Consumer price index

Consider an economy that produces just two goods—pizza and videos. The typical household in this economy initially spends 90% of its budget on pizza and the remaining 10% on videos. If the price of pizza increases by 10% over the next year, and the price of videos increases by just 2%, what is the rate of inflation? One possible measure is a simple average of the two price increases: ½ · 10% + ½ · 2% = 6%. Clearly, this would be a misleading indicator of the increased cost of living. Instead, the Consumer Price Index is calculated using a *weighted average* of the two price changes, where the weights are the shares of initial expenditures. In this example, the rate of inflation is calculated as 0.9 · 10% + 0.1· 2% = 9.2%.

A simple average of n items is found by summing all the items and dividing by n, or equivalently, summing $1/n$ times each item: $\sum_{i=1}^{n}\frac{1}{n}X_i$ where X_i is the value of item i. A weighted average is similar, except each item is weighted by an amount w_i: $\sum_{i=1}^{n}w_iX_i$, where the sum of the weights equals one. You will note that a simple average is a special case of the weighted average, in which all of the weights are the same and equal to $1/n$.

As stated in the text, the CPI is a ratio that measures the cost of a market basket of goods in a particular year relative to the cost of that same basket in the base year. Mathematically, the CPI is computed as CPI = $\frac{\sum_{i=1}^{n}P_i^1X_i^0}{\sum_{i=1}^{n}P_i^0X_i^0}$, where P_i^1 is the current price of good i, X_i^0 is the amount of good i consumed in the base year, and P_i^0 is its price in the base year. The denominator is the sum of expenditures on a market basket of goods in the base year (year 0), while the numerator is the cost of this same market basket when evaluated at current prices. Alternatively, this could be written as the sum of n distinct terms, as follows: CPI = $\sum_{i=1}^{n}\left(\frac{P_i^1X_i^0}{\sum_{i=1}^{n}P_i^0X_i^0}\right)$. Consider the first term in this series, $\frac{P_1^1X_1^0}{\sum_{i=1}^{n}P_i^0X_i^0}$. If we multiply both numerator and denominator by P_1^0 and rearrange, this can be written as $\frac{P_1^0X_1^0}{\sum_{i=1}^{n}P_i^0X_i^0}\left(\frac{P_1^1}{P_1^0}\right)$. The term in parentheses measures the ratio of the current price for good 1 relative to the base year price, while the first term measures the proportion of total base year expenditures accounted for by good 1.

If we similarly multiply the numerator and denominator of each term in the summation by the appropriate price, we arrive at the following: CPI = $\sum_{i=1}^{n}w_i\left(\frac{P_i^1}{P_i^0}\right)$ where w_i = $\frac{P_i^0X_i^0}{\sum_{i=1}^{n}P_i^0X_i^0}$. That is, the CPI is a weighted average of the relative price increases of the individual goods, where each weight is the share of that good's expenditures relative to total expenditures measured in the base year.