Show All Solutions

Rosen, Discrete Mathematics and Its Applications, 8th edition
Extra Examples
Section 1.1—Propositional Logic

Extra Examples — *Page references correspond to locations of Extra Examples icons in the textbook.*

---

p.2, icon at Example 1
#1. Is the following sentence a proposition? If it is a proposition, determine whether it is true or false.

"Portland is the capital of Maine."

See Solution

---

p.2, icon at Example 1
#2. Is the following sentence a proposition? If it is a proposition, determine whether it is true or false.

"Can Allen come with you?"

See Solution

---

p.2, icon at Example 1
#3. Is the following sentence a proposition? If it is a proposition, determine whether it is true or false.

$1 + 2 = 3$ or $2 + 3 = 5$.

See Solution

---

p.2, icon at Example 1

#4. Is the following sentence a proposition? If it is a proposition, determine whether it is true or false.

"Take two aspirin."

See Solution

---

p.2, icon at Example 1

#5. Is the following sentence a proposition? If it is a proposition, determine whether it is true or false.

"$x + 4 > 9$."

See Solution

---

p.3, icon at Example 3

#1. Write the negation of "George Washington was the first president of the United States."

See Solution

---

p.3, icon at Example 3

#2. Write the negation of "$1 + 5 = 7$."

See Solution

---

p.3, icon at Example 3
#3. Write the negation of "$1 + 5 \neq 7$."

See Solution

---

p.3, icon at Example 3
#4. Write the negation of "It is hot today."

See Solution

---

p.3, icon at Example 3
#5. Write the negation of "6 is negative."

See Solution

---

p.5, icon at Example 7
#1. The following proposition uses the English connective "or." Determine from the context whether "or" is intended to be used in the inclusive or exclusive sense.

> "Tonight I will stay home or go out to a movie."

See Solution

---

p.5, icon at Example 7

#2. The following proposition uses the English connective “or.” Determine from the context whether “or” is intended to be used in the inclusive or exclusive sense.

> “If you fail to make a payment on time or fail to pay the amount due, you will incur a penalty.”

See Solution

---

p.5, icon at Example 7

#3. The following proposition uses the English connective “or.” Determine from the context whether “or” is intended to be used in the inclusive or exclusive sense.

> “If I can’t schedule the airline flight or if I can’t get a hotel room, then I can’t go on the trip.”

See Solution

---

p.5, icon at Example 7

#4. The following proposition uses the English connective “or.” Determine from the context whether “or” is intended to be used in the inclusive or exclusive sense.

> “She has one or two brothers.”

See Solution

---

p.5, icon at Example 7

#5. The following proposition uses the English connective “or.” Determine from the context whether “or” is intended to be used in the inclusive or exclusive sense.

> “If you do not wear a shirt or do not wear shoes, then you will be denied service in the restaurant.”

See Solution

---

p.5, icon at Example 7
#6. The following proposition uses the English connective "or." Determine from the context whether "or" is intended to be used in the inclusive or exclusive sense.

"I will pass or fail the course."

See Solution

---

p.5, icon at Example 7
#7. The following proposition uses the English connective "or." Determine from the context whether "or" is intended to be used in the inclusive or exclusive sense.

"To register for ENL499 you must have passed the qualifying exam or be listed as an English major."

See Solution

---

p.8, icon at Example 10
#1. The following statement is a conditional proposition in one of its many alternate forms. Write it in English in the form "If . . . then . . . ."

"If it rains, I'll stay here."

See Solution

---

p.8, icon at Example 10
#2. The following statement is a conditional proposition in one of its many alternate forms. Write it in English in the form "If . . . then . . . ."

"I go walking whenever it rains."

See Solution

---

p.8, icon at Example 10
#3. The following statement is a conditional proposition in one of its many alternate forms. Write it in English in the form "If . . . then . . . ."

"To pass the course it is sufficient that you get a high grade on the final exam."

See Solution

---

p.8, icon at Example 10
#4. The following statement is a conditional proposition in one of its many alternate forms. Write it in English in the form "If . . . then . . . ."

"To pass the course it is necessary that you get a high grade on the final exam."

