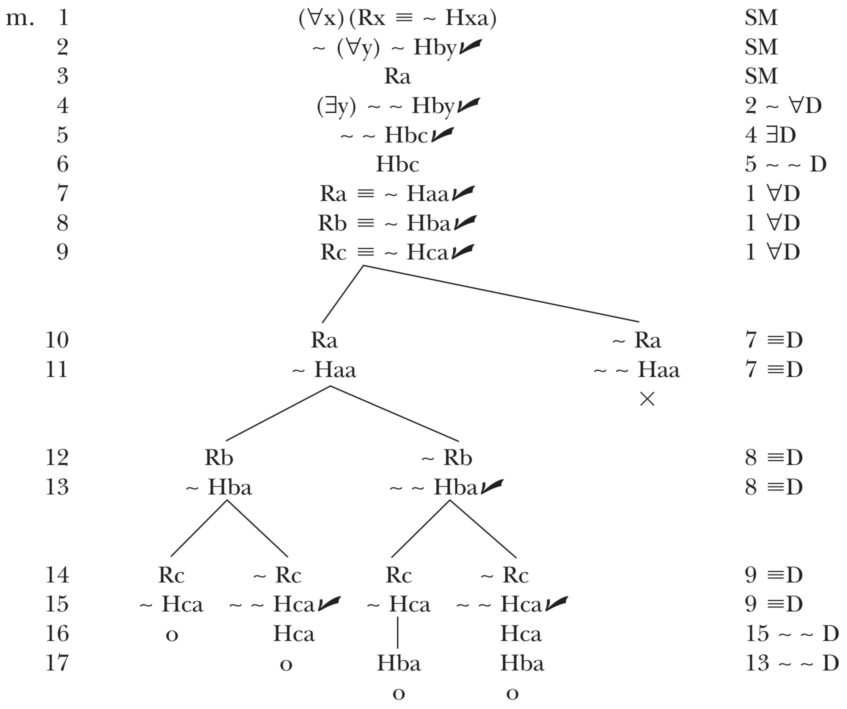
UD: The set {1}
a: 1
F: {<1>}
H: {<1>}

k. 1	(∀x)(∀y)Lxy		SM
2	(∃z) ~ Lza ⊃ (∀z) ~ Lza✓		SM
3	(∀y)Lay		1 ∀D
4	Laa		3 ∀D
5	~ (∃z) ~ Lza✓	(∀z) ~ Lza	2 ⊃D
6		~ Laa	5 ∀D
7	(∀z) ~ ~ Lza	×	5 ~ ∃D
8	~ ~ Laa✓		7 ∀D
9	Laa		8 ~ ~ D
	o		

The tree has one completed open branch. The set is quantificationally consistent.

The literal ‘Laa’ on the completed open branch will be true on any interpretation that makes the following assignments:

UD: The set {1}
L: {<1, 1>}



The tree has four completed open branches (the leftmost four). The set is quantificationally consistent.

The literals ‘Ra’, ‘Rb’, ‘Rc’, ‘Hbc’, ‘~ Haa’, ‘~ Hba’, and ‘~ Hca’ on the leftmost completed open branch will all be true on any interpretation that makes the following assignments:

UD: The set {1, 2, 3}
a: 1
b: 2
c: 3
R: {<1>, <2>, <3>}
H: {<2, 3>}

The literals ‘Ra’, ‘Rb’, ‘Rc’, ‘Hbc’, ‘~ Haa’, ‘~ Hba’, and ‘~ Hca’ on the second completed open branch will also all be true on any interpretation that makes these assignments, as will literals ‘Ra’, ‘Rb’, ‘Rc’, ‘Hbc’, ‘~ Haa’, ‘~ Hba’, and ‘~ Hca’ on the third completed open branch and the literals ‘Ra’, ‘Rb’, ‘Rc’, ‘Hbc’, ‘~ Haa’, ‘~ Hba’, and ‘~ Hca’ on the fourth completed open branch.

Section 9.3E

1.a.	1	~ ((∃x)Fx ∨ ~ (∃x)Fx)✓	SM
	2	~ (∃x)Fx✓	1 ~ ∨D
	3	~ ~ (∃x)Fx✓	1 ~ ∨D
	4	(∀x) ~ Fx	2 ~ ∃D
	5	(∃x)Fx✓	3 ~ ~ D
	6	Fa	5 ∃D
	7	~ Fa	4 ∀D
		×	

The tree is closed. The sentence '(∃x)Fx ∨ ~ (∃x)Fx' is quantificationally true.

c.	1	~ ((∀x)Fx ∨ (∀x) ~ Fx)✓	SM
	2	~ (∀x)Fx✓	1 ~ ∨D
	3	~ (∀x) ~ Fx✓	1 ~ ∨D
	4	(∃x) ~ Fx✓	2 ~ ∀D
	5	(∃x) ~ ~ Fx✓	3 ~ ∀D
	6	~ Fa	4 ∃D
	7	~ ~ Fb✓	5 ∃D
	8	Fb	7 ~ ~ D

The tree has a completed open branch. The sentence '(∀x)Fx ∨ (∀x) ~ Fx' is not quantificationally true.

e.	1	~ ((∀x)Fx ∨ (∃x) ~ Fx)✓	SM
	2	~ (∀x)Fx✓	1 ~ ∨D
	3	~ (∃x) ~ Fx✓	1 ~ ∨D
	4	(∃x) ~ Fx✓	2 ~ ∀D
	5	(∀x) ~ ~ Fx	3 ~ ∃D
	6	~ Fa	4 ∃D
	7	~ ~ Fa✓	5 ∀D
	8	Fa	7 ~ ~ D
		×	

The tree is closed. The sentence '(∀x)Fx ∨ (∃x) ~ Fx' is quantificationally true.

g.	1	~ ((∀x)(Fx ∨ Gx) ⊃ ((∃x) ~ Fx ⊃ (∃x)Gx))✓		SM
	2	(∀x)(Fx ∨ Gx)		1 ~ ⊃D
	3	~ ((∃x) ~ Fx ⊃ (∃x)Gx)✓		1 ~ ⊃D
	4	(∃x) ~ Fx✓		3 ~ ⊃D
	5	~ (∃x)Gx✓		3 ~ ⊃D
	6	(∀x) ~ Gx		5 ~ ∃D
	7	~ Fa		4 ∃D
	8	Fa ∨ Ga✓		2 ∀D
	9	Fa	Ga	8 ∨D
	10	×	~ Ga	6 ∀D
			×	

The tree is closed. The sentence '(∀x)(Fx ∨ Gx) ⊃ [(∃x) ~ Fx ⊃ (∃x)Gx]' is quantificationally true.

i.	1	~ (((∀x)Fx ∨ (∀x)Gx) ⊃ (∀x)(Fx ∨ Gx))✓			SM
	2	(∀x)Fx ∨ (∀x)Gx✓			1 ~ ⊃D
	3	~ (∀x)(Fx ∨ Gx)✓			1 ~ ⊃D
	4	(∃x) ~ (Fx ∨ Gx)✓			3 ~ ∀D
	5	~ (Fa ∨ Ga)✓			4 ∃D
	6	~ Fa			5 ~ ∨D
	7	~ Ga			5 ~ ∨D
	8		(∀x)Fx	(∀x)Gx	2 ∨D
	9		Fa	Ga	8 ∀D
			×	×	

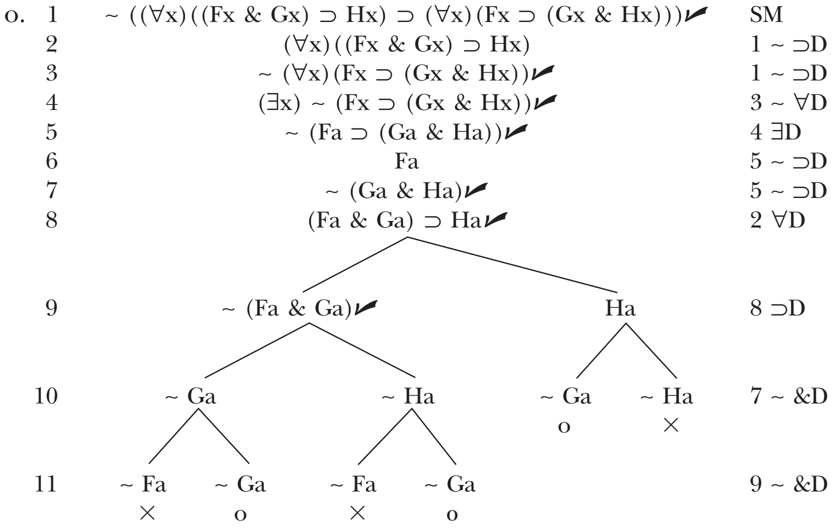
The tree is closed. The sentence ‘((∀x)Fx ∨ (∀x)Gx) ⊃ (∀x)(Fx ∨ Gx)’ is quantificationally true.

k.	1	~ ((∃x)(Fx & Gx) ⊃ ((∃x)Fx & (∃x)Gx))✓			SM
	2	(∃x)(Fx & Gx)✓			1 ~ ⊃D
	3	~ ((∃x)Fx & (∃x)Gx)✓			1 ~ ⊃D
	4	Fa & Ga✓			2 ∃D
	5	Fa			4 &D
	6	Ga			4 &D
	7		~ (∃x)Fx✓	~ (∃x)Gx✓	3 ~ &D
	8		(∀x) ~ Fx	(∀x) ~ Gx	7 ~ ∃D
	9		~ Fa	~ Ga	8 ∀D
			×	×	

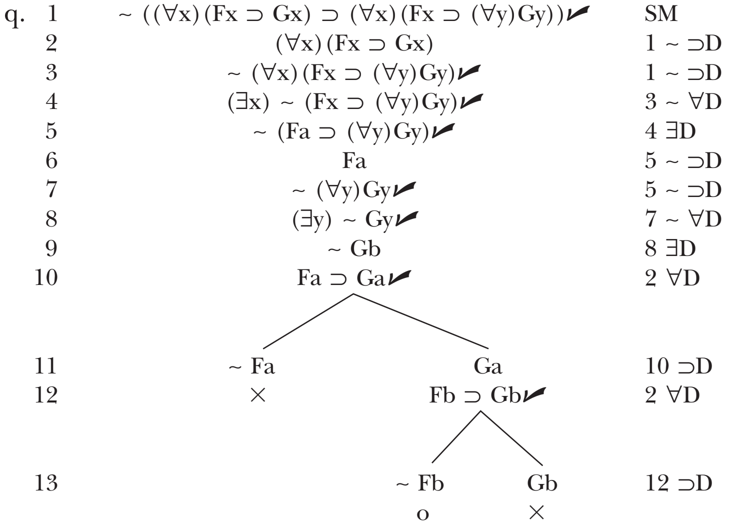
The tree is closed. The sentence ‘(∃x)(Fx & Gx) ⊃ ((∃x)Fx & (∃x)Gx)’ is quantificationally true.

m.	1	~ (~ (∃x)Fx ∨ (∀x) ~ Fx)✓	SM
	2	~ ~ (∃x)Fx✓	1 ~ ∨D
	3	~ (∀x) ~ Fx✓	1 ~ ∨D
	4	(∃x)Fx✓	2 ~ ~ D
	5	(∃x) ~ ~ Fx✓	3 ~ ∀D
	6	Fa	4 ∃D
	7	~ ~ Fb✓	5 ∃D
	8	Fb	7 ~ ~ D
		o	

The tree has a completed open branch. The sentence ‘~ (∃x)Fx ∨ (∀x) ~ Fx’ is not quantificationally true.



The tree has at least one completed open branch. The sentence '(∀x)((Fx & Gx) ⊃ Hx) ⊃ (∀x)(Fx ⊃ (Gx & Hx))' is not quantificationally true.



The tree has a completed open branch. The sentence '(∀x)(Fx ⊃ Gx) ⊃ (∀x)(Fx ⊃ (∀y)Gy)' is not quantificationally true.

s. 1	$\sim ((\forall x)Gxx \supset (\forall x)(\forall y)Gxy)$ ✓	SM
2	$(\forall x)Gxx$	$1 \sim \supset D$
3	$\sim (\forall x)(\forall y)Gxy$ ✓	$1 \sim \supset D$
4	$(\exists x) \sim (\forall y)Gxy$ ✓	$3 \sim \forall D$
5	$\sim (\forall y)Gay$ ✓	$4\ \exists D$
6	$(\exists y) \sim Gay$ ✓	$5 \sim \forall D$
7	$\sim Gab$	$6\ \exists D$
8	Gaa	$2\ \forall D$
9	Gbb	$2\ \forall D$
	o	

The tree has a completed open branch. The sentence '$(\forall x)Gxx \supset (\forall x)(\forall y)Gxy$' is not quantificationally true.

u. 1	$\sim ((\exists x)(\forall y)Gxy \supset (\forall x)(\exists y)Gyx)$ ✓	SM
2	$(\exists x)(\forall y)Gxy$ ✓	$1 \sim \supset D$
3	$\sim (\forall x)(\exists y)Gyx$ ✓	$1 \sim \supset D$
4	$(\exists x) \sim (\exists y)Gyx$ ✓	$3 \sim \forall D$
5	$(\forall y)Gay$	$2\ \exists D$
6	$\sim (\exists y)Gyb$ ✓	$4\ \exists D$
7	$(\forall y) \sim Gyb$	$6 \sim \exists D$
8	Gab	$5\ \forall D$
9	$\sim Gab$	$7\ \forall D$
	×	

The tree is closed. The sentence '$(\exists x)(\forall y)Gxy \supset (\forall x)(\exists y)Gyx$' is quantificationally true.

w. 1	$\sim (((\exists x)Lxx \supset (\forall y)Lyy) \supset (Laa \supset Lgg))$ ✓		SM
2	$(\exists x)Lxx \supset (\forall y)Lyy$ ✓		$1 \sim \supset D$
3	$\sim (Laa \supset Lgg)$ ✓		$1 \sim \supset D$
4	Laa		$3 \sim \supset D$
5	$\sim Lgg$		$3 \sim \supset D$
6	$\sim (\exists x)Lxx$ ✓	$(\forall y)Lyy$	$2 \supset D$
7	$(\forall x) \sim Lxx$		$6 \sim \exists D$
8	$\sim Laa$		$7\ \forall D$
9	×	Lgg	$6\ \forall D$
		×	

The tree is closed. The sentence '$[(\exists x)Lxx \supset (\forall y)Lyy] \supset (Laa \supset Lgg)$' is quantificationally true.

2.a. 1	(∀x)Fx & (∃x) ~ Fx✓	SM
2	(∀x)Fx	1 &D
3	(∃x) ~ Fx✓	1 &D
4	~ Fa	3 ∃D
5	Fa	2 ∀D
	×	

The tree is closed. The sentence is not quantificationally false.

c. 1	(∃x)Fx & (∃x) ~ Fx✓	SM
2	(∃x)Fx✓	1 &D
3	(∃x) ~ Fx✓	1 &D
4	Fa	2 ∃D
5	~ Fb	3 ∃D
	o	

The tree has at least one completed open branch. The sentence '(∀x)Fx & (∃x) ~ Fx' is not quantificationally false.

e. 1	(∀x)(Fx ⊃ (∀y) ~ Fy)		SM
2	Fa ⊃ (∀y) ~ Fy✓		1 ∀D
3	~ Fa	(∀y) ~ Fy	2 ⊃D
4	o	~ Fa	3 ∀D
		o	

The tree has at least one completed open branch. The sentence '(∀x)(Fx ⊃ (∀y) ~ Fy)' is not quantificationally false.

g. 1	(∀x)(Fx ≡ ~ Fx)		SM
2	Fa ≡ ~ Fa✓		1 ∀D
3	Fa	~ Fa	2 ≡D
4	~ Fa	~ ~ Fa✓	2 ≡D
5	×	Fa	4 ~ ~ D
		×	

The tree is closed. The sentence '(∀x)(Fx ≡ ~ Fx)' is quantificationally false.

i. 1	(∃x)(∃y)(Fxy & ~ Fyx)✓	SM
2	(∃y)(Fay & ~ Fya)✓	1 ∃D
3	Fab & ~ Fba✓	2 ∃D
4	Fab	3 &D
5	~ Fba	3 &D
	o	

The tree has a completed open branch. The sentence '(∃x)(∃y)(Fxy & ~ Fyx)' is not quantificationally false.

k. 1	(∀x) (∀y) (Fxy ⊃ ~ Fyx)		SM
2	(∀y) (Fay ⊃ ~ Fya)		1 ∀D
3	Faa ⊃ ~ Faa✓		2 ∀D
4	~ Faa	~ Faa	3 ⊃D
	o	o	

The tree has at least one completed open branch. The sentence '(∀x) (∀y) (Fxy ⊃ ~ Fyx)' is not quantificationally false.

m. 1	(∃x) (∀y) Gxy & ~ (∀y) (∃x) Gxy✓	SM
2	(∃x) (∀y) Gxy✓	1 &D
3	~ (∀y) (∃x) Gxy✓	1 &D
4	(∃y) ~ (∃x) Gxy✓	3 ~ ∀D
5	(∀y) Gay	2 ∃D
6	~ (∃x) Gxb✓	4 ∃D
7	(∀x) ~ Gxb	6 ~ ∃D
8	Gab	5 ∀D
9	~ Gab	7 ∀D
	×	

The tree is closed. The sentence '(∃x) (∀y) Gxy & ~ (∀y) (∃x) Gxy' is quantificationally false.

