Monopoly price exceeds marginal revenue

For a firm with monopoly power, the increase in revenue from selling one more unit (MR) is less than the price at which this unit is sold. The proof of this assertion follows from a simple calculation of marginal revenue.

Suppose the firm's demand curve is given by the relation $P = f(Q)$. Because the firm possesses monopoly power, it faces a downsloping demand curve: $f'(Q) < 0$. Revenue is $R(Q) = QP = Qf(Q)$ and marginal revenue is $\mathrm{d}R(Q)/\mathrm{d}Q = R'(Q)$. Using the product rule, $R'(Q) = f(Q) + Qf'(Q)$. Noting that $P = f(Q)$, $R'(Q) = P + Qf'(Q)$. The second term is clearly negative, indicating that $MR = R'(Q) < P$ as was asserted.

What is the meaning of the term $Qf'(Q)$ in MR? Consider what happens when a monopolist wishes to sell an additional unit of output. The price falls by $\mathrm{d}P/\mathrm{d}Q = f'(Q)$, and this price decrease applies to the first Q units sold. Hence, the increase in revenue is equal to the price at which the last unit is sold, P, minus the loss in revenue from selling Q units at this lower price, $Qf'(Q)$.

Consider the special case of a linear, downsloping demand curve: $f(Q) = a - bQ$ where the vertical intercept, a, is the price at which zero units are sold and $f'(Q) = -b$ is the slope of the demand curve. Using our formula for marginal revenue, $MR = f(Q) + Qf'(Q) = (a - bQ) - bQ = a - 2bQ$. We see that the marginal revenue relation is also linear, has the same price-axis intercept, and is twice as steep. Graphically, this implies that the marginal revenue curve is everywhere half the distance between the price axis and the demand curve. (Note that we could also form $R(Q)$ directly as $PQ = f(Q)Q = (a - bQ)Q = aQ - bQ^2$ and $R'(Q) = a - 2bQ$ as before.)