See Solution

---

p.8, icon at Example 10
#5. The following statement is a conditional proposition in one of its many alternate forms. Write it in English in the form "If . . . then . . . ."

"I will buy the tickets only if you call."

See Solution

---

p.8, icon at Example 10
#6. The following statement is a conditional proposition in one of its many alternate forms. Write it in English in the form "If . . . then . . . ."

"To be able to go on the trip, it is necessary that you get written permission."

See Solution

---

p.8, icon at Example 10
#7. The following sign is at the entrance of a restaurant: "No shoes, no shirt, no service." Write this sentence as a conditional proposition.

See Solution

---

p.8, icon at Example 10
#8. Write the compound proposition $s \to v$ in English, using the variables:

v: "I take a vacation"
s: "it is summer"

See Solution

---

p.8, icon at Example 10
#9. Write the compound proposition $s \to \neg w$ in English, using the variables:

s: "it is summer"
w: "I work

See Solution

---

p.8, icon at Example 10
#10. Write the compound proposition $\neg v \to w$ in English, using the variables:

v: "I take a vacation"
w: "I work"

See Solution

---

p.8, icon at Example 10
#11. "Tell me what you eat and I will tell you what you are" is a quote by Jean-Anthelme Brillat-Savarin (French gastronomist, 1755–1829). Express this as a compound proposition.

See Solution

---

p.8, icon at Example 10
#12. Write the negation of "If it rains, I stay home."

See Solution

---

p.8, icon at Example 10
#13. Find the negation of the statement "If you pay your membership dues, then if you come to the club, you can enter free."

See Solution

p.9, icon at Example 12

#1. Write the contrapositive, converse, and inverse of the following proposition:

"If the number is positive, then its square is positive."

See Solution

---

p.9, icon at Example 12

#2. Write the contrapositive, converse, and inverse of the following proposition:

"I stay home whenever it is stormy."

See Solution

---

p.10, icon at Example 13

#1. Write the following proposition in the form "... if and only if ...."

"It rains exactly when I plan a picnic."

See Solution

---

p.10, icon at Example 13

#2. Write the following proposition in the form "... if and only if ...."

"I attend class when I have a quiz and I have a quiz when I attend class."

See Solution

---

p.10, icon at Example 13

#3. Write the following proposition in the form "... if and only if ...."

"I visit the library whenever I have a paper to write, and conversely."

See Solution

---

p.10, icon at Example 13

#4. The following English statement can be written in the form "if ..., then ...". Yet in some cases there is an implied "only if"; that is, the converse is implied. Do you think that the following statement has an implied converse?

"If you study hard, then you will pass the course."

See Solution

---

p.10, icon at Example 13

#5. The following English statement can be written in the form "if . . . , then . . . ". Yet in some cases there is an implied "only if"; that is, the converse is implied. Do you think that the following statement has an implied converse?

"If you have a red ink cartridge in your printer, then you can use the printer to print the report in red."

See Solution

---

p.10, icon at Example 13

#6. The following English statement can be written in the form "if . . . , then . . . ". Yet in some cases there is an implied "only if"; that is, the converse is implied. Do you think that the following statement has an implied converse?

"If you pay the electric bill, then the electric company will turn on your power."

See Solution

---

p.10, icon at Example 13

#7. The following English statement can be written in the form "if . . . , then . . . ". Yet in some cases there is an implied "only if"; that is, the converse is implied. Do you think that the following statement has an implied converse?

"You must be a resident in order to vote."

See Solution

---

p.10, icon at Example 13

#8. The following English statement can be written in the form "if . . . , then . . . ". Yet in some cases there is an implied "only if"; that is, the converse is implied. Do you think that the following statement has an implied converse?

"If you have a dollar, then you can buy coffee from the vending machine."

See Solution

---

p.10, icon at Example 13

#9. The following English statement can be written in the form "if . . . , then . . . ". Yet in some cases there is an implied "only if"; that is, the converse is implied. Do you think that the following statement has an implied converse?

> "You need a ticket in order to enter the theater."

See Solution

Show All Solutions

Rosen, Discrete Mathematics and Its Applications, 8th edition
Extra Examples
Section 1.2—Applications of Propositional Logic

Extra Examples — *Page references correspond to locations of Extra Examples icons in the textbook.*

---

p.18, icon at Example 1

#1. Suppose u represents "you understand the material", s represents "you study the theory", and w represents "you work on exercises". Write the following compound proposition using u, s, w, and appropriate connectives.