3.a. 1	~ ((∃x) Fxx ⊃ (∃x) (∃y) Fxy)✓	SM
2	(∃x) Fxx✓	1 ~ ⊃D
3	~ (∃x) (∃y) Fxy✓	1 ~ ⊃D
4	(∀x) ~ (∃y) Fxy	3 ~ ∃D
5	Faa	2 ∃D
6	~ (∃y) Fay✓	4 ∀D
7	(∀y) ~ Fay	6 ~ ∃D
8	~ Faa	7 ∀D
	×	

The tree for the negation of '(∃x) Fxx ⊃ (∃x) (∃y) Fxy' is closed. The latter sentence is quantificationally true.

c. 1	~ ((∃x) (∀y) Lxy ⊃ (∃x) Lxx)✓	SM
2	(∃x) (∀y) Lxy✓	1 ~ ⊃D
3	~ (∃x) Lxx✓	1 ~ ⊃D
4	(∀x) ~ Lxx	3 ~ ∃D
5	(∀y) Lay	2 ∃D
6	~ Laa	4 ∀D
7	Laa	5 ∀D
	×	

The tree for the negation of '(∃x) (∀y) Lxy ⊃ (∃x) Lxx' is closed. The latter sentence is quantificationally true.

e.	1	~ ((∀x)(Fx ⊃ (∃y)Gya)⊃ (Fb ⊃ (∃y)Gya))✓		SM
	2	(∀x)(Fx ⊃ (∃y)Gya)		1 ~ ⊃D
	3	~ (Fb ⊃ (∃y)Gya)✓		1 ~ ⊃D
	4	Fb		3 ~ ⊃D
	5	~ (∃y)Gya✓		3 ~ ⊃D
	6	(∀y) ~ Gya		5 ~ ∃D
	7	Fb ⊃ (∃y)Gya✓		2 ∀D
	8	~ Fb	(∃y)Gya✓	7 ⊃D
	9	×	Gca	8 ∃D
	10		~ Gca	6 ∀D
			×	

The tree for the negation of '(∀x)(Fx ⊃ (∃y)Gya) ⊃ (Fb ⊃ (∃y)Gya)' is closed. The latter sentence is quantificationally true.

g.	1	~ ((∀x)(Fx ⊃ (∀y)Gxy) ⊃ (∃x)(Fx ⊃ ~ (∀y)Gxy))✓		SM
	2	(∀x)(Fx ⊃ (∀y)Gxy)		1 ~ ⊃D
	3	~ (∃x)(Fx ⊃ ~ (∀y)Gxy)✓		1 ~ ⊃D
	4	(∀x) ~ (Fx ⊃ ~ (∀y)Gxy)		3 ~ ∃D
	5	~ (Fa ⊃ ~ (∀y)Gay)✓		4 ∀D
	6	Fa		5 ~ ⊃D
	7	~ ~ (∀y)Gay✓		5 ~ ⊃D
	8	(∀y)Gay		7 ~ ~ D
	9	Fa ⊃ (∀y)Gay✓		2 ∀D
	10	~ Fa	(∀y)Gay	9 ⊃D
	11	×	Gaa	10 ∀D
			o	

1	(∀x)(Fx ⊃ (∀y)Gxy)⊃ (∃x)(Fx ⊃ ~ (∀y)Gxy)✓			SM
2	~ (∀x)(Fx ⊃ (∀y)Gxy)✓	(∃x)(Fx ⊃ ~ (∀y)Gxy)✓		1 ⊃D
3	(∃x) ~ (Fx ⊃ (∀y)Gxy)✓			2 ~ ∀D
4	~ (Fa ⊃ (∀y)Gay)✓			3 ∃D
5	Fa			4 ~ ⊃D
6	~ (∀y)Gay✓			4 ~ ⊃D
7	(∃y) ~ Gay✓			6 ~ ∀D
8	~ Gab			7 ∃D
9	o	Fa ⊃ ~ (∀y)Gay✓		2 ∃D
10		~ Fa	~ (∀y)Gay✓	9 ⊃D
11		o	(∃y) ~ Gay✓	10 ~ ∀D
12			~ Gab	11 ∃D
			o	

Both the tree for the sentence '(∀x)(Fx ⊃ (∀y)Gxy)⊃ (∃x)(Fx ⊃ ~ (∀y)Gxy)' and the tree for its negation have at least one completed open branch. The sentence is quantificationally indeterminate.

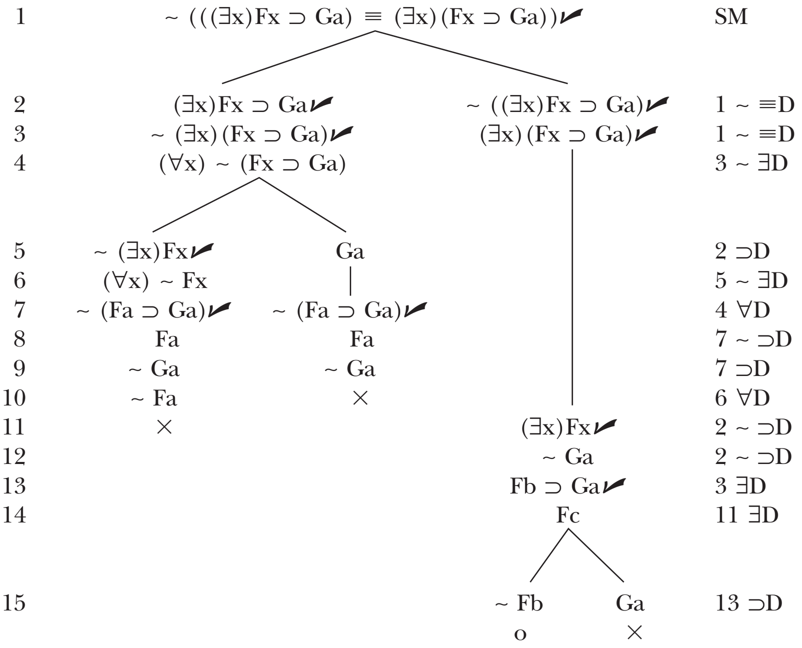
4.a.	1	~ ((∀x)Mxx ≡ ~ (∃x) ~ Mxx)✓		SM
	2	(∀x)Mxx	~ (∀x)Mxx✓	1 ~ ≡D
	3	~ ~ (∃x) ~ Mxx✓	~ (∃x) ~ Mxx✓	1 ~ ≡D
	4	(∃x) ~ Mxx✓		3 ~ ~ D
	5	~ Maa		4 ∃D
	6	Maa		2 ∀D
	7	×	(∃x) ~ Mxx✓	2 ~ ∀D
	8		(∀x) ~ ~ Mxx	3 ~ ∃D
	9		~ Mbb	7 ∃D
	10		~ ~ Mbb✓	8 ∀D
	11		Mbb	10 ~ ~ D
			×	

The tree is closed. The sentences '(∀x)Mxx' and '~ (∃x) ~ Mxx' are quantificationally equivalent.

c.	1	~ ((∀x)(Fa ⊃ Gx) ≡ (Fa ⊃ (∀x)Gx))✓				SM
	2	(∀x)(Fa ⊃ Gx)		~ (∀x)(Fa ⊃ Gx)✓		1 ~ ≡D
	3	~ (Fa ⊃ (∀x)Gx)✓		Fa ⊃ (∀x)Gx✓		1 ~ ≡D
	4	Fa				3 ~ ⊃D
	5	~ (∀x)Gx✓				3 ~ ⊃D
	6	(∃x) ~ Gx✓				5 ~ ∀D
	7	~ Gb				6 ∃D
	8	Fa ⊃ Gb✓				2 ∀D
	9	~ Fa	Gb			8 ⊃D
	10	×	×	(∃x) ~ (Fa ⊃ Gx)✓		2 ~ ∀D
	11			~ (Fa ⊃ Gc)✓		10 ∃D
	12			Fa		11 ~ ⊃D
	13			~ Gc		11 ~ ⊃D
	14			~ Fa	(∀x)Gx	3 ⊃D
	15			×	Gc	14 ∀D
					×	

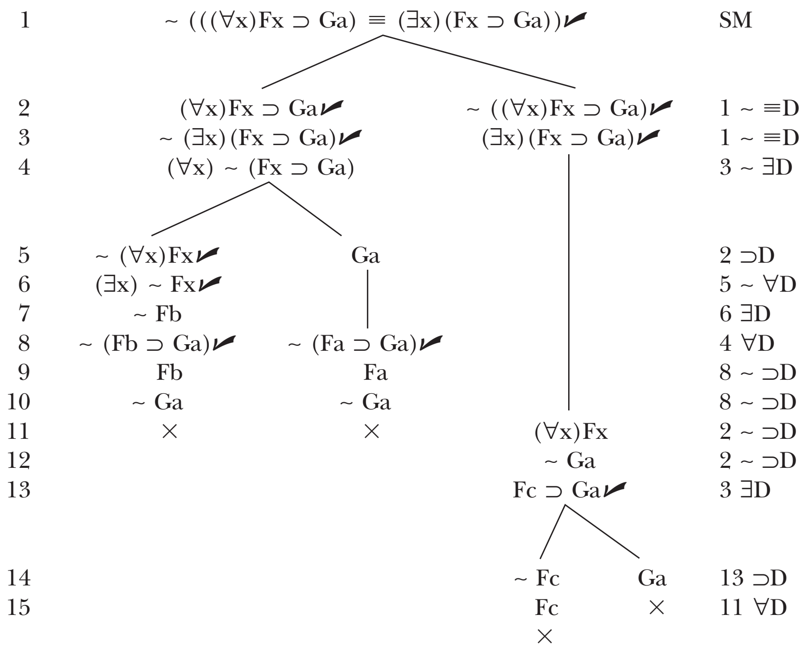
The tree is closed. The sentences '(∀x)(Fa ⊃ Gx)' and 'Fa ⊃ (∀x)Gx' are quantificationally equivalent.

e.



The tree has a completed open branch. The sentences '(∃x)Fx ⊃ Ga' and '(∃x)(Fx ⊃ Ga)' are not quantificationally equivalent.

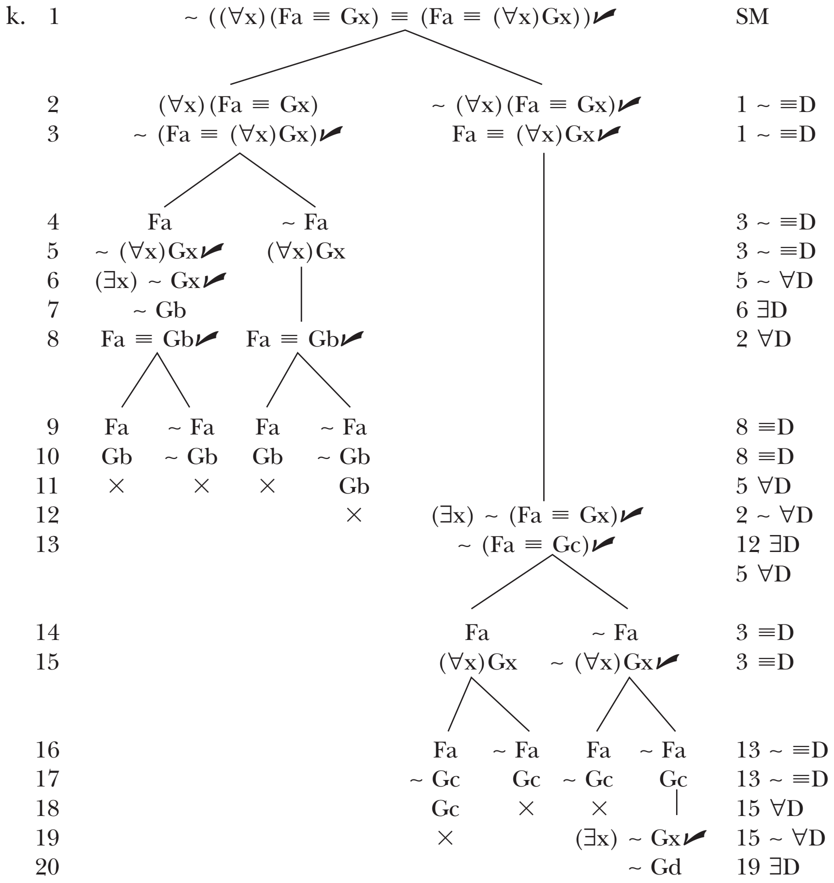
g.



The tree is closed. The sentences '(∀x)Fx ⊃ Ga' and '(∃x)(Fx ⊃ Ga)' are quantificationally equivalent.

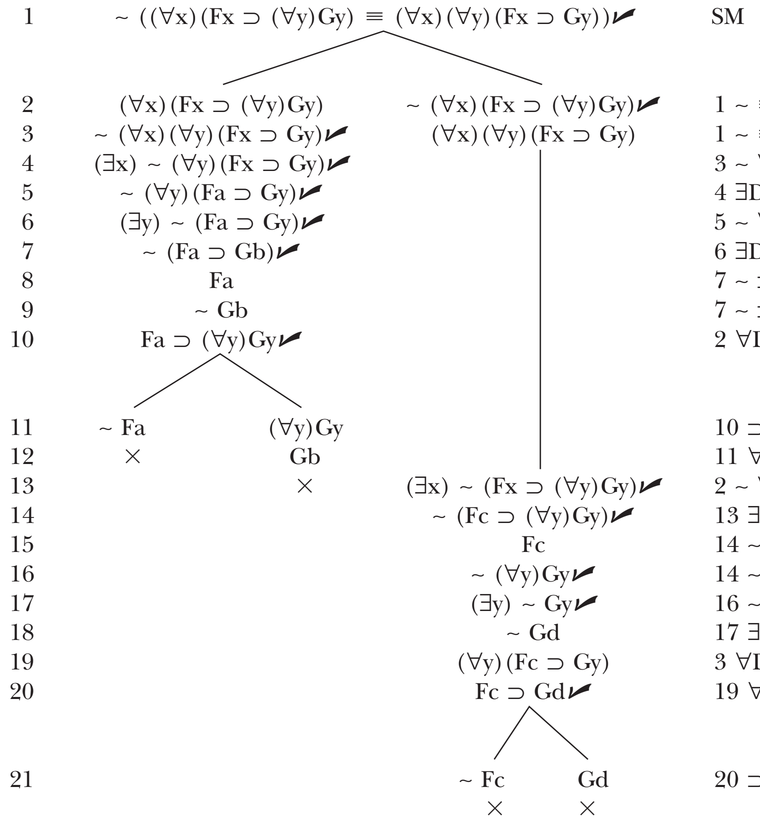
i.	1	~ ((∀x)(∀y)(Fx ⊃ Gy) ≡ (∀x)(Fx ⊃ (∀y)Gy))✓				SM
	2	(∀x)(∀y)(Fx ⊃ Gy)		~ (∀x)(∀y)(Fx ⊃ Gy)✓		1 ~ ≡D
	3	~ (∀x)(Fx ⊃ (∀y)Gy)✓		(∀x)(Fx ⊃ (∀y)Gy)		1 ~ ≡D
	4	(∃x) ~ (Fx ⊃ (∀y)Gy)✓				3 ~ ∀D
	5	~ (Fa ⊃ (∀y)Gy)✓				4 ∃D
	6	Fa				5 ~ ⊃D
	7	~ (∀y)Gy✓				5 ~ ⊃D
	8	(∃y) ~ Gy✓				7 ~ ∀D
	9	~ Gb				8 ∃D
	10	(∀y)(Fa ⊃ Gy)				2 ∀D
	11	Fa ⊃ Gb✓				10 ∀D
	12	~ Fa	Gb			11 ⊃D
	13	×	×	(∃x) ~ (∀y)(Fx ⊃ Gy)✓		2 ~ ∀D
	14			~ (∀y)(Fc ⊃ Gy)✓		13 ∃D
	15			(∃y) ~ (Fc ⊃ Gy)✓		14 ~ ∀D
	16			~ (Fc ⊃ Gd)✓		15 ∃D
	17			Fc		16 ~ ⊃D
	18			~ Gd		16 ~ ⊃D
	19			Fc ⊃ (∀y)Gy✓		3 ∀D
	20			~ Fc	(∀y)Gy	19 ⊃D
	21			×	Gd	20 ∀D
					×	

The tree is closed. The sentences ‘(∀x)(∀y)(Fx ⊃ Gy)’ and ‘(∀x)(Fx ⊃ (∀y)Gy)’ are quantificationally equivalent.



The tree has a completed open branch. The sentences ‘(∀x)(Fa ≡ Gx)’ and ‘Fa ≡ (∀x)Gx’ are not quantificationally equivalent.

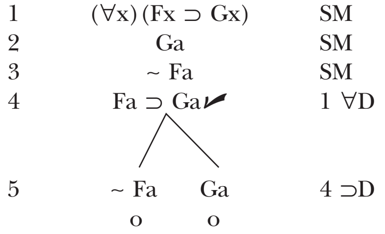
m.



1	~ ((∀x)(Fx ⊃ (∀y)Gy) ≡ (∀x)(∀y)(Fx ⊃ Gy))✓				SM
2	(∀x)(Fx ⊃ (∀y)Gy)		~ (∀x)(Fx ⊃ (∀y)Gy)✓		1 ~ ≡D
3	~ (∀x)(∀y)(Fx ⊃ Gy)✓		(∀x)(∀y)(Fx ⊃ Gy)		1 ~ ≡D
4	(∃x) ~ (∀y)(Fx ⊃ Gy)✓				3 ~ ∀D
5	~ (∀y)(Fa ⊃ Gy)✓				4 ∃D
6	(∃y) ~ (Fa ⊃ Gy)✓				5 ~ ∀D
7	~ (Fa ⊃ Gb)✓				6 ∃D
8	Fa				7 ~ ⊃D
9	~ Gb				7 ~ ⊃D
10	Fa ⊃ (∀y)Gy✓				2 ∀D
11	~ Fa	(∀y)Gy			10 ⊃D
12	×	Gb			11 ∀D
13		×	(∃x) ~ (Fx ⊃ (∀y)Gy)✓		2 ~ ∀D
14			~ (Fc ⊃ (∀y)Gy)✓		13 ∃D
15			Fc		14 ~ ⊃D
16			~ (∀y)Gy✓		14 ~ ⊃D
17			(∃y) ~ Gy✓		16 ~ ∀D
18			~ Gd		17 ∃D
19			(∀y)(Fc ⊃ Gy)		3 ∀D
20			Fc ⊃ Gd✓		19 ∀D
21			~ Fc	Gd	20 ⊃D
			×	×	

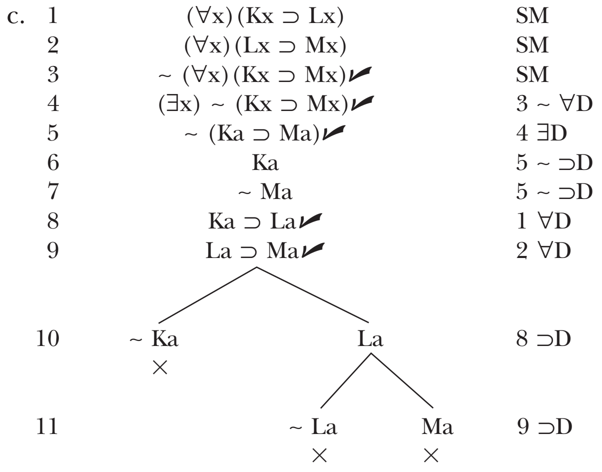
The tree is closed. The sentences '(∀x)(Fx ⊃ (∀y)Gy)' and '(∀x)(∀y)(Fx ⊃ Gy)' are quantificationally equivalent.

5.a.