> "You study the theory and work on exercises, but you don't understand the material."

See Solution

---

p.18, icon at Example 1
#2. Suppose u represents "you understand the material" and s represents "you study the theory". Write the following compound proposition using u, s, and appropriate connectives.

> "Studying the theory is sufficient for understanding the material."

See Solution

---

p.18, icon at Example 1
#3. Suppose s represents "you study the theory" and w represents "you work on exercises". Write the following compound proposition using s, w, and appropriate connectives.

> "In order to work on exercises, you need to study the theory."

See Solution

---

p.18, icon at Example 1

#4. Suppose u represents "you understand the material", s represents "you study the theory", and w represents "you work on exercises". Write the following compound proposition using u, s, w, and appropriate connectives.

"When you study the theory and work on exercises, you understand the material."

See Solution

---

p.18, icon at Example 1

#5. Suppose u represents "you understand the material", s represents "you study the theory", and w represents "you work on exercises". Write the following compound proposition using u, s, w, and appropriate connectives.

"You don't understand the material unless you study the theory and work on exercises."

See Solution

---

p.18, icon at Example 3

#1. Translate this system specification into symbols:

"The online user is sent a notification of a link error if the network link is down."

See Solution

---

p.18 icon at Example 3

#2. Translate this system specification into symbols:

> "Whenever the file is locked or the system is in executive clearance mode, the user cannot make changes in the data."

See Solution

---

p.18, icon at Example 3

#3. Write these system specifications in symbols using the propositions

> v: "The user enters a valid password,"
> a: "Access is granted to the user,"
> c: "The user has contacted the network administrator,"

and logical connectives. Then determine if the system specifications are consistent.

(i) "The user has contacted the network administrator, but does not enter a valid password."
(ii) "Access is granted whenever the user has contacted the network administrator or enters a valid password."
(iii) "Access is denied if the user has not entered a valid password or has not contacted the network administrator."

See Solution

---

p.19, icon at Example 6

#1. How would you do a Boolean search for the appropriate Web pages for each of these:

(a) hotels in New England.
(b) hotels in England.
(c) hotels in England or New England.

See Solution

---

p.20, icon at Example 8

#1. Suppose you have three cards: one red on both sides (red/red), one green on both sides (green/green), and one red on one side and green on the other side (red/green). The three cards are placed in a row on a table. Explain how to determine the identity of all three cards by selecting one card and turning it over.

See Solution

---

p.20 icon at Example 8

#2. Another of Smullyan's puzzles poses this problem. You meet two people, A and B. Each person either always tells the truth (i.e., the person is a knight) or always lies (i.e., the person is a knave). Person A tells you, "We are not both truthtellers."
Determine, if possible, which type of person each one is.

See Solution

Show All Solutions

Rosen, Discrete Mathematics and Its Applications, 8th edition
Extra Examples
Section 1.3—Propositional Equivalences

Extra Examples — *Page references correspond to locations of Extra Examples icons in the textbook.*

---

p.27, icon below Table 2
#1. Prove that $\neg[r \vee (q \wedge (\neg r \rightarrow \neg p))] \equiv \neg r \wedge (p \vee \neg q)$ by using a truth table.

See Solution

---

p.27, icon below Table 2
#2. Show that $\neg(p \vee q) \not\equiv \neg p \vee \neg q$.

See Solution

p.31, icon at Example 6

#1. Prove that $\neg[r \vee (q \wedge (\neg r \rightarrow \neg p))] \equiv \neg r \wedge (p \vee \neg q)$ by using a series of logical equivalences.

See Solution

p.31, icon at Example 6

#2. Here is a newspaper headline:

> "Legislature Fails to Override Governor's Veto of Bill to Cancel Sales Tax Reform."

Did the legislature vote in favor of or against sales tax reform?

See Solution

p.31, icon at Example 6
#3. Suppose you want to prove a theorem of the form $p \to (q \vee r)$. Prove that this is equivalent to showing that $(p \wedge \neg q) \to r$.

See Solution

---

p.31, icon at Example 6
#4. Write the statement $p \to (\neg q \wedge r)$ using only the connectives $\neg$ and $\wedge$.

See Solution

Show All Solutions

Rosen, Discrete Mathematics and Its Applications, 8th edition
Extra Examples
Section 1.4—Predicates and Quantifiers

Extra Examples — *Page references correspond to locations of Extra Examples icons in the textbook.*

---

p.41, icon at Example 3
#1. Let $P(x)$ be the statement

$$x^2 < x$$

where the universe for x is all real numbers.

(a) Determine the truth value of $P(0)$.
(b) Determine the truth value of $P(1/3)$.
(c) Determine the truth value of $P(2)$.
(d) Determine the set of all real numbers for which $P(x)$ is true.

See Solution

---

p.41, icon at Example 3
#2. Let $Q(x, y)$ be the statement

$$x + y = x - y$$

where the universe for x and y is the set of all real numbers. Determine the truth value of:

(a) $Q(5, -2)$.
(b) $Q(4.7, 0)$.
(c) Determine the set of all pairs of numbers, x and y, such that $Q(x, y)$ is true.

See Solution

---

p.41, icon at Example 3

#3. Find all real numbers x and y such that $R(x, y)$ is true, where $R(x, y)$ is the predicate "$xy = y$."

See Solution

---

p.44, icon at Example 8

#1. Suppose $P(x)$ is the predicate "$x < |x|$." Determine the truth value of $\forall x\, P(x)$, where the universe for x is:

(a) the three numbers -3, -2, -1.
(b) all real numbers.

See Solution

---

p.44, icon at Example 8

#2. Find a universe for x such that $\forall x\, (x^2 < x)$ is true.

See Solution

---

p.46, icon at Example 13

#1. Suppose $P(x)$ is the predicate "$x < |x|$." Determine the truth value of $\exists x\, P(x)$ where the universe for x is:

(a) the three numbers 1, 2, 3.
(b) the six numbers -2, -1, 0, 1, 2, 3.

See Solution

---

p.46, icon at Example 13

#2. Determine whether $\exists t\,(t^2 + 12 = 7t)$ is true, where the universe for t consists of all real numbers.

See Solution

---

p.46, icon at Example 13

#3. Write the following statement in English, using the predicates

$F(x)$: "x is a Freshman"
$T(x, y)$: "x is taking y"

where x represents students and y represents courses:

$$\exists x\,(F(x) \wedge T(x, \text{Calculus 3})).$$

See Solution

---

p.51, icon at Example 20

#1. Negate "There is a person who walked on the moon."

See Solution

p.51, icon at Example 20
#2. Negate "Everyone in the class has a laptop computer."

See Solution

p.51, icon at Example 20
#3. Negate "Some integer x is positive and all integers y are negative."

See Solution

p.51, icon at Example 20
#4. Negate "There is a student who came late to class and there is a student who is absent from class."

See Solution

p.52, icon at Example 23

#1. Write in symbols using predicates and quantifiers: "Everyone who visited France stayed in Paris."

See Solution

---

p.52, icon at Example 23

#2. Express this statement in symbols, using predicates and any needed quantifiers:

"Every freshman at the College is taking CS 101."

See Solution

p.52, icon at Example 23

#3. Express this statement in symbols, using predicates and any needed quantifiers:

"Every freshman at the College is taking some Computer Science course."

See Solution

p.52, icon at Example 23

#4. Consider this sentence, which is the final sentence of 12th Amendment of U. S. Constitution: "No person constitutionally ineligible to the office of President shall be eligible to the office of Vice President of the United States."

(a) Rewrite the sentence in English in the form "If ..., then ...."

(b) Using the predicates $P(x)$: "x is constitutionally eligible to the office of President" and $V(x)$: "x is constitutionally eligible to the office of Vice President of the United States," where the universe for x consists of all people, write the sentence using quantifiers and these predicates.

See Solution

---

p.52, icon at Example 23

#5. Consider this sentence, which is Section 2 of Article I of the U. S. Constitution: "No person shall be a Representative who shall not have attained the age of twenty-five years, and been seven years a citizen of the United States, and who shall not, when elected, be an inhabitant of that state in which he shall be chosen."