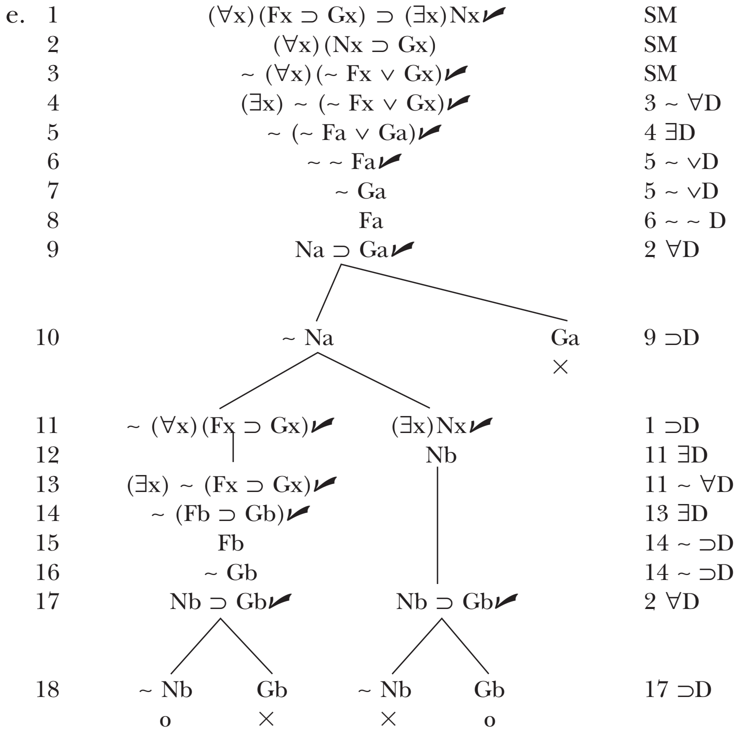
1	(∀x)(Fx ⊃ Gx)		SM
2	Ga		SM
3	~ Fa		SM
4	Fa ⊃ Ga✓		1 ∀D
5	~ Fa	Ga	4 ⊃D
	o	o	

The tree has at least one completed open branch. The argument is quantificationally invalid.



c.	1	(∀x)(Kx ⊃ Lx)			SM
	2	(∀x)(Lx ⊃ Mx)			SM
	3	~ (∀x)(Kx ⊃ Mx)✓			SM
	4	(∃x) ~ (Kx ⊃ Mx)✓			3 ~ ∀D
	5	~ (Ka ⊃ Ma)✓			4 ∃D
	6	Ka			5 ~ ⊃D
	7	~ Ma			5 ~ ⊃D
	8	Ka ⊃ La✓			1 ∀D
	9	La ⊃ Ma✓			2 ∀D
	10	~ Ka ×	La		8 ⊃D
	11		~ La ×	Ma ×	9 ⊃D

The tree is closed. The argument is quantificationally valid.



e.	1	(∀x)(Fx ⊃ Gx) ⊃ (∃x)Nx✓				SM
	2	(∀x)(Nx ⊃ Gx)				SM
	3	~ (∀x)(~ Fx ∨ Gx)✓				SM
	4	(∃x) ~ (~ Fx ∨ Gx)✓				3 ~ ∀D
	5	~ (~ Fa ∨ Ga)✓				4 ∃D
	6	~ ~ Fa✓				5 ~ ∨D
	7	~ Ga				5 ~ ∨D
	8	Fa				6 ~ ~ D
	9	Na ⊃ Ga✓				2 ∀D
	10	~ Na	Ga ×			9 ⊃D
	11	~ (∀x)(Fx ⊃ Gx)✓	(∃x)Nx✓			1 ⊃D
	12		Nb			11 ∃D
	13	(∃x) ~ (Fx ⊃ Gx)✓				11 ~ ∀D
	14	~ (Fb ⊃ Gb)✓				13 ∃D
	15	Fb				14 ~ ⊃D
	16	~ Gb				14 ~ ⊃D
	17	Nb ⊃ Gb✓	Nb ⊃ Gb✓			2 ∀D
	18	~ Nb o	Gb ×	~ Nb ×	Gb o	17 ⊃D

The tree has at least one completed open branch. The argument is quantificationally invalid.

g.	1	(∀x) (~ Ax ⊃ Kx)		SM
	2	(∃y) ~ Ky✓		SM
	3	~ (∃w) (Aw ∨ ~ Lwf)✓		SM
	4	(∀w) ~ (Aw ∨ ~ Lwf)		3 ~ ∃D
	5	~ Ka		2 ∃D
	6	~ Aa ⊃ Ka✓		1 ∀D
	7	~ (Aa ∨ ~ Laf)✓		4 ∀D
	8	~ Aa		7 ~ ∨D
	9	~ ~ Laf✓		7 ~ ∨D
	10	Laf		9 ~ ~ D
	11	~ ~ Aa✓	Ka	6 ⊃D
	12	Aa	×	11 ~ ~ D
		×		

The tree is closed. The argument is quantificationally valid.

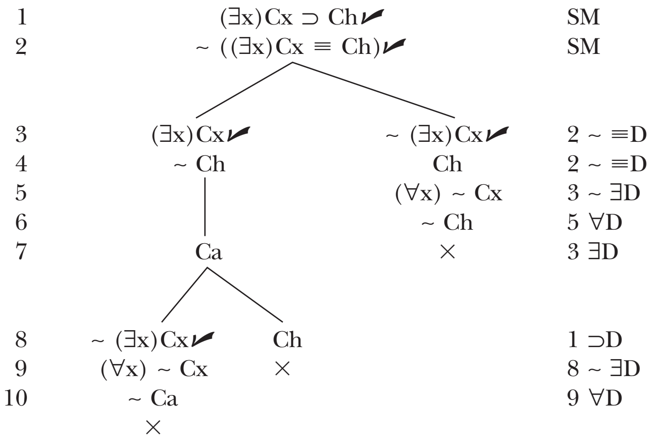
i.	1	(∀x) (∀y) Cxy				SM
	2	~ ((Caa & Cab) & (Cba & Cbb))✓				SM
	3	(∀y) Cay				1 ∀D
	4	(∀y) Cby				1 ∀D
	5	Caa				3 ∀D
	6	Cab				3 ∀D
	7	Cba				4 ∀D
	8	Cbb				4 ∀D
	9	~ (Caa & Cab)✓		~ (Cba & Cbb)✓		2 ~ &D
	10	~ Caa	~ Cab	~ Cba	~ Cbb	9 ~ &D
		×	×	×	×	

The tree is closed. The argument is quantificationally valid.

k.	1	(∀x) (Fx ⊃ Gx)		SM
	2	~ (∃x)Fx✓		SM
	3	~ ~ (∃x)Gx✓		SM
	4	(∃x)Gx✓		3 ~ ~ D
	5	Ga		4 ∃D
	6	(∀x) ~ Fx		2 ~ ∃D
	7	Fa ⊃ Ga✓		1 ∀D
	8	~ Fa		6 ∀D
	9	~ Fa	Ga	7 ⊃D
		o	o	

The tree has at least one completed open branch. The argument is quantificationally invalid.

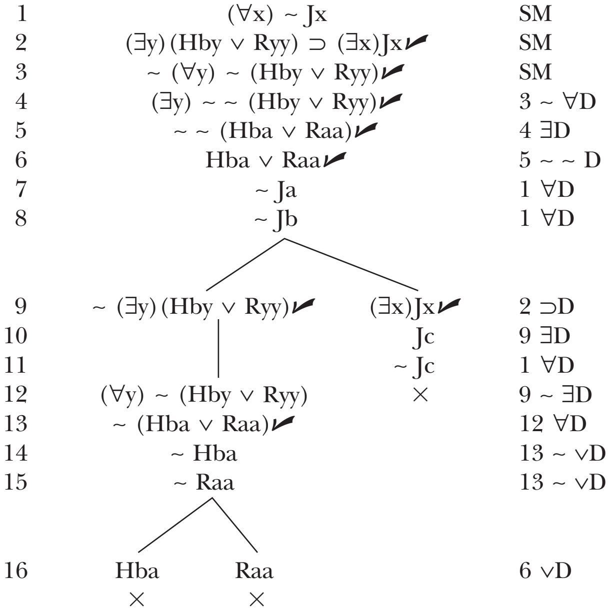
m.



1	$(\exists x)Cx \supset Ch$ ✓			SM
2	$\sim((\exists x)Cx \equiv Ch)$ ✓			SM
3	$(\exists x)Cx$ ✓		$\sim(\exists x)Cx$ ✓	2 ~ ≡D
4	$\sim Ch$		Ch	2 ~ ≡D
5			$(\forall x) \sim Cx$	3 ~ ∃D
6			$\sim Ch$	5 ∀D
7	Ca		×	3 ∃D
8	$\sim(\exists x)Cx$ ✓	Ch		1 ⊃D
9	$(\forall x) \sim Cx$	×		8 ~ ∃D
10	$\sim Ca$			9 ∀D
	×			

The tree is closed. The argument is quantificationally valid.

6.a.



1	$(\forall x) \sim Jx$			SM
2	$(\exists y)(Hby \vee Ryy) \supset (\exists x)Jx$ ✓			SM
3	$\sim(\forall y) \sim (Hby \vee Ryy)$ ✓			SM
4	$(\exists y) \sim\sim (Hby \vee Ryy)$ ✓			3 ~ ∀D
5	$\sim\sim(Hba \vee Raa)$ ✓			4 ∃D
6	$Hba \vee Raa$ ✓			5 ~ ~ D
7	$\sim Ja$			1 ∀D
8	$\sim Jb$			1 ∀D
9	$\sim(\exists y)(Hby \vee Ryy)$ ✓		$(\exists x)Jx$ ✓	2 ⊃D
10			Jc	9 ∃D
11			$\sim Jc$	1 ∀D
12	$(\forall y) \sim (Hby \vee Ryy)$		×	9 ~ ∃D
13	$\sim(Hba \vee Raa)$ ✓			12 ∀D
14	$\sim Hba$			13 ~ ∨D
15	$\sim Raa$			13 ~ ∨D
16	Hba	Raa		6 ∨D
	×	×		

The tree is closed. The entailment does hold.

c.	1	(∀y)((Hy & Fy) ⊃ Gy)			SM
	2	(∀z)Fz & ~ (∀x)Kxb✓			SM
	3	~ (∀x)(Hx ⊃ Gx)✓			SM
	4	(∀z)Fz			2 &D
	5	~ (∀x)Kxb✓			2 &D
	6	(∃x) ~ (Hx ⊃ Gx)✓			3 ~ ∀D
	7	(∃x) ~ Kxb✓			5 ~ ∀D
	8	~ Kab			7 ∃D
	9	~ (Hc ⊃ Gc)✓			6 ∃D
	10	Hc			9 ~ ⊃D
	11	~ Gc			9 ~ ⊃D
	12	(Hc & Fc) ⊃ Gc✓			1 ∀D
	13	~ (Hc & Fc)✓		Gc	12 ⊃D
				×	
	14	~ Hc	~ Fc		13 ~ &D
	15	×	Fc		4 ∀D
			×		

The tree is closed. The entailment does hold.

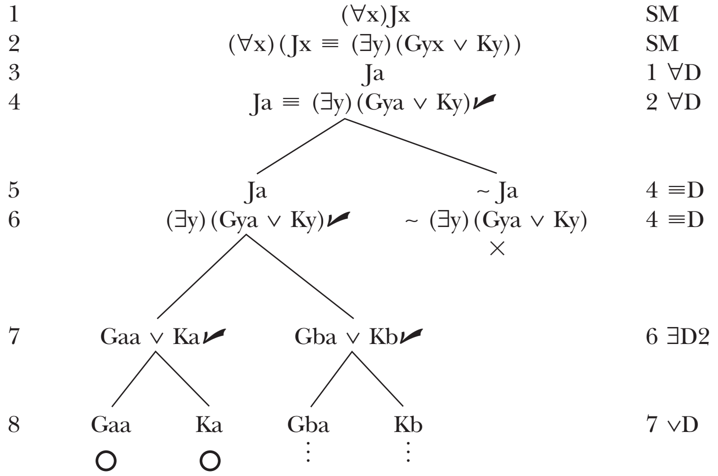
e.	1	(∀z)(Lz ≡ Hz)		SM
	2	(∀x) ~ (Hx ∨ ~ Bx)		SM
	3	~ ~ Lb✓		SM
	4	Lb		3 ~ ~ D
	5	Lb ≡ Hb✓		1 ∀D
	6	Lb	~ Lb	5 ≡D
	7	Hb	~ Hb	5 ≡D
	8	~ (Hb ∨ ~ Bb)✓	×	2 ∀D
	9	~ Hb		8 ~ ∨D
	10	~ ~ Bb		8 ~ ∨D
		×		

The tree is closed. The entailment does hold.

Section 9.4E

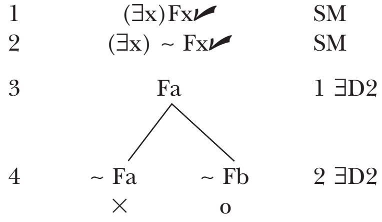
Note: Branches that are open but not completed are so indicated by a series of dots below the branch.

1.a.

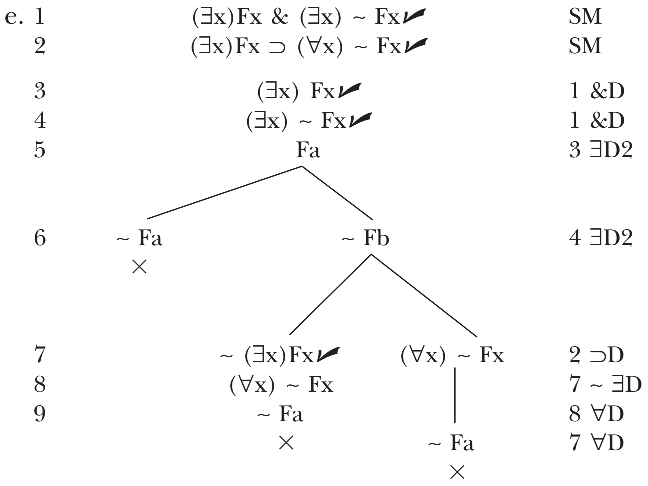


The tree has at least one completed open branch. The set is quantificationally consistent.

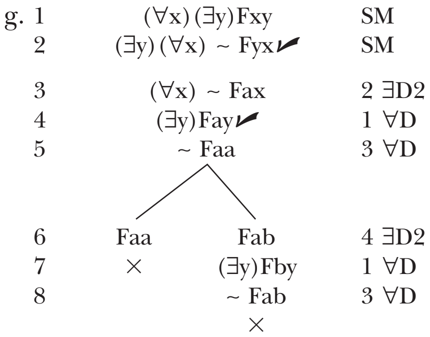
c.



The tree has a completed open branch. The set is quantificationally consistent.

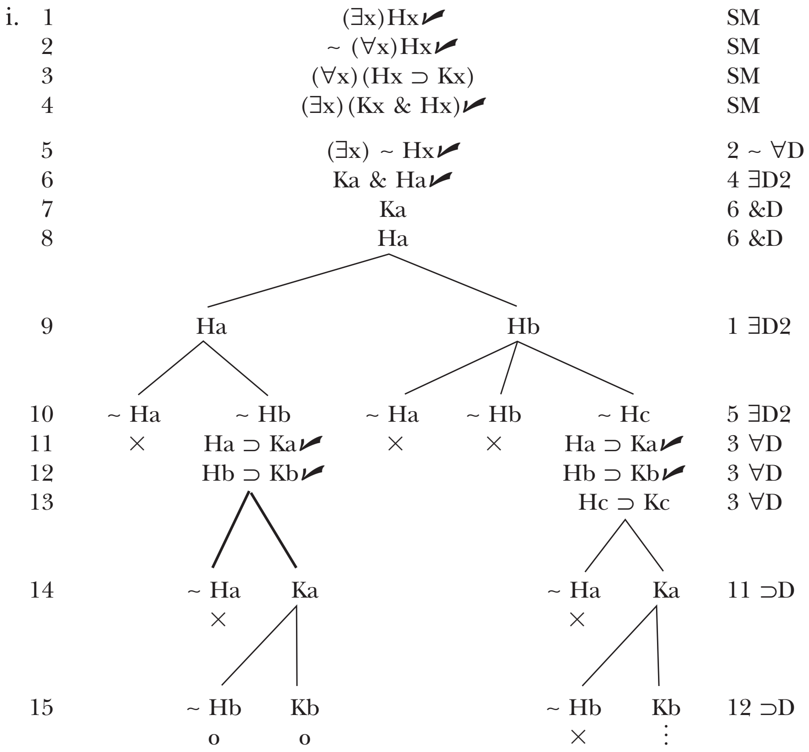


The tree is closed. The set is quantificationally inconsistent.



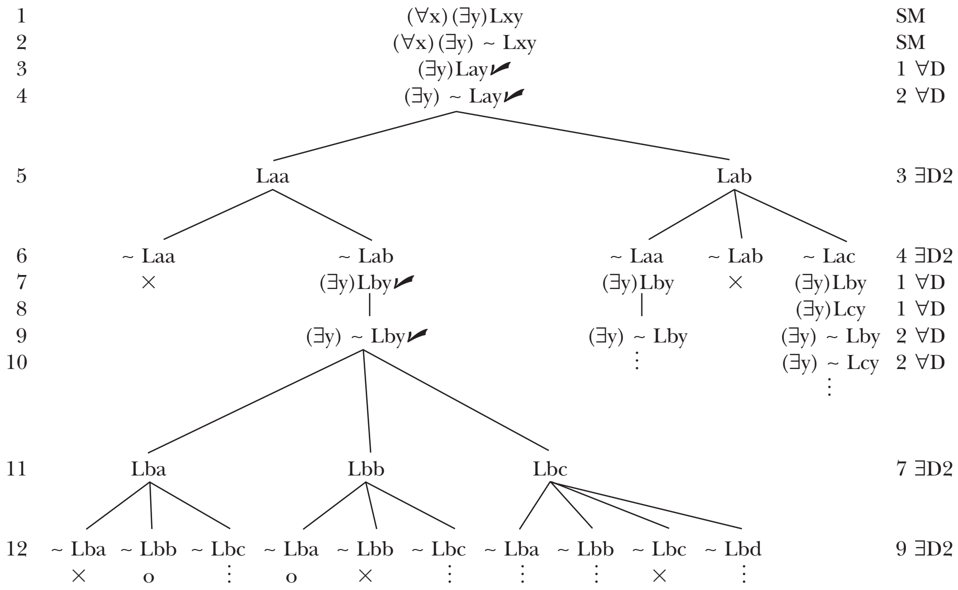
The tree is closed. The set is quantificationally inconsistent.

i.



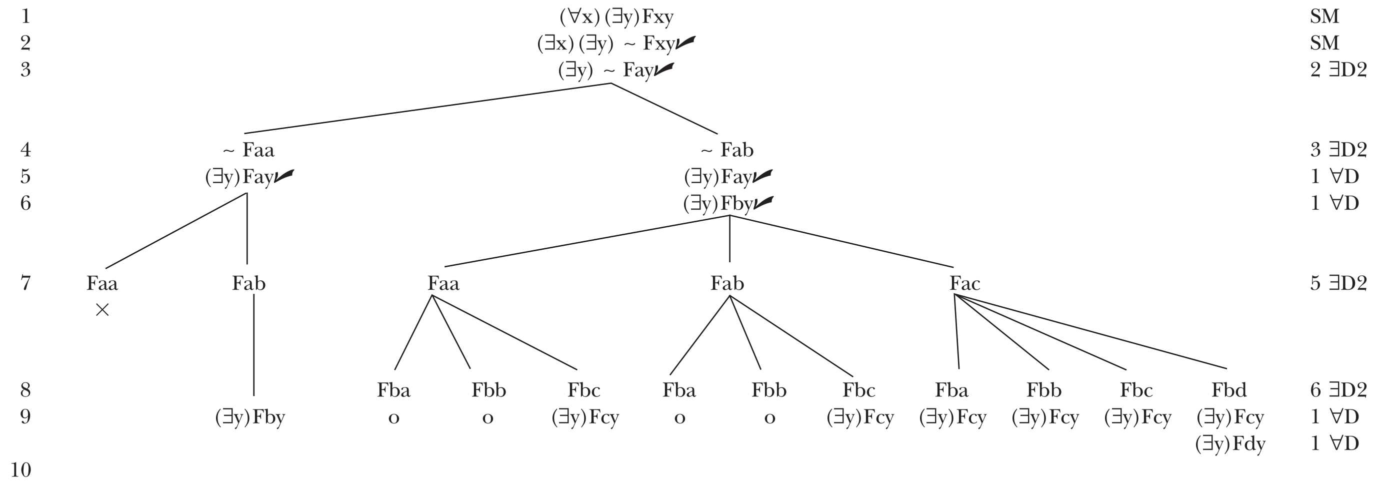
The tree has at least one completed open branch. The set is quantificationally consistent.

k.