(a) Rewrite the sentence in English in the form "If ..., then ...".

(b) Using the predicates $A(x)$: "x is at least twenty-five years old," $C(x)$: "x has been a citizen of the United States for at least seven years," $I(x)$: "x, when elected, is an inhabitant of the state in which he is chosen," and $R(x)$: "x can be a Representative," where the universe for x in all four predicates consists of all people, rewrite the sentence using quantifiers and these predicates. [Note: At the time at which the U. S. Constitution was ratified, the universe for x consisted of landowning males.]

See Solution

---

p.53, icon at Example 25

#1. Express the specification "Whenever at least one network link is operating, a 10 megabyte file can be transmitted" using predicates and quantifiers.

See Solution

Show All Solutions

Rosen, Discrete Mathematics and Its Applications, 8th edition
Extra Examples
Section 1.5—Nested Quantifiers

Extra Examples — *Page references correspond to locations of Extra Examples icons in the textbook.*

---

p.61, icon at Example 1
#1. Write the following statements in English, using the predicate $S(x, y)$: "x shops in y", where x represents people and y represents stores:

(a) $\forall y\, S(Margaret, y)$.
(b) $\exists x \forall y\, S(x, y)$.

See Solution

---

p.61, icon at Example 1
#2. Write in symbols using predicates and quantifiers: "Every Junior in this class scored above 90 on the first exam."

See Solution

p.61, icon at Example 1

#3. Write the following statement in English, using the predicates

$S(x, y)$: "x shops in y"
$T(x)$: "x is a student"

where x represents people and y represents stores:

$$\exists y \, \forall x \, (T(x) \rightarrow \neg S(x, y)).$$

See Solution

p.61, icon at Example 1

#4. Write the following statement in English, using the predicates

$S(x, y)$: "x shops in y"
$T(x)$: "x is a student"

where x represents people and y represents stores:

$$\forall y \, \exists x \, (T(x) \wedge S(x, y)).$$

See Solution

p.61, icon at Example 1

#5. Write the following statement in English, using the predicate $S(x, y)$ for "x shops in y", where x represents people and y represents stores:

$$\exists x_1 \, \exists y \, \forall x_2 \, [S(x_1, y) \wedge (x_1 \neq x_2 \rightarrow \neg S(x2, y))].$$

See Solution

---

p.61, icon at Example 1

#6. Write the following statement in English, using the predicates

$C(x)$: "x is a Computer Science major"
$M(y)$: "y is a math course"
$T(x, y)$: "x is taking y"

where x represents students and y represents courses:

$$\forall x\, \exists y\, (C(x) \rightarrow M(y) \wedge T(x, y)).$$

See Solution

---

p.61, icon at Example 1

#7. Write the following statement in English, using the predicates

$C(x)$: "x is a Computer Science major"
$T(x, y)$: "x is taking y"

where x represents students and y represents courses:

$$\forall y\, \exists x\, (\neg C(x) \wedge T(x, y)).$$

See Solution

---

p.61, icon at Example 1

#8. Write the following statement in English, using the predicates

$F(x)$: "x is a Freshman"
$M(y)$: "y is a math course"
$T(x, y)$: "x is taking y"

where x represents students and y represents courses:

$$\neg\exists x\,[F(x) \wedge \forall y\,(M(y) \rightarrow T(x, y))].$$

See Solution

---

p.61, icon at Example 1
#9. Write the following statement using quantifiers and the predicate $S(x, y)$ for "x shops in y", where the universe for x consists of people and the universe for y consists of stores:

"Will shops in Al's Record Shoppe."

See Solution

---

p.61, icon at Example 1
#10. Write the following statement using quantifiers and the predicates

$S(x, y)$: "x shops in y"
$T(x)$: "x is a student"

where the universe for x consists of people and the universe for y consists of stores:

"There is no store that has no students who shop there."

See Solution

---

p.61, icon at Example 1

#11. Write the following statement using quantifiers and the predicates

$S(x, y)$: "x shops in y"
$T(x)$: "x is a student"

where the universe for x consists of people and the universe for y consists of stores:

"The only shoppers in some stores are students."