The tree has at least one completed open branch. The set is quantificationally consistent.

m.



The tree has two completed open branch. The set is quantificationally consistent.

2.a. 1	(∀x) (Fax ⊃ (∃y)Fya)		SM
2	Faa ⊃ (∃y)Fya ✓		1 ∀D
3	~ Faa	(∃y)Fya	2 ⊃D
	○		

1	~ (∀x) (Fax ⊃ (∃y)Fya) ✓		SM
2	(∃x) ~ (Fax ⊃ (∃y)Fya) ✓		1 ~ ∀D
3	~ (Faa ⊃ (∃y)Fya) ✓	~ (Fab ⊃ (∃y)Fya) ✓	2 ∃D2
4	Faa	Fab	3 ~ ⊃D
5	~ (∃y)Fya ✓	~ (∃y)Fya ✓	3 ~ ⊃D
6	(∀y) ~ Fya	(∀y) ~ Fya	5 ~ ∃D
7	~ Faa	~ Faa	6 ∀D
8	×	~ Fba	6 ∀D
		o	

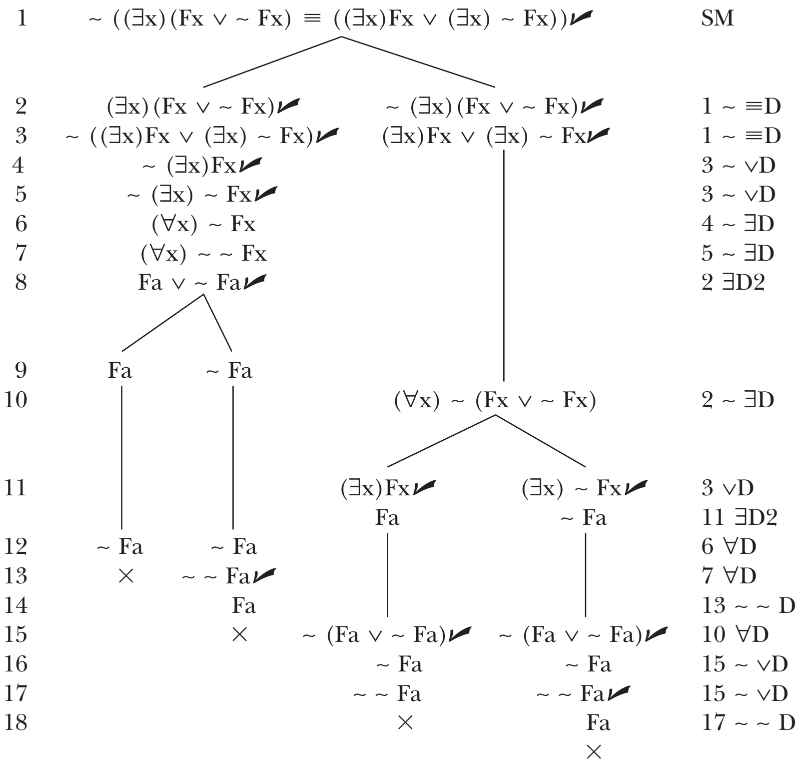
Both the tree for the sentence and the tree for its negation have at least one completed open branch. The sentence is quantificationally indeterminate.

c. 1	~ (∀x) (Fx ⊃ (∀y) (Hy ⊃ Fy)) ✓		SM
2	(∃x) ~ (Fx ⊃ (∀y) (Hy ⊃ Fy)) ✓		1 ~ ∀D
3	~ (Fa ⊃ (∀y) (Hy ⊃ Fy)) ✓		2 ∃D2
4	Fa		3 ~ ⊃D
5	~ (∀y) (Hy ⊃ Fy) ✓		3 ~ ⊃D
6	(∃y) ~ (Hy ⊃ Fy) ✓		5 ~ ∀D
7	~ (Ha ⊃ Fa) ✓	~ (Hb ⊃ Fb) ✓	6 ∃D2
8	Ha	Hb	7 ~ ⊃D
9	~ Fa	~ Fb	7 ~ ⊃D
	×	○	

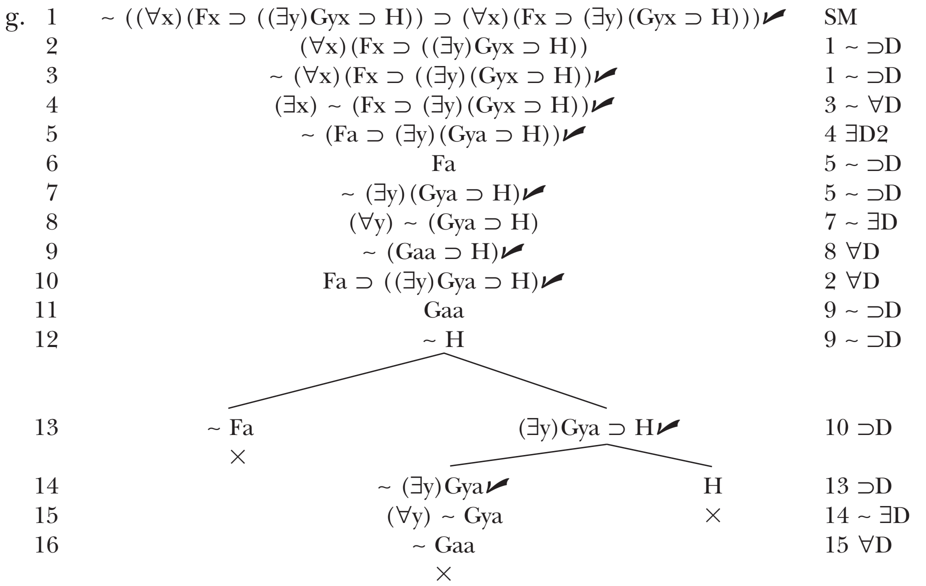
1	(∀x) (Fx ⊃ (∀y) (Hy ⊃ Fy))		SM
2	Fa ⊃ (∀y) (Hy ⊃ Fy) ✓		1 ∀D
3	~ Fa	(∀y) (Hy ⊃ Fy)	2 ⊃D
	o		

Both the tree for the sentence and the tree for its negation have at least one completed open branch. The sentence is quantificationally indeterminate.

e.

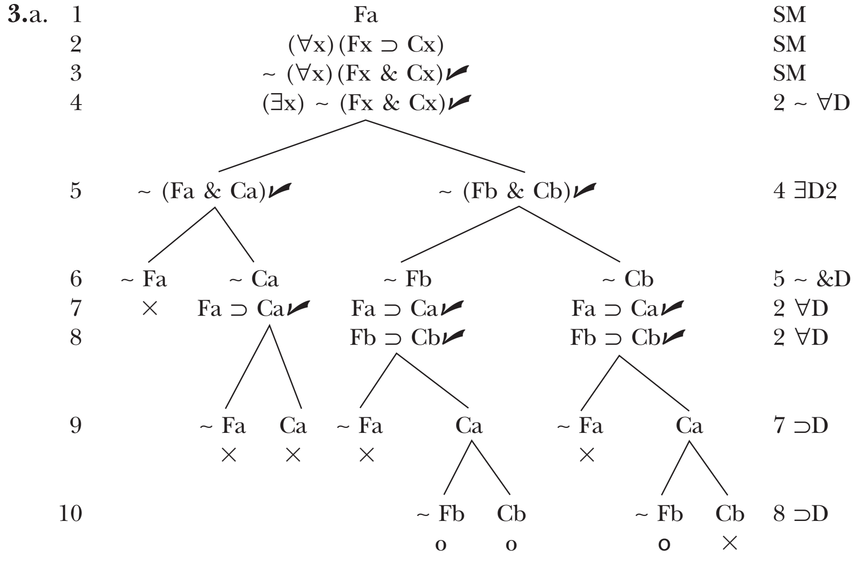


The tree for the negation of the sentence is closed. The sentence is quantificationally true.



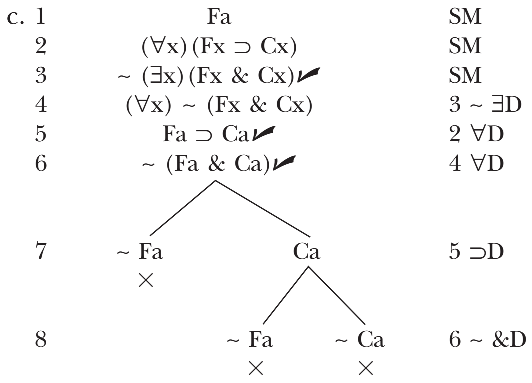
g.	1	~ ((∀x)(Fx ⊃ ((∃y)Gyx ⊃ H)) ⊃ (∀x)(Fx ⊃ (∃y)(Gyx ⊃ H)))✓			SM
	2	(∀x)(Fx ⊃ ((∃y)Gyx ⊃ H))			1 ~ ⊃D
	3	~ (∀x)(Fx ⊃ ((∃y)(Gyx ⊃ H))✓			1 ~ ⊃D
	4	(∃x) ~ (Fx ⊃ (∃y)(Gyx ⊃ H))✓			3 ~ ∀D
	5	~ (Fa ⊃ (∃y)(Gya ⊃ H))✓			4 ∃D2
	6	Fa			5 ~ ⊃D
	7	~ (∃y)(Gya ⊃ H)✓			5 ~ ⊃D
	8	(∀y) ~ (Gya ⊃ H)			7 ~ ∃D
	9	~ (Gaa ⊃ H)✓			8 ∀D
	10	Fa ⊃ ((∃y)Gya ⊃ H)✓			2 ∀D
	11	Gaa			9 ~ ⊃D
	12	~ H			9 ~ ⊃D
	13	~ Fa	(∃y)Gya ⊃ H✓		10 ⊃D
		×			
	14		~ (∃y)Gya✓	H	13 ⊃D
	15		(∀y) ~ Gya	×	14 ~ ∃D
	16		~ Gaa		15 ∀D
			×		

The tree for the negation of the sentence is closed. The sentence is quantificationally true.

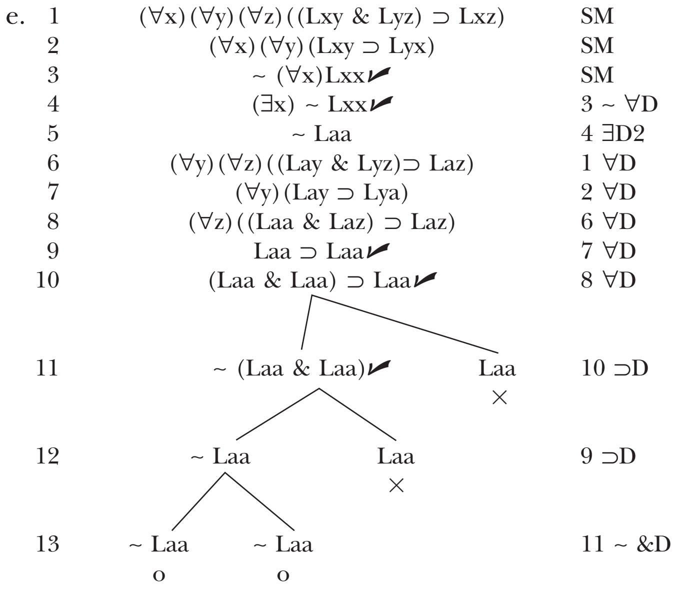


3.a.	1	Fa									SM
	2	(∀x)(Fx ⊃ Cx)									SM
	3	~ (∀x)(Fx & Cx)✓									SM
	4	(∃x) ~ (Fx & Cx)✓									2 ~ ∀D
	5	~ (Fa & Ca)✓			~ (Fb & Cb)✓						4 ∃D2
	6	~ Fa	~ Ca		~ Fb			~ Cb			5 ~ &D
	7	×	Fa ⊃ Ca✓		Fa ⊃ Ca✓			Fa ⊃ Ca✓			2 ∀D
	8				Fb ⊃ Cb✓			Fb ⊃ Cb✓			2 ∀D
	9		~ Fa	Ca	~ Fa	Ca		~ Fa	Ca		7 ⊃D
			×	×	×			×			
	10					~ Fb	Cb		~ Fb	Cb	8 ⊃D
						o	o		o	×	

The tree for the premises and the negation of the conclusion has at least one completed open branch. The argument is quantificationally invalid.

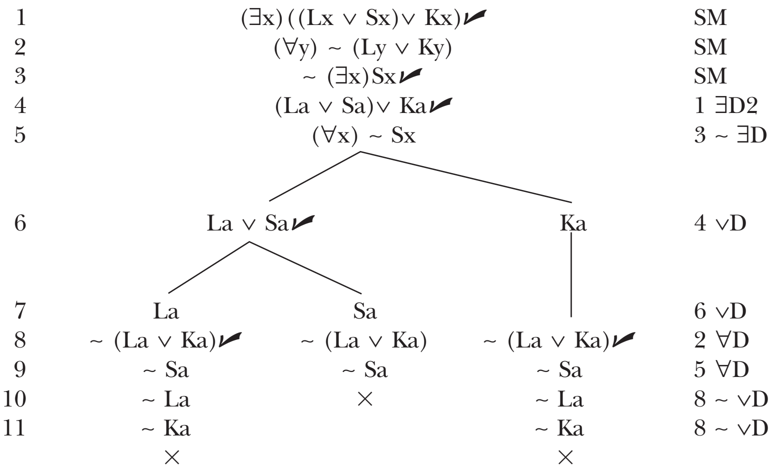


The tree for the premises and the negation of the conclusion is closed. The argument is quantificationally valid.



The tree for the premises and the negation of the conclusion has at least one completed open branch. The argument is quantificationally invalid.

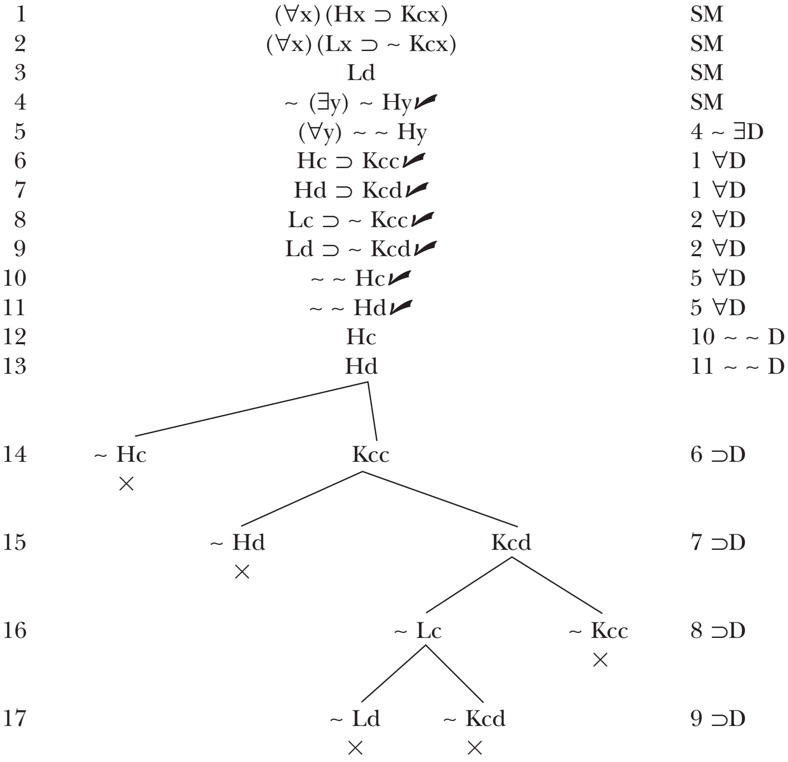
g.



1		(∃x)((Lx ∨ Sx)∨ Kx)✓		SM
2		(∀y) ~ (Ly ∨ Ky)		SM
3		~ (∃x)Sx✓		SM
4		(La ∨ Sa)∨ Ka✓		1 ∃D2
5		(∀x) ~ Sx		3 ~ ∃D
6	La ∨ Sa✓		Ka	4 ∨D
7	La	Sa		6 ∨D
8	~ (La ∨ Ka)✓	~ (La ∨ Ka)	~ (La ∨ Ka)✓	2 ∀D
9	~ Sa	~ Sa	~ Sa	5 ∀D
10	~ La	×	~ La	8 ~ ∨D
11	~ Ka		~ Ka	8 ~ ∨D
	×		×	

The tree for the premises and the negation of the conclusion is closed. The argument is quantificationally valid.

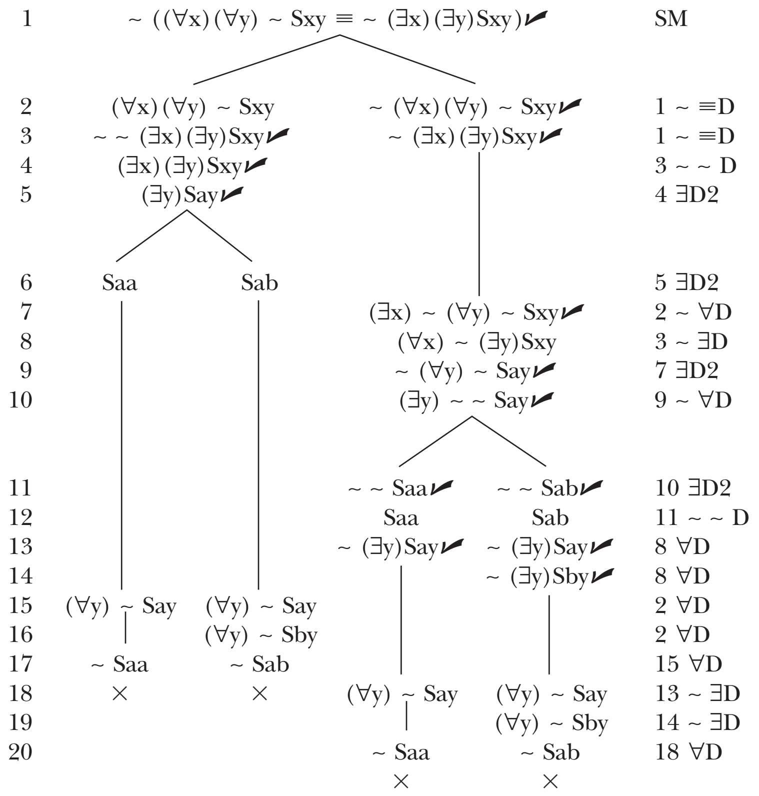
i.



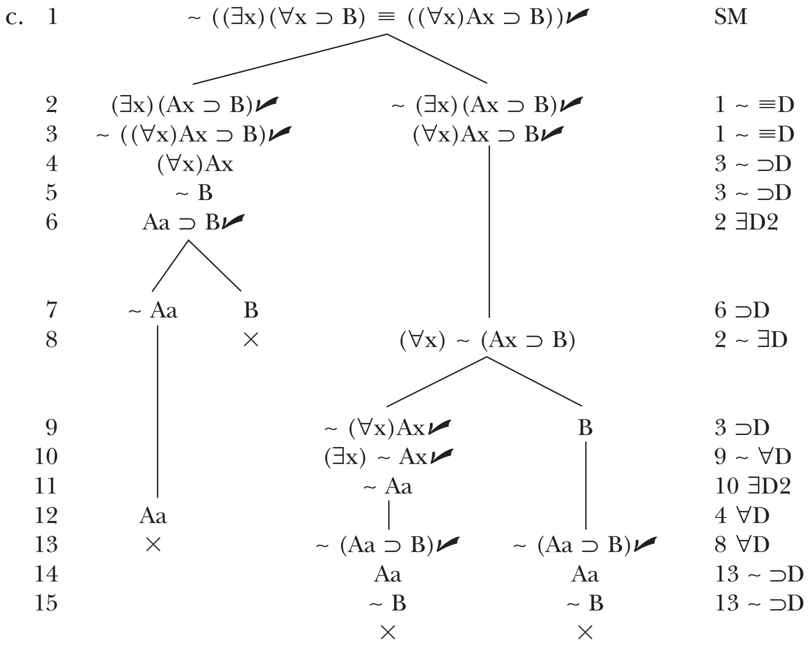
1			(∀x)(Hx ⊃ Kcx)			SM
2			(∀x)(Lx ⊃ ~ Kcx)			SM
3			Ld			SM
4			~ (∃y) ~ Hy✓			SM
5			(∀y) ~ ~ Hy			4 ~ ∃D
6			Hc ⊃ Kcc✓			1 ∀D
7			Hd ⊃ Kcd✓			1 ∀D
8			Lc ⊃ ~ Kcc✓			2 ∀D
9			Ld ⊃ ~ Kcd✓			2 ∀D
10			~ ~ Hc✓			5 ∀D
11			~ ~ Hd✓			5 ∀D
12			Hc			10 ~ ~ D
13			Hd			11 ~ ~ D
14	~ Hc ×		Kcc			6 ⊃D
15		~ Hd ×		Kcd		7 ⊃D
16				~ Lc	~ Kcc ×	8 ⊃D
17			~ Ld ×	~ Kcd ×		9 ⊃D

The tree for the premises and the negation of the conclusion is closed. The argument is quantificationally valid.

4. a.

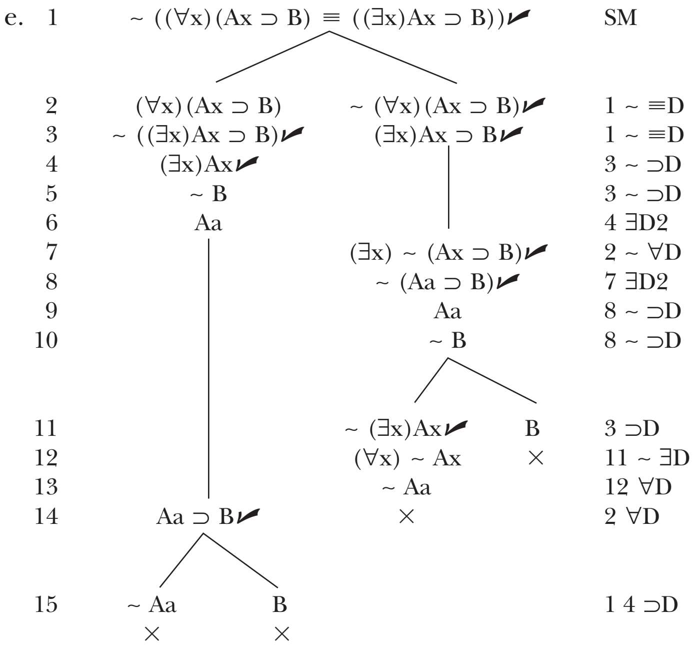


The tree for the negation of the corresponding biconditional is closed. The sentences are quantificationally equivalent.



c.	1	~ ((∃x)(∀x ⊃ B) ≡ ((∀x)Ax ⊃ B))✓				SM
	2	(∃x)(Ax ⊃ B)✓		~ (∃x)(Ax ⊃ B)✓		1 ~ ≡D
	3	~ ((∀x)Ax ⊃ B)✓		(∀x)Ax ⊃ B✓		1 ~ ≡D
	4	(∀x)Ax				3 ~ ⊃D
	5	~ B				3 ~ ⊃D
	6	Aa ⊃ B✓				2 ∃D2
	7	~ Aa	B			6 ⊃D
	8		×	(∀x) ~ (Ax ⊃ B)		2 ~ ∃D
	9			~ (∀x)Ax✓	B	3 ⊃D
	10			(∃x) ~ Ax✓		9 ~ ∀D
	11			~ Aa		10 ∃D2
	12	Aa				4 ∀D
	13	×		~ (Aa ⊃ B)✓	~ (Aa ⊃ B)✓	8 ∀D
	14			Aa	Aa	13 ~ ⊃D
	15			~ B	~ B	13 ~ ⊃D
				×	×	

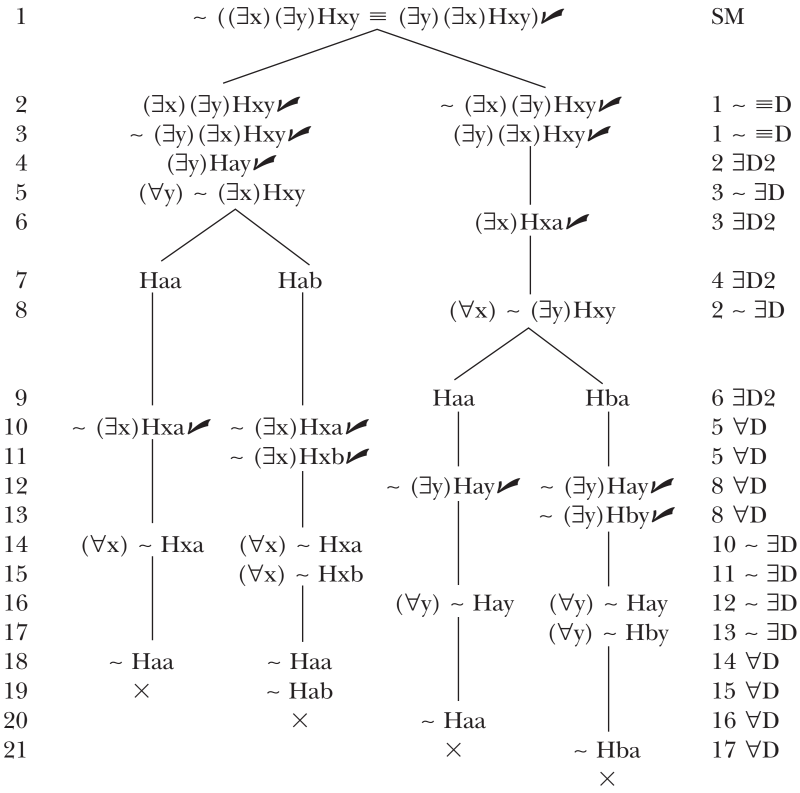
The tree for the negation of the corresponding biconditional is closed. The sentences are quantificationally equivalent.



e.	1	~ ((∀x)(Ax ⊃ B) ≡ ((∃x)Ax ⊃ B))✓				SM
	2	(∀x)(Ax ⊃ B)		~ (∀x)(Ax ⊃ B)✓		1 ~ ≡D
	3	~ ((∃x)Ax ⊃ B)✓		(∃x)Ax ⊃ B✓		1 ~ ≡D
	4	(∃x)Ax✓				3 ~ ⊃D
	5	~ B				3 ~ ⊃D
	6	Aa				4 ∃D2
	7			(∃x) ~ (Ax ⊃ B)✓		2 ~ ∀D
	8			~ (Aa ⊃ B)✓		7 ∃D2
	9			Aa		8 ~ ⊃D
	10			~ B		8 ~ ⊃D
	11			~ (∃x)Ax✓	B	3 ⊃D
	12			(∀x) ~ Ax	×	11 ~ ∃D
	13			~ Aa		12 ∀D
	14	Aa ⊃ B✓		×		2 ∀D
	15	~ Aa	B			1 4 ⊃D
		×	×			

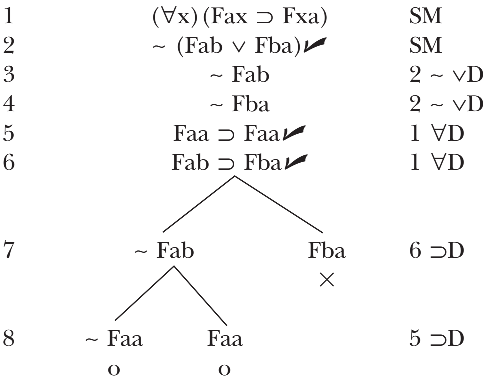
The tree for the negation of the corresponding biconditional is closed. The sentences are quantificationally equivalent.

g.



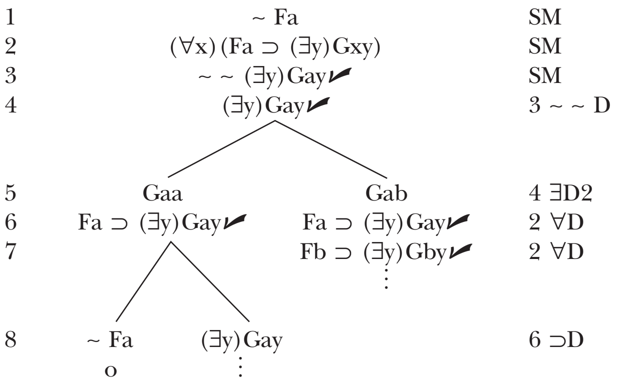
The tree for the negation of the corresponding biconditional is closed. The sentences are quantificationally equivalent.

5.a.



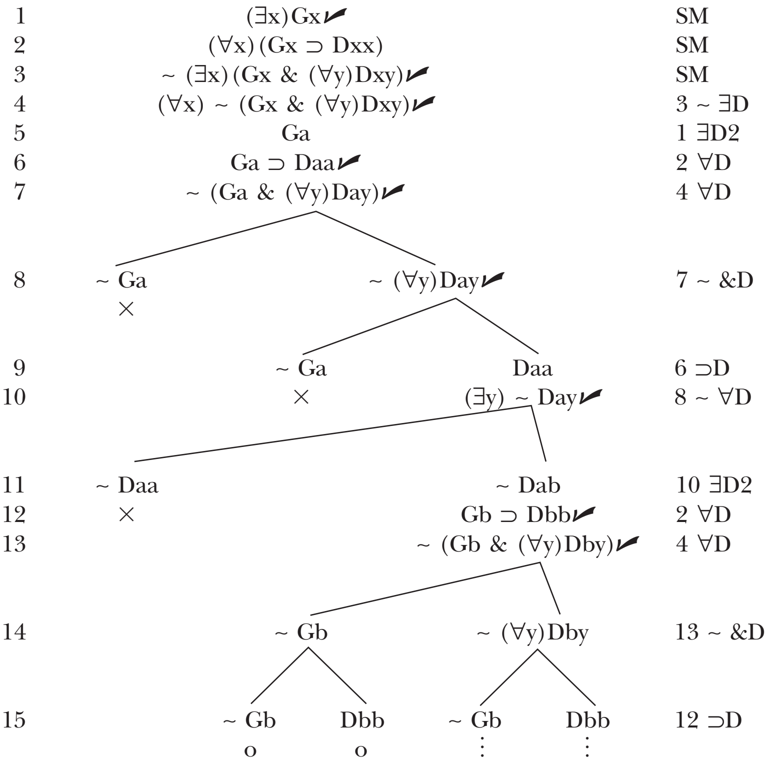
The tree has at least one completed open branch. The entailment does not hold.

c.



The tree has at least one completed open branch. The entailment does not hold.

e.



The tree has at least one completed open branch. The entailment does not hold.

7. If a tree is closed, then on each branch of that tree there is a contradictory pair of literals **P** and ~ **P**. One of these sentences occurs subsequent to the other on the branch in question. Let **Q** be the latter of the two sentences and let **n** be the number of the line on which **Q** occurs. Then **n** is either the last line of the branch or the second to the last line of the branch. The reason is that once both an atomic sentence and its negation have been added to a branch, that branch is closed and no further sentences can be added to the branch after the current decomposition has been completed. (Some decomposition rules do add two sentences to each branch passing through the sentence being decomposed.) Hence such a branch is finite—for no infinite branch can have a last member.

9. No. For example, consider the sentence ‘(∃x)(Fx & ~ Fb)’ and its substitution instance ‘Fb & ~ Fb’. Clearly, every tree for the unit set of the latter sentence closes, but the systematic tree for the unit set of ‘(∃x)(Fx & ~ Fb)’ does not close. Rather, it has a completed open branch:

1	(∃x)(Fx & ~ Fb)✓		SM
2	Fa & ~ Fb✓	Fb & ~ Fb✓	1 ∃D2
3	Fa	Fb	2 &D
4	~ Fb	~ Fb	2 &D
	o	×	

11. Since it is already specified that stage 1 is done before stage 2 and stage 2 before stage 3, and stage 3 before stage 4, we would have to specify the order in which work within each stage is to be done, and what constants are to be used in what order.

Section 9.5E

1.a.	1	(∀x)Fxx	SM
	2	(∃x)(∃y) ~ Fxy✓	SM
	3	(∀x)x = a	SM
	4	(∃y) ~ Fby✓	2 ∃D
	5	~ Fbc	4 ∃D
	6	Faa	1 ∀D
	7	c = a	3 ∀D
	8	Fac	6, 7 =D
	9	b = a	3 ∀D
	10	Fbc	8, 9 =D
		×	

The tree is closed. The set is quantificationally inconsistent.

c. 1	$(\forall x)(x = a \supset Gxb)$		SM
2	$\sim (\exists x)Gxx$ ✓		SM
3	$a = b$		SM
4	$(\forall x) \sim Gxx$		$2 \sim \exists D$
5	$a = a \supset Gab$ ✓		$1\ \forall D$
6	$\sim a = a$	Gab	$5 \supset D$
7	×	$\sim Gaa$	$4\ \forall D$
8		Gaa	3, 6 =D
		×	

The tree is closed. The set is quantificationally inconsistent.

e. 1	$(\forall x)((Fx \,\&\, \sim Gx) \supset \sim x = a)$			SM
2	$Fa \,\&\, \sim Ga$ ✓			SM
3	Fa			2 &D
4	$\sim Ga$			2 &D
5	$(Fa \,\&\, \sim Ga) \supset \sim a = a$ ✓			$1\ \forall D$
6	$\sim (Fa \,\&\, \sim Ga)$ ✓		$\sim a = a$	$5 \supset D$
			×	
7	$\sim Fa$	$\sim \sim Ga$ ✓		$6 \sim \& D$
8	×	Ga		$7 \sim \sim D$
		×		

The tree is closed. The set is quantificationally inconsistent.

g. 1	$(\forall x)(x = a \supset Gxf(b))$		SM
2	$\sim (\exists x)Gxf(x)$ ✓		SM
3	$f(a) = f(b)$		SM
4	$(\forall x) \sim Gxf(x)$		$2 \sim \exists D$
5	$a = a \supset Gaf(b)$ ✓		$1\ \forall D$
6	$\sim a = a$	$Gaf(b)$	$5 \supset D$
7	×	$\sim Gaf(a)$	$4\ \forall D$
8		$Gaf(a)$	3, 6 =D
		×	

The tree is closed. The set is quantificationally inconsistent.

i. 1	(∃x) ~ x = *g*(x)✓	SM
2	(∀x)(∀y)x = *g*(y)	SM
3	~ a = *g*(a)	1 ∃D
4	(∀y)a = *g*(y)	2 ∀D
5	a = *g*(a)	4 ∀D
	×	

The tree is closed. The set is quantificationally inconsistent.

k. 1	(∀x)[Hx ⊃ (∀y)Txy]		SM
2	(∃x)H*f*(x)✓		SM
3	~ (∃x)Txx✓		SM
4	H*f*(a)		2 ∃D
5	(∀x) ~ Txx		3 ~ ∃D
6	H*f*(a) ⊃ (∀y)T*f*(a)y✓		1 ∀D
			6 ⊃D
7	~ H*f*(a)	(∀y)T*f*(a)y)	
	×	T*f*(a)*f*(a)	7 ∀D
		~ T*f*(a)*f*(a)	5 ∀D
		×	

The tree is closed. The set is quantificationally inconsistent.

m. 1	(∃x)Fx ⊃ (∃x)(∃y)*f*(y)= x✓		SM
2	(∃x)Fx✓		SM
3	Fa		2 ∃D
4	~ (∃x)Fx✓	(∃x)(∃y)*f*(y)= x✓	1 ⊃D
5	(∀x) ~ Fx		4 ~ ∃
6	~ Fa		5 ∀D
7	×	(∃y)*f*(y)= b✓	4 ∃D
8		*f*(c)= b	7 ∃D
		o	

The tree has a completed open branch. The set is quantificationally consistent.