See Solution

---

p.61, icon at Example 1

#12. Suppose that the universe for x and y is $\{1, 2, 3\}$. Also, assume that $P(x, y)$ is a predicate that is true in the following cases, and false otherwise: $P(1, 3)$, $P(2, 1)$, $P(2, 2)$, $P(3, 1)$, $P(3, 2)$, $P(3, 3)$. Determine whether each of the following is true or false:

(a) $\forall y \exists x\,(x \neq y \wedge P(x, y))$.
(b) $\forall x \exists y\,(x \neq y \wedge \neg P(x, y))$.
(c) $\forall y \exists x\,(x \neq y \wedge \neg P(x, y))$.

See Solution

---

p.61, icon at Example 1

#13. Suppose that the universe for x and y is $\{1, 2, 3, 4\}$. Assume that $P(x, y)$ is a predicate that is true in the following cases and false otherwise: $P(1, 4)$, $P(2, 1)$, $P(2, 2)$, $P(3, 4)$, $P(4, 1)$, $P(4, 4)$. Determine whether each of the following is true or false:

(a) $\exists y \forall x\, P(x, y)$.
(b) $\forall x\, P(x, x)$.
(c) $\forall x \exists y\,(x \neq y \wedge P(x, y))$.

See Solution

p.61, icon at Example 1

#14. Consider this sentence, which is Amendment 3 to the U.S. Constitution: "No soldier shall, in time of peace, be quartered in any house, without the consent of the owner, nor in time of war, but in a manner to be prescribed by law."

(a) The sentence has the form of a conjunction of two conditional sentences. Write the given sentence in this form.

(b) Using the six predicates, $S(x)$: "x is a soldier," $P(t)$ "t is a peaceful time," $Q(x, y, h)$: "x is required to allow y to be quartered in h," $O(x, h)$: "x owns h," $C(x, y, h)$: "x consents to quarter y in h,', $A(x, h)$: "the law allows x to be quartered in h," where the universe for x and y consists of all people, the universe for t consists of all points in time, and the universe for h consists of all houses, rewrite the sentence using quantifiers and predicates.

See Solution

p.61, icon at Example 1

#15. Consider these lines of code from a C++ program:

```
if (!(x!=0 && y/x < 1) || x==0)
  cout « "True";
else
  cout « "False"
```

(a) Express the code in this statement as a compound statement using the logical connectives $\neg$, $\vee$, $\wedge$, $\rightarrow$, and these predicates

$E(x)$: $x = 0$
$RL(x, y)$: $y/x < 1$
$A(z)$: "z is assigned to `cout`"

where x and y are integers and z is a Boolean variable (with values True and False).

(b) Use the laws of propositional logic to simplify the statement by expressing it in a simpler form.

(c) Translate the answer in part (b) back into C++.

See Solution

p.62, icon at Example 3
#1. What are the truth values of each of these? Assume that in each case the universe consists of all real numbers.

(a) $\exists x\, \exists y\, (xy = 2)$
(b) $\exists x\, \forall y\, (xy = 2)$
(c) $\forall x\, \exists y\, (xy = 2)$
(d) $\forall x\, \forall y\, (xy = 2)$

See Solution

---

p.62, icon at Example 3
#2. Write the following statements in English, using the predicate $S(x, y)$: "x shops in y", where x represents people and y represents stores:

$(a)\ \exists y\, \forall x\, S(x, y).$
$(b)\ \forall x\, \exists y\, S(x, y).$

See Solution

p.62, icon at Example 3

#3. Suppose that the universe for x and y is $\{1, 2, 3\}$. Also, assume that $P(x, y)$ is a predicate that is true in the following cases, and false otherwise: $P(1, 3)$, $P(2, 1)$, $P(2, 2)$, $P(3, 1)$, $P(3, 2)$, $P(3, 3)$. Determine whether each of the following is true or false:

(a) $\exists x \forall y\,(y < x \rightarrow P(x, y))$.
(b) $\forall y \exists x\,(y < x \vee P(x, y))$.
(c) $\exists x \exists y\,(P(x, y) \wedge P(y, x))$.
(d) $\forall y \exists x\,(P(x, y) \rightarrow \neg P(y, x))$.