The literals ‘Fa’, and ‘*f*(c) = b’ on the completed open branch will be be true on any interpretation that makes the following assignments:

UD: The set {1, 2, 3}
~~a~~: 1
b: 2
c: 3
f: {<1, 1>, <2, 2>, <3, 2>}
F: {<1>}

2.a. 1	~ (a = b ≡ b = a)✓		SM
2	a = b	~ a = b	1 ~ ≡D
3	~ b = a	b = a	1 ~ ≡D
4	~ a = a	~ b = b	2, 3 =D
	×	×	

The tree is closed. The sentence ‘a = b ≡ b = a’ is quantificationally true.

c. 1	~ ((Gab & ~ Gba) ⊃ ~ a = b)✓	SM
2	Gab & ~ Gba✓	1 ~ ⊃D
3	~ ~ a = b✓	1 ~ ⊃D
4	Gab	2 &D
5	~ Gba	2 &D
6	a = b	3 ~ ~ D
7	Gaa	4, 6 =D
8	~ Gaa	5, 6 =D
	×	

The tree is closed. The sentence ‘(Gab & ~ Gba) ⊃ ~ a = b’ is quantificationally true.

e. 1	~ (Fa ≡ (∃x) (Fx & x = a))✓			SM
2	Fa		~ Fa	1 ~ ≡D
3	~ (∃x) (Fx & x = a)✓		(∃x) (Fx & x = a)✓	1 ~ ≡D
4	(∀x) ~ (Fx & x = a)			3 ~ ∃D
5	~ (Fa & a = a)✓			4 ∀D
6	~ Fa	~ a = a		5 ~ &D
7	×	×	Fb & b = a✓	3 ∃D
8			Fb	7 &D
9			b = a	7 &D
10			~ Fb	2, 9 =D
			×	

The tree is closed. The sentence ‘Fa ≡ (∃x) (Fx & x = a)’ is quantificationally true.

g.	1	~ ((∀x)x = a ⊃ ((∃x)Fx ⊃ (∀x)Fx))✓	SM
	2	(∀x)x = a	1 ~ ⊃D
	3	~ ((∃x)Fx ⊃ (∀x)Fx)✓	1 ~ ⊃D
	4	(∃x)Fx✓	3 ~ ⊃D
	5	~ (∀x)Fx✓	3 ~ ⊃D
	6	(∃x) ~ Fx✓	5 ~ ∀D
	7	Fb	4 ∃D
	8	~ Fc	6 ∃D
	9	c = a	2 ∀D
	10	b = a	2 ∀D
	11	c = b	9, 10 =D
	12	Fc	7, 11 =D
		×	

The tree is closed. The sentence '(∀x)x = a ⊃ ((∃x)Fx ⊃ (∀x)Fx)' is quantificationally true.

i.	1	(∀x)(∀y) ~ x = y	SM
	2	(∀y) ~ a = y	1 ∀D
	3	~ a = a	2 ∀D
		×	

The tree is closed. The sentence '(∀x)(∀y) ~ x = y' is quantificationally false.

k.	1	(∃x)(∃y) ~ x = y✓	SM
	2	(∃y) ~ a = y✓	1 ∃D
	3	~ a = b	2 ∃D

1	~ (∃x)(∃y) ~ x = y✓	SM
2	(∀x) ~ (∃y) ~ x = y	1 ~ ∃D
3	~ (∃y) ~ a = y✓	2 ∀D
4	(∀y) ~ ~ a = y	3 ~ ∃D
5	~ ~ a = a✓	4 ∀D
6	a = a	5 ~ ~ D
	o	

Both the tree for the sentence '(∃x)(∃y) ~ x = y' and the tree for its negation have at least one completed open branch. The sentence is quantificationally indeterminate.

m. 1	(∀x)(∀y)((Fx ≡ Fy) ⊃ x = y)			SM
2	(∀y)((Fa ≡ Fy) ⊃ a = y)			1 ∀D
3	(Fa ≡ Fa) ⊃ a = a✓			2 ∀D
4	~ (Fa ≡ Fa)✓		a = a	3 ⊃D
			o	
5	Fa	~ Fa		4 ~ ≡D
6	~ Fa	Fa		4 ~ ≡D
	×	×		

1	~ (∀x)(∀y)((Fx ≡ Fy) ⊃ x = y)✓		SM
2	(∃x) ~ (∀y)((Fx ≡ Fy) ⊃ x = y)✓		1 ~ ∀D
3	~ (∀y)((Fa ≡ Fy) ⊃ a = y)✓		2 ∃D
4	(∃y) ~ ((Fa ≡ Fy) ⊃ a = y)✓		3 ~ ∀D
5	~ ((Fa ≡ Fb) ⊃ a = b)✓		4 ∃D
6	Fa ≡ Fb✓		5 ~ ⊃D
7	~ a = b		5 ~ ⊃D
8	Fa	~ Fa	6 ≡D
9	Fb	~ Fb	6 ≡D
	o	o	

Both the tree for the given sentence ‘(∀x)(∀y)((Fx ≡ Fy) ⊃ x = y)’ and the tree for its negation have at least one completed open branch. The sentence is quantificationally indeterminate.

o. 1	~ (((∃x)Gax & ~ (∃x)Gxa) ⊃ (∀x)(Gxa ⊃ ~ x = a))✓	SM
2	(∃x)Gax & ~ (∃x)Gxa✓	1 ~ ⊃D
3	~ (∀x)(Gxa ⊃ ~ x = a)✓	1 ~ ⊃D
4	(∃x)Gax✓	2 &D
5	~ (∃x)Gxa✓	2 &D
6	(∀x) ~ Gxa	5 ~ ∃D
7	(∃x) ~ (Gxa ⊃ ~ x = a)✓	3 ~ ∀D
8	~ (Gba ⊃ ~ b = a)✓	7 ∃D
9	Gac	4 ∃D
10	Gba	8 ~ ⊃D
11	~ ~ b = a	8 ~ ⊃D
12	~ Gba	6 ∀D
	×	

The tree is closed. The sentence ‘[(∃x)Gax & ~ (∃x)Gxa] ⊃ (∀x)(Gxa ⊃ ~ x = a)’ is quantificationally true.

3.a.	1	$\sim (\exists x)x = f(a)$	SM
	2	$(\forall x) \sim x = f(a)$	1 $\sim \exists$D
	3	$\sim f(a) = f(a)$	2 $\forall$D
		×	

The tree is closed. The sentence ‘$(\exists x)x = f(a)$’ is quantificationally true.

c.	1	$\sim (\exists x)(\exists y)x = y$	SM
	2	$(\forall x) \sim (\exists y)x = y$	1 $\sim \exists$D
	3	$\sim (\exists y)a = y$	2 $\forall$D
	4	$(\forall y) \sim a = y$	3 $\sim \exists$D
	5	$\sim a = a$	4 $\forall$D
		×	

The tree is closed. The sentence ‘$(\exists x)(\exists y)x = y$’ is quantificationally true.

e.	1	$\sim (\forall x)[Gx \supset (\exists y)f(x) = y]$	SM
	2	$(\exists x) \sim [Gx \supset (\exists y)f(x) = y]$	1 $\sim \forall$D
	3	$\sim [Ga \supset (\exists y)f(a) = y]$	2 $\exists$D
	4	Ga	3 $\sim \supset$D
	5	$\sim (\exists y)f(a) = y$	3 $\sim \supset$D
	6	$(\forall y) \sim f(a) = y$	5 $\sim \exists$D
	7	$\sim f(a) = f(a)$	7 $\forall$D
		×	

The tree is closed. The sentence ‘$(\forall x)[Gx \supset (\exists y)f(x) = y]$’ is quantificationally true.

g.	1	$\sim (\forall y) \sim [(\forall x)x = y \vee (\forall x)f(x) = y]$ ✓		SM
	2	$(\exists y) \sim\sim [(\forall x)x = y \vee (\forall x)f(x) = y]$ ✓		1 $\sim \forall$D
	3	$\sim\sim [(\forall x)x = a \vee (\forall x)f(x) = a]$ ✓		2 $\exists$D
	4	$[(\forall x)x = a \vee (\forall x)f(x) = a]$ ✓		3 $\sim\sim$ D
	5	$(\forall x)x = a$	$(\forall x)f(x) = a$	4 $\vee$D
	6	$a = a$	$f(a) = a$	5 $\forall$D
		o		

The tree has a completed open branch. The sentence ‘$(\forall y) \sim [(\forall x)x = y \vee (\forall x) f(x) = y]$’ is not quantificationally true.

4.a.	1	~ (~ a = b ≡ ~ b = a)✓		SM
	2	~ a = b	~ ~ a = b✓	1 ~ ≡D
	3	~ ~ b = a✓	~ b = a	1 ~ ≡D
	4	b = a		3 ~ ~ D
	5	~ b = b		2, 4 = D
	6	×	a = b	2 ~ ~ D
	7		~ a = a	6, 3 =D
			×	

The tree is closed. The sentences '~ a = b' and '~ b = a' are quantificationally equivalent.

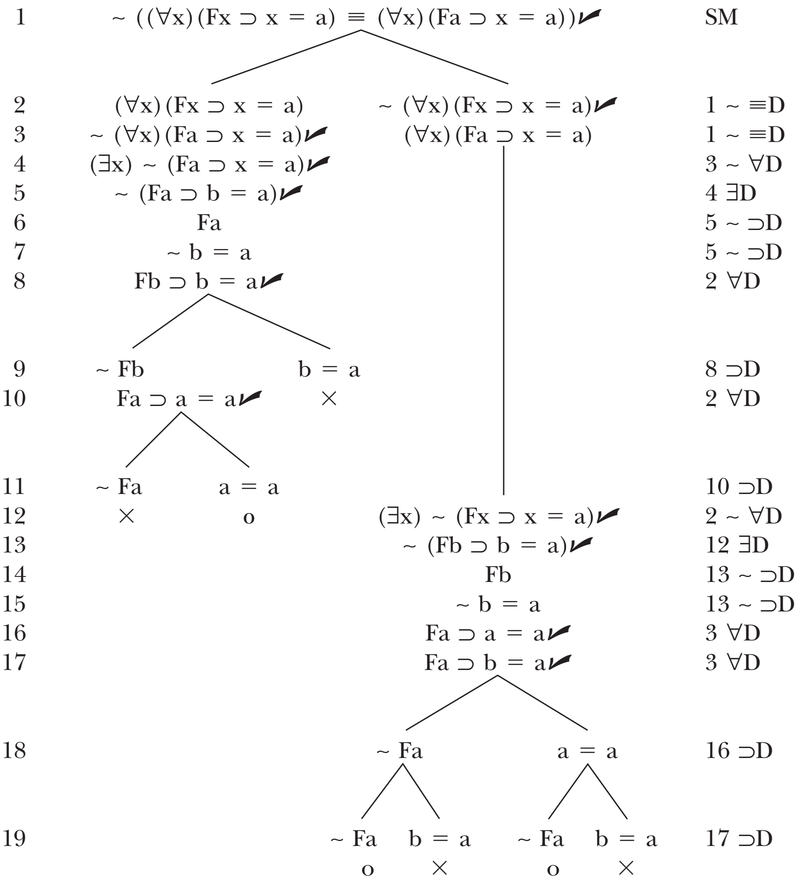
c.	1	~ ((∀x)x = a ≡ (∀x)x = b)✓		SM
	2	(∀x)x = a	~ (∀x)x = a✓	1 ~ ≡D
	3	~ (∀x)x = b✓	(∀x)x = b	1 ~ ≡D
	4	(∃x) ~ x = b✓		3 ~ ∀D
	5	~ c = b		4 ∃D
	6	b = a		2 ∀D
	7	c = a		2 ∀D
	8	c = b		6, 7 =D
	9	×	(∃x) ~ x = a✓	2 ~ ∀D
	10		~ c = a	9 ∃D
	11		c = b	3 ∀D
	12		a = b	3 ∀D
	13		c = a	11, 12 =D
			×	

The tree is closed. The sentences '(∀x)x = a' and '(∀x)x = b' are quantificationally equivalent.

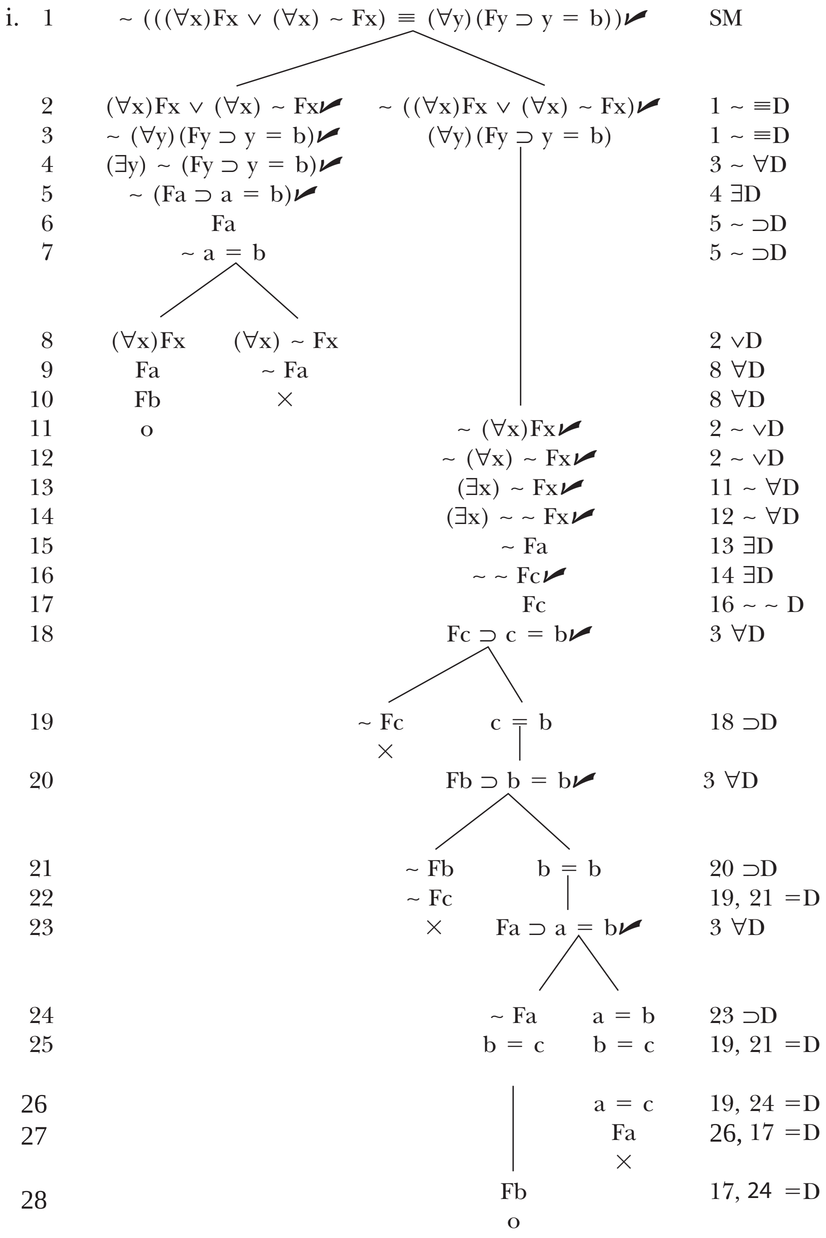
e.	1	~ ((∀x)(∀y)x = y ≡ (∀x)x = a)✓		SM
	2	(∀x)(∀y)x = y	~ (∀x)(∀y)x = y✓	1 ~ ≡D
	3	~ (∀x)x = a✓	(∀x)x = a	1 ~ ≡D
	4	(∃x) ~ x = a✓		3 ~ ∀D
	5	~ b = a		4 ∃D
	6	(∀y)b = y		2 ∀D
	7	b = a		6 ∀D
	8	×	(∃x) ~ (∀y)x = y✓	2 ~ ∀D
	9		~ (∀y)b = y✓	8 ∃D
	10		(∃y) ~ b = y✓	9 ~ ∀D
	11		~ b = c	10 ∃D
	12		b = a	3 ∀D
	13		c = a	3 ∀D
	14		b = c	12, 13 =D
			×	

The tree is closed. The sentences '(∀x)(∀y)x = y' and '(∀x)x = a' are quantificationally equivalent.

g.



The tree has at least one completed open branch. The sentences ‘(∀x)(Fx ⊃ x = a)’ and ‘(∀x)(Fa ⊃ x = a)’ are not quantificationally equivalent.



The tree has at least one completed open branch. The sentences '(∀x)Fx ∨ (∀x) ~ Fx)' and '(∀y)(Fy ⊃ y = b)' are not quantificationally equivalent.

k.	1	~ ((∃x)(x = a & x = b) ≡ a = b)✓			SM
	2	(∃x)(x = a & x = b)✓	~ (∃x)(x = a & x = b)✓		1 ~ ≡D
	3	~ a = b	a = b		1 ~ ≡D
	4		(∀x) ~ (x = a & x = b)		2 ~ ∃D
	5		~ (a = a & a = b)✓		4 ∀D
	6		~ (b = a & b = b)		4 ∀D
	7		~ a = a	~ a = b	5 ~ &D
	8	c = a & c = b✓	×	×	2 ∃D
	9	c = a			8 &D
	10	c = b			8 &D
	11	~ c = b			3, 9 =D
		×			

The tree is closed. The sentences '(∃x)(x = a & x = b)' and 'a = b' are quantificationally equivalent.

5.a.	1	a = b & ~ Bab✓	SM
	2	~ ~ (∀x)Bxx✓	SM
	3	(∀x)Bxx	2 ~ ~ D
	4	a = b	1 &D
	5	~ Bab	1 &D
	6	Bbb	3 ∀D
	7	Bab	4, 6 =D
		×	

The tree is closed. The argument is quantificationally valid.

c.	1	(∀z)(Gz ⊃ (∀y)(Ky ⊃ Hzy))			SM
	2	(Ki & Gj) & i = j✓			SM
	3	~ Hii			SM
	4	Ki & Gj✓			2 &D
	5	i = j			2 &D
	6	Ki			4 &D
	7	Gj			4 &D
	8	Gj ⊃ (∀y)(Ky ⊃ Hjy)✓			1 ∀D
	9	~ Gj	(∀y)(Ky ⊃ Hjy)		8 ⊃D
	10	×	Ki ⊃ Hji✓		9 ∀D
	11		~ Ki	Hji	10 ⊃D
	12		×	Hii	5, 11 =D
				×	

The tree is closed. Therefore the argument is quantificationally valid.

e. 1	a = b	SM
2	~ (Ka ∨ ~ Kb)✓	SM
3	~ Ka	2 ~ ∀D
4	~ ~ Kb✓	2 ~ ∀D
5	Kb	4 ~ ~ D
6	Ka	1, 5 =D
	×	

The tree is closed. The argument is quantificationally valid.

g. 1	(∀x) (x = a ∨ x = b)		SM
2	(∃x) (Fxa & Fbx)✓		SM
3	~ (∃x)Fxx		SM
4	(∀x) ~ Fxx		3 ~ ∃D
5	Fca & Fbc✓		2 ∃D
6	Fca		5 &D
7	Fbc		5 &D
8	c = a ∨ c = b✓		1 ∀D
9			
10	c = a	c = b	8 ∨D
11	Fcc		6, 10 =D
12		Fcc	7, 10 =D
13	~ Fcc	~ Fcc	4 ∀D
	×	×	

The tree is closed. The argument is quantificationally valid.

i. 1	(∀x) (∀y) (Fxy ∨ Fyx)		SM
2	a = b		SM
3	~ (∀x) (Fxa ∨ Fbx)✓		SM
4	(∃x) ~ (Fxa ∨ Fbx)✓		3 ~ ∀D
5	~ (Fca ∨ Fbc)✓		4 ∃D
6	~ Fca		5 ~ ∀D
7	~ Fbc		5 ~ ∀D
8	(∀y) (Fay ∨ Fya)		1 ∀D
9	Fac ∨ Fca✓		8 ∀D
10	Fac	Fca	9 ∀D
11	~ Fac	×	2, 7 =D
	×		

The tree is closed. The argument is quantificationally valid.

k.	1	$(\forall x)(Fx \equiv {\sim} Gx)$		SM
	2	Fa		SM
	3	Gb		SM
	4	${\sim}{\sim}\, a = b$ ✓		SM
	5	$a = b$		$4\ {\sim}{\sim}D$
	6	$Fa \equiv {\sim} Ga$ ✓		$1\ \forall D$
	7	Fa	${\sim} Fa$	$6 \equiv D$
	8	${\sim} Ga$	${\sim}{\sim} Ga$	$6 \equiv D$
	9	Ga	×	$3, 5 = D$
		×		

The tree is closed. The argument is quantificationally valid.

m.	1	$(\forall x)(\forall y)x = y$	SM
	2	${\sim}{\sim}(\exists x)(\exists y)(Fx \,\&\, {\sim} Fy)$ ✓	SM
	3	$(\exists x)(\exists y)(Fx \,\&\, {\sim} Fy)$ ✓	$2\ {\sim}{\sim}D$
	4	$(\exists y)(Fa \,\&\, {\sim} Fy)$ ✓	$3\ \exists D$
	5	$Fa \,\&\, {\sim} Fb$ ✓	$4\ \exists D$
	6	Fa	$5\ \&D$
	7	${\sim} Fb$	$5\ \&D$
	8	$(\forall y)a = y$	$1\ \forall D$
	9	$a = b$	$8\ \forall D$
	10	${\sim} Fa$	$7, 9 = D$
		×	

The tree is closed. The argument is quantificationally valid.

o.	1	$(\forall x)(Hx \supset Hf(x))$		SM
	2	$(\exists z)\ {\sim} Hf(z)$ ✓		SM
	3	${\sim}{\sim}(\forall x)Hx$ ✓		SM
	4	$(\forall x)Hx$		$3\ {\sim}{\sim}D$
	5	${\sim} Hf(a)$		$2\ \exists D$
	6	Ha		$4\ \forall D$
	7	$Ha \supset Hf(a)$ ✓		$1\ \forall D$
	8	${\sim} Ha$	$Hf(a)$	$7 \supset D$
		×	×	

The tree is closed. The argument is quantificationally valid.

q.	1	$(\forall x)(\forall y)(Hxy \equiv \sim Hyx)$		SM
	2	$(\exists x)[Hxf(x) \,\&\, \sim Hf(x)x]$ ✓		SM
	3	$\sim \sim (\forall x)f(x) = x$ ✓		SM
	4	$(\forall x)f(x) = x$		3 ~ ~ D
	5	$Haf(a) \,\&\, \sim Hf(a)a$ ✓		2 ∃D
	6	$Haf(a)$		5 &D
	7	$\sim Hf(a)a$		5 &D
	8	$(\forall y)(Hay \equiv \sim Hya)$		1 ∀D
	9	$Haa \equiv \sim Haa$ ✓		5 ∀D
	10	Haa	$\sim Haa$	9 ≡D
	11	$\sim Haa$	$\sim \sim Haa$	9 ≡D
	12	×	Haa	11 ~ ~ D
			×	

The tree is closed. The argument is quantificationally valid.

s.	1	$(\forall x)[Px \supset (Ox \vee \sim x = f(b))]$			SM
	2	$(\exists x)[(Px \,\&\, \sim Ox) \,\&\, x = f(b)]$ ✓			SM
	3	$\sim Ob$			SM
	4	$(Pa \,\&\, \sim Oa) \,\&\, a = f(b)$ ✓			2 ∃D
	5	$Pa \,\&\, \sim Oa$ ✓			4 &D
	6	$a = f(b)$			4 &D
	7	Pa			5 &D
	8	$\sim Oa$			5 &D
	9	$Pa \supset (Oa \vee \sim a = f(b)$ ✓			1 ∀D
	10	$Pb \supset (Ob \vee \sim b = f(b))$			1 ∀D
	11	$\sim Pa$	$Oa \vee \sim a = f(b)$		9 ⊃D
		×			
			Oa	$\sim a = f(b)$	11 ∨D
			×	×	

The tree is closed. The argument is quantificationally valid.

6.a.

1	(∀x) (Fx ⊃ (∃y) (Gyx & ~ y = x))		SM
2	(∃x)Fx✓		SM
3	~ (∃x) (∃y) ~ x = y✓		SM
4	(∀x) ~ (∃y) ~ x = y		3 ~ ∃D
5	Fa		2 ∃D
6	Fa ⊃ (∃y) (Gya & ~ y = a)✓		1 ∀D
7	~ Fa	(∃y) (Gya & ~ y = a)✓	6 ⊃D
8	×	Gba & ~ b = a✓	7 ∃D
9		Gba	8 &D
10		~ b = a	8 &D
11		~ (∃y) ~ a = y✓	4 ∀D
12		~ (∃y) ~ b = y✓	4 ∀D
13		(∀y) ~ ~ a = y	11 ~ ∃D
14		(∀y) ~ ~ b = y	12 ~ ∃D
15		~ ~ a = a✓	13 ∀D
16		~ ~ a = b✓	13 ∀D
17		~ ~ b = a✓	14 ∀D
18		~ ~ b = b✓	14 ∀D
19		a = a	15 ~ ~ D
20		a = b	16 ~ ~ D
21		b = a	17 ~ ~ D
22		b = b	18 ~ ~ D
23		~ b = b	10, 21 =D
		×	

The tree is closed. The entailment does hold.

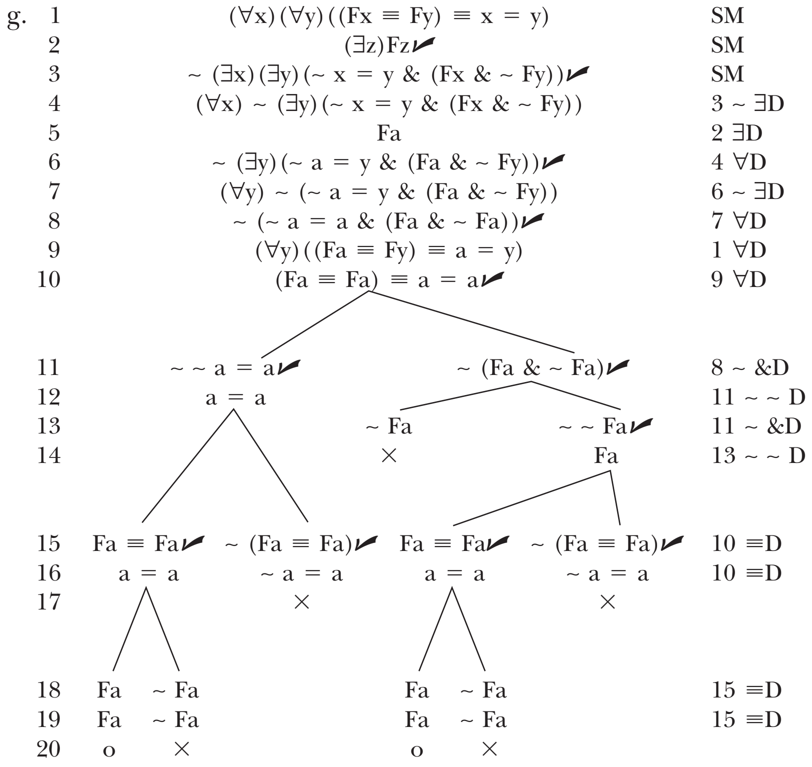
c.

1	(∀x) (Fx ⊃ ~ x = a)		SM
2	(∃x)Fx✓		SM
3	~ (∃x) (∃y) ~ x = y✓		SM
4	Fb		2 ∃D
5	(∀x) ~ (∃y) ~ x = y		3 ~ ∃D
6	Fb ⊃ ~ b = a✓		1 ∀D
7	~ Fb	~ b = a	6 ⊃D
8	×	~ (∃y) ~ a = y✓	5 ∀D
9		(∀y) ~ ~ a = y	8 ~ ∃D
10		~ ~ a = b✓	9 ∀D
11		a = b	10 ~ ~ D
12		~ a = a	7, 11 =D
		×	

The tree is closed. The entailment does hold.

e.	1	(∃w)(∃z) ~ w = z✓	SM
	2	(∃w)Hw✓	SM
	3	~ (∃w) ~ Hw✓	SM
	4	(∀w) ~ ~ Hw	3 ~ ∃D
	5	(∃z) ~ a = z✓	1 ∃D
	6	Hb	2 ∃D
	7	~ a = c	5 ∃D
	8	~ ~ Ha✓	4 ∀D
	9	~ ~ Hb✓	4 ∀D
	10	~ ~ Hc✓	4 ∀D
	11	Ha	8 ~ ~ D
	12	Hb	9 ~ ~ D
	13	Hc	10 ~ ~ D
		o	

The tree has a completed open branch. The entailment does not hold.



The tree has at least one completed open branch. The entailment does not hold.

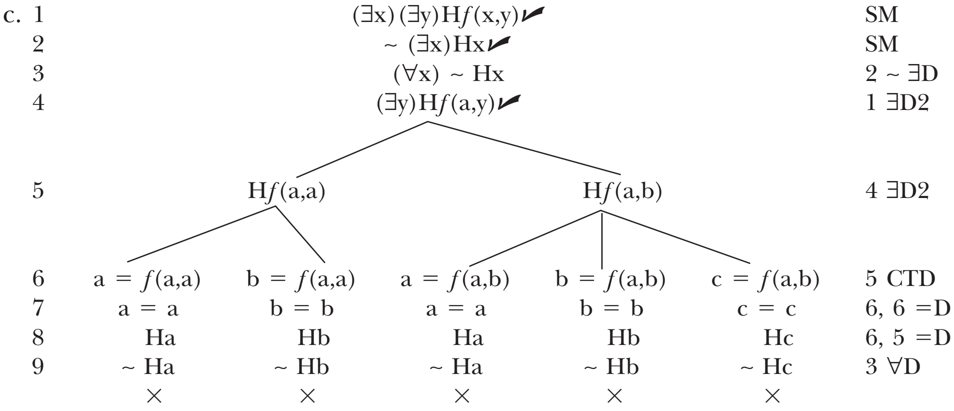
i. 1	$(\forall x)(\forall y)[\sim x = g(y) \supset Gxy]$		SM
2	$\sim (\exists x)Gax$ ✓		SM
3	$\sim (\exists x)a = g(x)$ ✓		SM
4	$(\forall x) \sim a = g(x)$		3 ~ ∃D
5	$\sim a = g(a)$		4 ∀D
6	$(\forall x) \sim Gax$		2 ~ ∃D
7	$(\forall y)[\sim a = g(y) \supset Gay]$		1 ∀D
8	$\sim a = g(a) \supset Gaa$ ✓		7 ∀D
9	$\sim Gaa$		6 ∀D
10	$\sim \sim a = g(a)$ ✓	Gaa	8 ⊃D
11	$a = g(a)$	×	10 ~ ~ D
	×		

The tree is closed. The entailment holds.

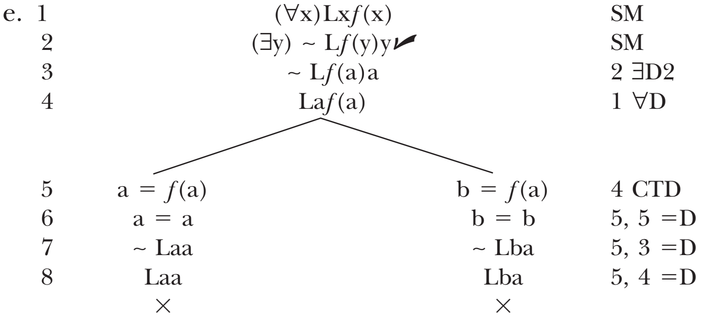
Section 9.6E

1. a. 1	$(\forall x)(\forall y)[\sim x = g(y) \supset Gxy]$			SM
2	$\sim (\exists x)Gax$ ✓			SM
3	$(\forall x) \sim Gax$			2 ~ED
4	$(\forall y)[\sim a = g(y) \supset Gay]$			1 ∀D
5	$\sim Gaa$			3 ∀D
6	$\sim a = g(a) \supset Gaa$ ✓			4 ∀D
7	$\sim \sim a = g(a)$ ✓		Gaa	6 ⊃D
8	$a = g(a)$		×	7 ~ ~ D
9	$a = g(a)$	$b = g(a)$		8 CTD
10	$a = a$	$a = a$		8, 8 =D
11	o	$a = b$		9, 8 =D
12		$b = a$		8, 9 =D

This systematic tree has a completed open branch. The set is quantificationally consistent.

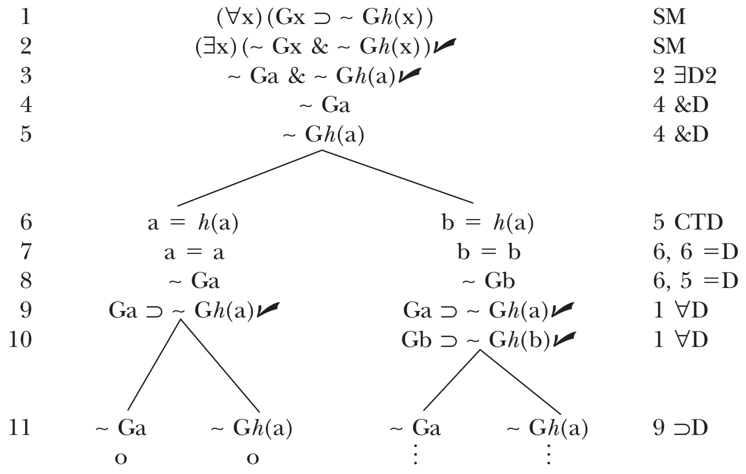


This systematic tree is closed. The set is quantificationally inconsistent.



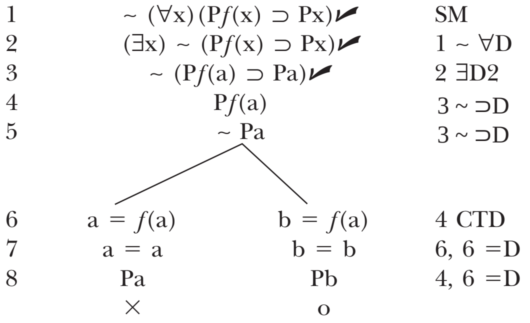
This systematic tree is closed. The set is quantificationally inconsistent.

g.

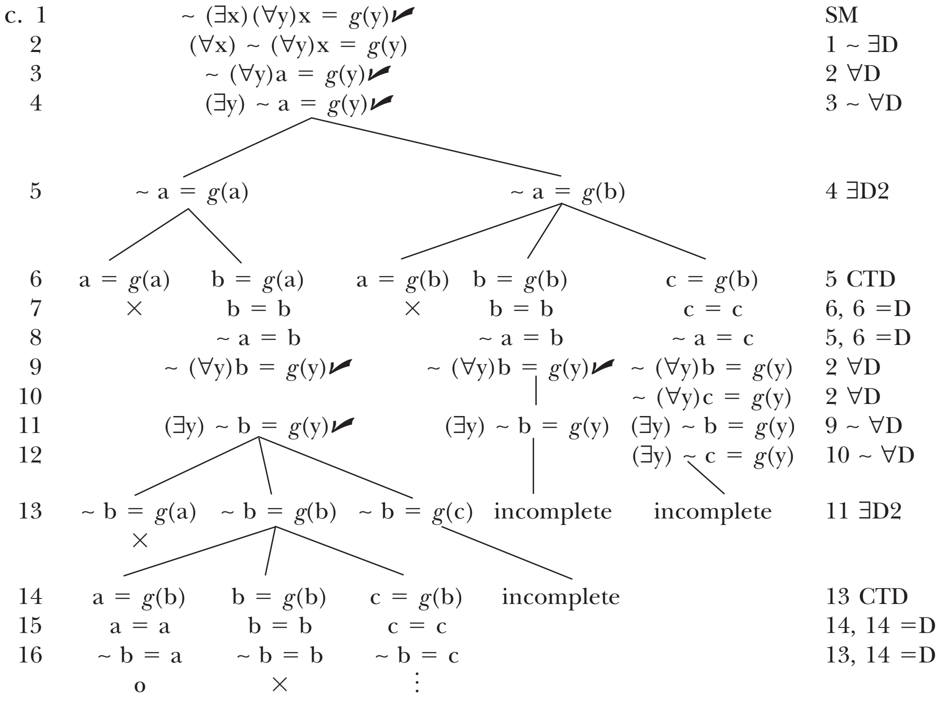


This systematic tree has at least one completed open branch (in fact it has two, the left two). The set is quantificationally consistent.

2.a.

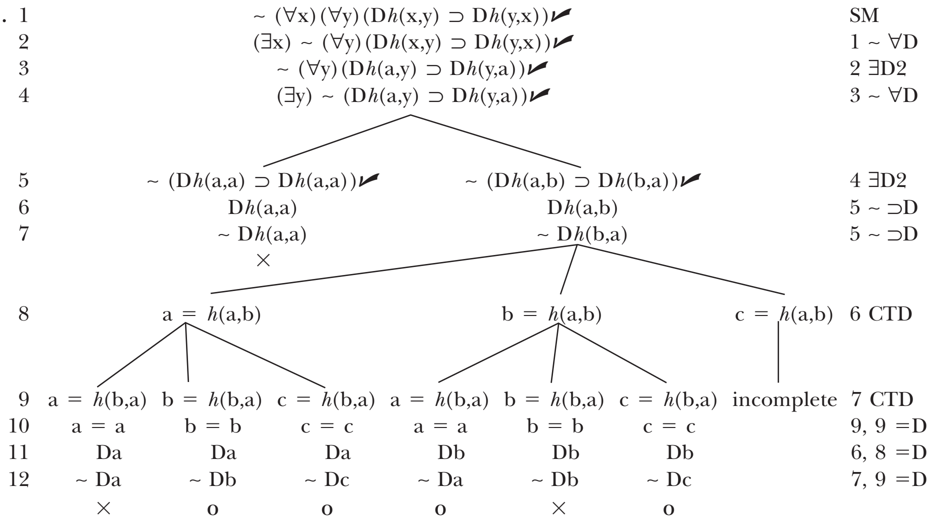


The tree has a completed open branch. Therefore, the sentence '(∀x) (Pf(x) ⊃ Px)' is not quantificationally true.



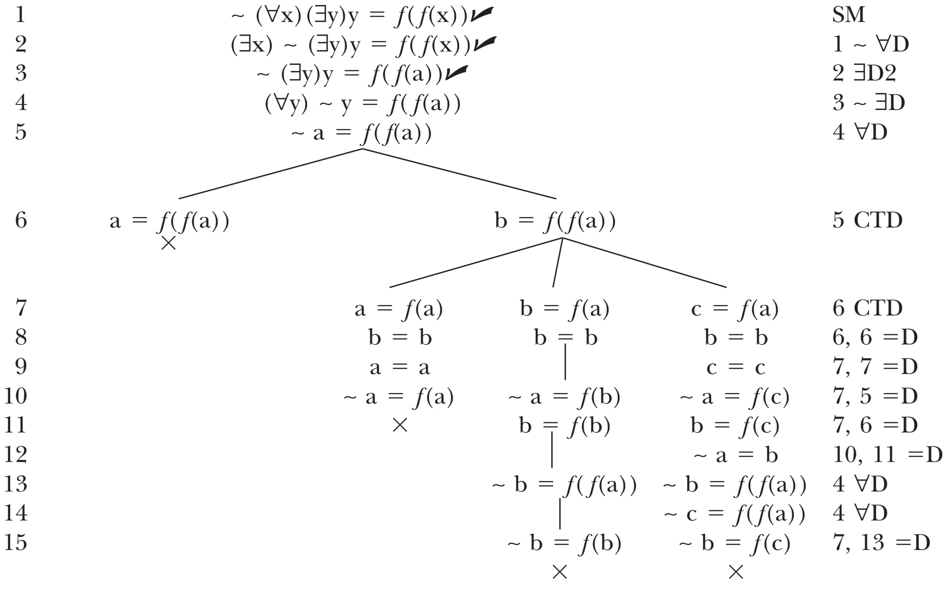
If we were to complete the indicated missing work, we would have a systematic tree with at least one completed open branch (the left-most branch). Therefore, the sentence being '(∃x)(∀y)x = g(y)' is not quantificationally true.

e.



If we were to complete the application of CTD and =D on the far right branch we would have a systematic tree with at least one completed open branch. Therefore, the sentence '$(\forall x)(\forall y)(Dh(x,y) \supset Dh(y,x))$' is not quantificationally true.

3.a.