See Solution

---

p.62, icon at Example 3

#4. Suppose $P(x, y, z)$ is a predicate where the universe for x, y, and z is $\{1, 2\}$. Also suppose that the predicate is true in the following cases $P(1, 1, 1)$, $P(1, 2, 1)$, $P(1, 2, 2)$, $P(2, 1, 1)$, $P(2, 2, 2)$, and false otherwise. Determine the truth value of each of the following quantified statements:

(a) $\forall x\, \exists y\, \exists z\, P(x, y, z)$.
(b) $\forall x\, \forall y\, \exists z\, P(x, y, z)$.
(c) $\forall y\, \forall z\, \exists x\, P(x, y, z)$.
(d) $\forall x\, \exists y\, \forall z\, P(x, y, z)$.

See Solution

---

p.62, icon at Example 3

#5. Suppose $P(x, y, z)$ is a predicate where the universe for x, y, and z is $\{1, 2\}$. Also suppose that the predicate is true in the following cases $P(1, 1, 1)$, $P(1, 2, 1)$, $P(1, 2, 2)$, $P(2, 1, 1)$, $P(2, 2, 2)$, and false otherwise. Determine the truth value of each of the following quantified statements:

(a) $\exists x\, \forall y\, \forall z\, P(x, y, z)$. (b) $\forall x\, \exists z\, \forall y\, P(x, y, z)$.

(c) $\forall y\, \exists x\, \exists z\, \neg P(x, y, z)$. (d) $\exists x\, \forall z\, \neg \forall y\, P(x, y, z)$.

See Solution

---

p.62, icon at Example 3

#6. Suppose that the universe for x and y is $\{1, 2, 3, 4\}$. Assume that $P(x, y)$ is a predicate that is true in the following cases and false otherwise: $P(1, 4)$, $P(2, 1)$, $P(2, 2)$, $P(3, 4)$, $P(4, 1)$, $P(4, 4)$. Determine whether each of the following is true or false:

(a) $\forall x \exists y P(x, y)$.

(b) $\forall y \exists x P(x, y)$.

(c) $\exists x \forall y P(x, y)$.

See Solution

---

p.64, icon at Example 6

#1. Write this fact about numbers using predicates and quantifiers: “Given a number, there is a number greater than it.”

See Solution

---

p.64, icon at Example 6
#2. Express the following statement using predicates and quantifiers: "The product of two positive numbers is positive."

See Solution

---

p.64, icon at Example 6
#3. Write these statements in symbols using the predicates:

$S(x)$: x is a perfect square; $N(x)$: x is negative.

Assume that the variable x is an integer.

(a) No perfect squares are negative.
(b) No negative numbers are perfect squares.

See Solution

p.64, icon at Example 6

#4. Write the following statement in symbols using the predicates

$S(x)$: x is a perfect square $P(x)$: x is positive

where the universe for x is the set of all integers:

"Perfect squares are positive."

See Solution

p.64, icon at Example 6

#5. Write the following statement in symbols using the predicate $P(x)$ to mean "x is positive", where the universe for x is the set of all integers.

"Exactly one number is positive."

See Solution

p.64, icon at Example 6

#6. Write the following statements in symbols, using $P(x)$ to mean "x is positive" and $F(x)$ to mean "x ends in the digit 5". Assume that the universe for x is the set of all integers.

(a) Some positive integers end in the digit 5.
(b) Some positive integers end in the digit 5, while others do not.

See Solution

---

p.64, icon at Example 6

#7. Write in symbols: There is no smallest positive number.

See Solution

---

p.64, icon at Example 6

#8. Write in symbols: If $a < b$, then $\frac{a+b}{2}$ lies between a and b.

See Solution

---

p.64, icon at Example 6

#9. Write in symbols: For all choices of a and b, $\frac{a+b}{2}$ lies between a and b.

See Solution

---

p.67, icon at Example 14

#1. Write the negation of the statement $\exists x\, \forall y\, (xy = 0)$ in symbols and in English. Determine the truth or falsity of the statement and its negation. Assume that the universe for x and y is the set of all real numbers.

See Solution

---

p.67, icon at Example 14

#2. Write the statement "There is a largest number" using predicates and quantifiers. Then give its negation in symbols.