The tree is closed. The sentence '$(\forall x)(\exists y)y = f(f(x))$' is quantificationally true.

c. 1	$\sim [(\forall x)Lf(x) \supset (\forall x)Lf(f(x))]$ ✓	SM
2	$(\forall x)Lf(x)$	$1 \sim \supset D$
3	$\sim (\forall x)Lf(f(x))$ ✓	$1 \sim \supset D$
4	$(\exists x) \sim Lf(f(x))$ ✓	$3 \sim \forall D$
5	$\sim Lf(f(a))$	$4\ \exists D2$
6	$Lf(a)$	$2\ \forall D$
7	$Lf(f(a))$	$2\ \forall D$
	×	

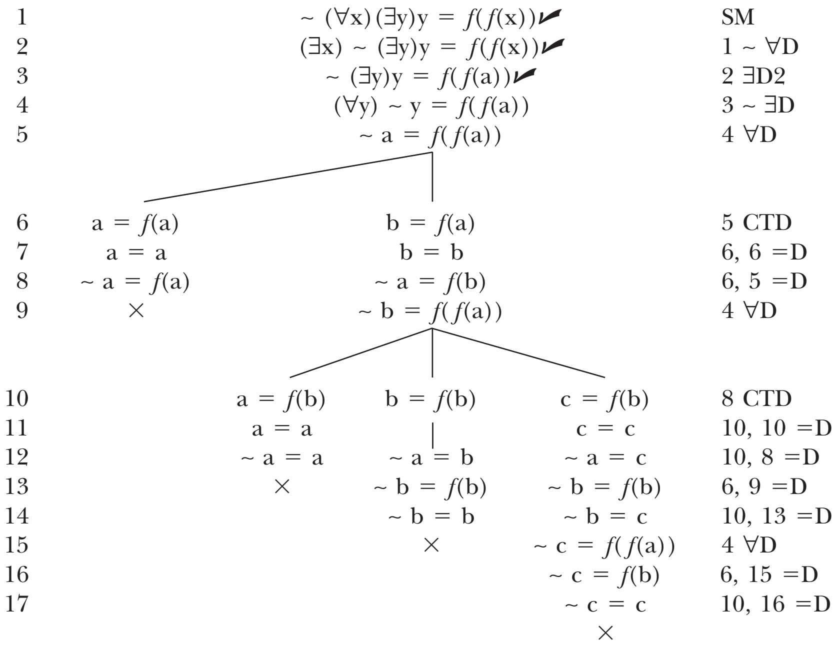
4.a. 1	$(\exists x)f(x) = x$ ✓		SM
2	$f(a) = a$		$1\ \exists D2$
			2 CTD
3	$a = f(a)$	$b = f(a)$	3,3 =D
4	$a = a$	$b = b$	
	o	o	

1	$\sim (\exists x)f(x) = x$ ✓					
2	$(\forall x) \sim f(x) = x$					$1 \sim \exists D$
3	$\sim f(a) = a$					$2\ \forall D$
4	$a = f(a)$	$b = f(a)$				2 CTD
5	$\sim a = a$	$\sim b = a$				3, 5 =D
6	×	$b = b$				4, 4 =D
7		$\sim f(b) = b$				$2\ \forall D$
8			$a = f(b)$	$b = f(b)$	$c = f(b)$	7 CTD
9			$\sim a = b$	$\sim b = b$	$\sim c = b$	7, 8 =D
10			$a = a$	×	$c = c$	8, 8 =D
11			o			

The tree for '$(\exists x)f(x) = x$' has at least one completed open branch, and the tree for the negation of '$(\exists x)f(x) = x$' has at least one completed open branch. The sentence is quantificationally indeterminate.

c. 1	$(\forall x)(\exists y)y = f(f(x))$				SM
2	$(\exists y)y = f(f(a))$ ✓				$1\ \forall D$
3	$a = f(f(a))$		$b = f(f(a))$		$2\ \exists D2$
4	$a = f(a)$	$b = f(a)$	$a = f(a)$	$b = f(a)$	3 CTD
5	$a = a$	$b = b$	$a = a$	$b = b$	4, 4 =D
	o	⋮	⋮	⋮	

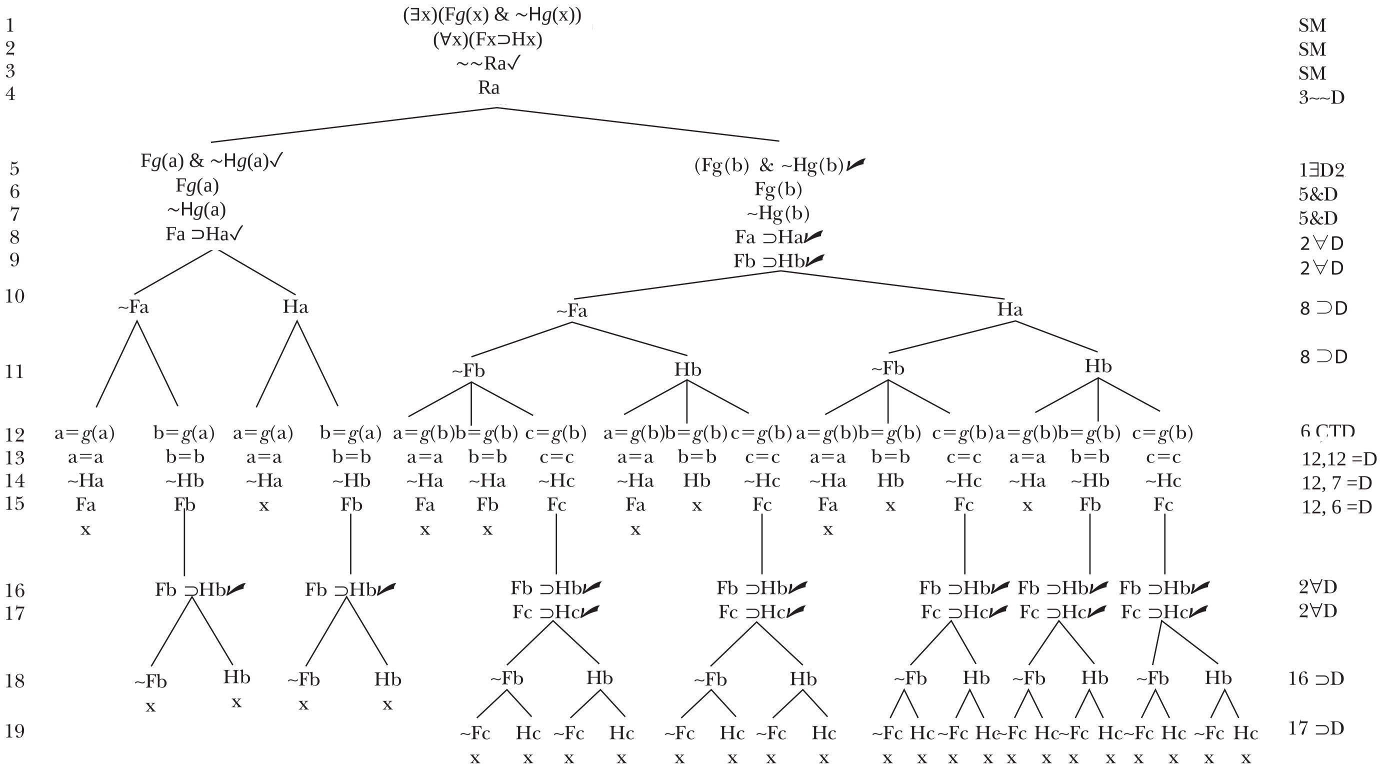
The tree has one completed open branch. The sentence '$(\forall x)(\exists y)y = f(f(x))$' is not quantificationally false.



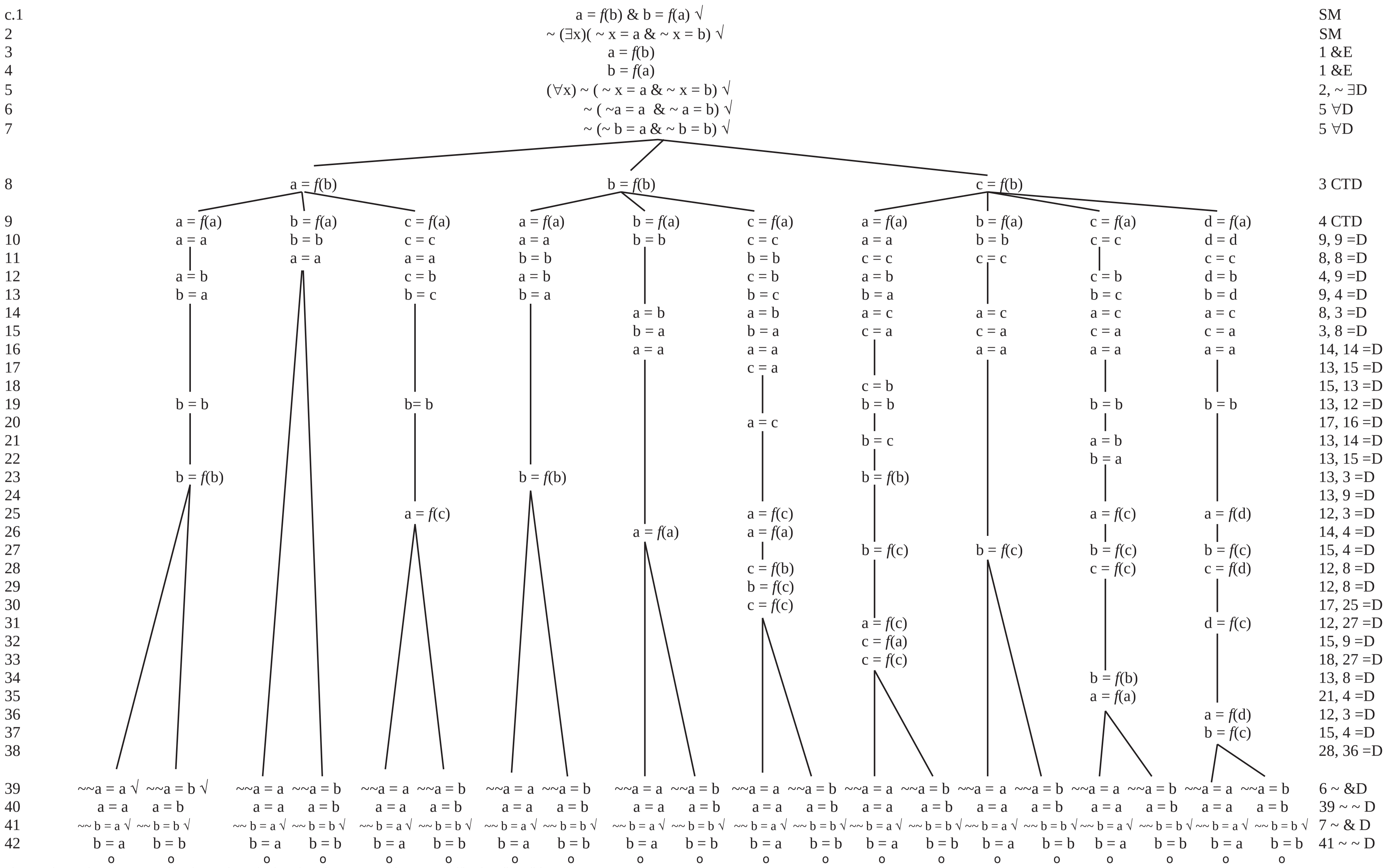
1			$\sim (\forall x)(\exists y)y = f(f(x))$ ✓		SM
2			$(\exists x) \sim (\exists y)y = f(f(x))$ ✓		1 $\sim \forall$D
3			$\sim (\exists y)y = f(f(a))$ ✓		2 $\exists$D2
4			$(\forall y) \sim y = f(f(a))$		3 $\sim \exists$D
5			$\sim a = f(f(a))$		4 $\forall$D
6	$a = f(a)$		$b = f(a)$		5 CTD
7	$a = a$		$b = b$		6, 6 =D
8	$\sim a = f(a)$		$\sim a = f(b)$		6, 5 =D
9	×		$\sim b = f(f(a))$		4 $\forall$D
10		$a = f(b)$	$b = f(b)$	$c = f(b)$	8 CTD
11		$a = a$		$c = c$	10, 10 =D
12		$\sim a = a$	$\sim a = b$	$\sim a = c$	10, 8 =D
13		×	$\sim b = f(b)$	$\sim b = f(b)$	6, 9 =D
14			$\sim b = b$	$\sim b = c$	10, 13 =D
15			×	$\sim c = f(f(a))$	4 $\forall$D
16				$\sim c = f(b)$	6, 15 =D
17				$\sim c = c$	10, 16 =D
				×	

The tree is closed. The sentence '$(\forall x)(\exists y)y = f(f(x))$' is quantificationally true.

5.a.

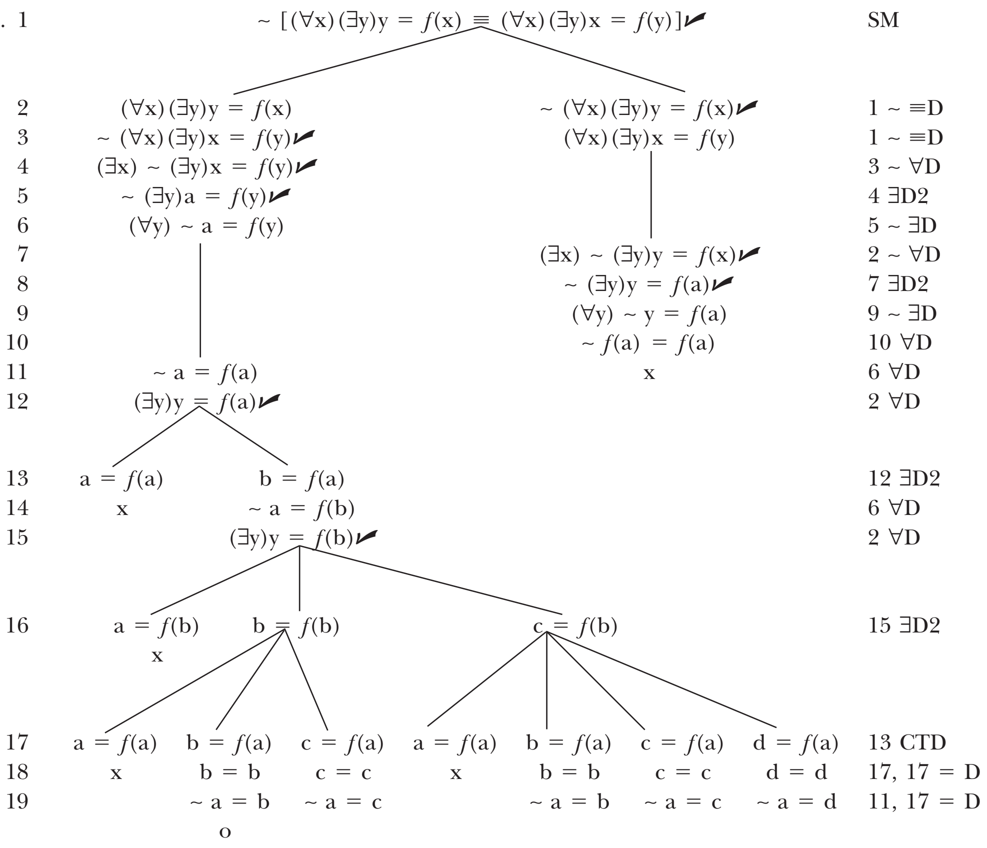


The tree for the premises and the negation of the conclusion is closed. The argument being tested is quantificationally valid.



The tree has at least one completed open branch; therefore the argument is not quantificationally valid.

6. a.



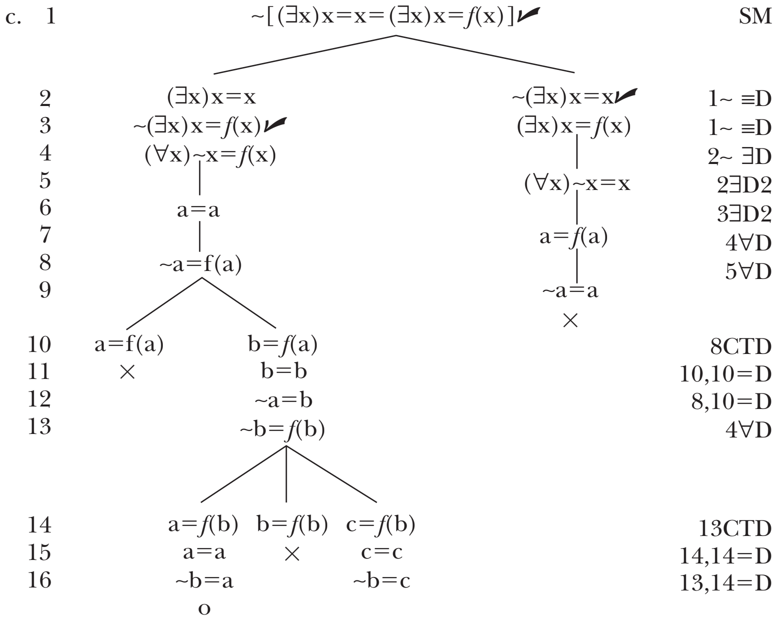
The tree has a completed open branch. The sentences are not quantificationally equivalent.

c. 1		~ [(∃x) x = x ≡ (∃x) x = f(x) √			SM
2		(∃x) x = x √		~ (∃x) x = x √	1 ~ ≡D
3		~ (∃x) x = f(x) √		(∃x) x = f(x) √	1 ~ ≡D
4		(∀x) ~ x = f(x)			3 ~ ∃D
5				(∀x) ~ x = x	2 ~ ∃D
6		a = a			2 ∃D2
7				a = f(a)	3 ∃D2
8		a = f(a)			4 ∀D
9				~ a = a	5 ∀D
				x	
10	a = f(a)	b = f(a)			8 CTD
11	x	b = b			10, 10 =D
12		a = b			10, 8 =D
13		~ b = f(b)			4 ∀D
14	a = f(b)	b = f(b)	c = f(b)		13 CTD
15	a = a	x	c = c		14, 14 =D
16	~ b = a		~ b = a		14, 13 =D

The systematic tree has at least one cpompleted open branch. The sentences are not quantificationally equivalent.

7. a. 1.	(∀x)(∀y)x = g(x,y)	SM
2.	~ (∀x)x = g(x,x)✓	SM
3.	(∃x) ~ x = g(x,x)✓	2 ~ ∀D
4.	~ a = g(a,a)	3 ∃D2
5.	(∀y)a = g(a,y)	1 ∀D
6.	a = g(a,a)	5 ∀D
	×	

This tree is closed. The entailment does hold.



The systematic tree has at least one completed open branch. The sentences are not quantificationally equivalent.

7. a. 1. (∀x)(∀y)x = *g*(x,y) SM
2. ~ (∀x)x = *g*(x,x)✓ SM
3. (∃x) ~ x = *g*(x,x)✓ 2 ~ ∀D
4. ~ a = *g*(a,a) 3 ∃D2
5. (∀y)a = *g*(a,y) 1 ∀D
6. a = *g*(a,a) 1 ∀D
×

This tree is closed. The entailment does hold.

c. 1			$(\forall x)x = f(f(x))$		SM
2			$\sim (\forall x)x = f(x)$ ✓		SM
3			$(\exists x) \sim x = f(x)$ ✓		2 $\sim \forall$D
4			$\sim a = f(a)$		3 $\exists$D 2
5			$a = f(f(a))$		1 $\forall$D
6	$a = f(a)$		$b = f(a)$		4 CTD
7	×		$\sim a = b$		4, 6 =D
8			$a = f(b)$		5, 6 =D
9			$b = b$		6, 6 =D
10			$a = a$		8, 8 =D
11			$b = f(f(b))$		1 $\forall$D
12		$a = f(b)$	$b = f(b)$	$c = f(b)$	8 CTD
13		$b = f(b)$			12, 11 =D
14		$a = b$			13, 8 =D
15		x	$a = b$		12, 8 =D
16			x	$c = c$	12, 12 =D
17				$b = f(c)$	12, 11 =D
18				$a = c$	12, 8 =D
19				$c = a$	8, 12 =D
				o	