See Solution

Show All Solutions

Rosen, Discrete Mathematics and Its Applications, 8th edition
Extra Examples
Section 1.6—Rules of Inference

Extra Examples — *Page references correspond to locations of Extra Examples icons in the textbook.*

---

p.77, icon at Example 6
#1. The proposition $(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$ is a tautology, as the reader can check. It is the basis for the rule of inference *modus tollens*:

$$\begin{array}{l} \neg q \\ p \rightarrow q \\ \hline \therefore \neg p \end{array}$$

Suppose we are given the propositions: "If the class finishes Chapter 2, then they have a quiz" and "The class does not have a quiz." Find a conclusion that can be drawn using modus tollens.

See Solution

---

p.77, icon at Example 6
#2. Suppose that "it is snowing" is true and that "it is windy" is true. Using the conjunction rule, what conclusion can be drawn?

See Solution

---

p.77, icon at Example 6
#3. Suppose "I have a dime or a quarter in my pocket" and "I do not have a dime in my pocket." According to the disjunctive syllogism rule, what can we conclude?

See Solution

---

p.77, icon at Example 6

#4. Determine whether this argument is valid by using a truth table:

I play golf or tennis.
If it is not Sunday, I play golf and tennis.
If it is Saturday or Sunday, then I don't play golf.
Therefore, I don't play golf.

See Solution

p.77, icon at Example 6

#5. Determine whether this argument is valid:

Lynn works part time or full time.
If Lynn does not play on the team, then she does not work part time.
If Lynn plays on the team, she is busy.
Lynn does not work full time.
Therefore, Lynn is busy.

See Solution

p.78, icon at Example 8

#1. Suppose we have the two propositions (with symbols to represent them):

"It is raining (r) or I work in the yard (w)."
"It is not raining ($\neg r$) or I go to the library (l)."

What conclusion can we draw from these propositions?

See Solution

p.80, icon at Example 12

#1. Suppose we have:

"Every student in this class is a Junior."
"Every Junior in this class passed the final exam."
"Allen is a student in this class."

Explain why we can draw the conclusion “Allen passed the final exam.”

See Solution

Show All Solutions

Rosen, Discrete Mathematics and Its Applications, 8th edition
Extra Examples
Section 1.7—Introduction to Proofs

Extra Examples — *Page references correspond to locations of Extra Examples icons in the textbook.*

---

p.85, icon below start of "Understanding How Theorems Are State" subsection
#1. Sometimes quantifiers in statements are understood, but do not actually appear in the words of the statement. Explain what quantifiers are understood in the statement "The product of two negative numbers is positive."

See Solution

---

p.85, icon below start of "Understanding How Theorems Are State" subsection
#2. Consider the theorem "If x ends in the digit 3, then x^3 ends in the digit 7." What quantifier is understood, but not written?

See Solution

---

p.85, icon below start of "Understanding How Theorems Are State" subsection
#3. Consider the theorem "No squares of integers end in the digit 8." What quantifier is understood, but not written?

See Solution

---

p.85, icon below start of "Understanding How Theorems Are State" subsection

#4. Consider the theorem "The average of two numbers can be 0." What quantifier is understood, but not written?

See Solution

---

p.87, icon at Example 1

#1. Using the definitions of even integer and odd integer, give a direct proof that this statement is true for all integers n:

> If n is odd, then $5n + 3$ is even.

See Solution

---

p.87, icon at Example 3

#1. Using the definitions of even integer and odd integer, give a proof by contraposition that this statement is true for all integers n:

> If $3n - 5$ is even, then n is odd.

See Solution

---

p.87, icon at Example 3

#2. Suppose we need to prove that this statement is true for all integers n, using the definitions of even integer and odd integer:

> If $7n - 5$ is odd, then n is even.

See Solution

---

p.89, icon at Example 8

#1. Suppose a, b, and c are odd integers. Prove that the roots of $ax^2 + bx + c = 0$ are not rational.

See Solution

---

p.90, icon at Example 10

#1. Give a proof by contradiction of: “If n is an even integer, then $3n+7$ is odd.”

See Solution

---

p.92, icon at Example 13

#1. Prove that this statement is true for all integers n: n is odd if and only if $5n + 3$ is even.

See Solution

---

p.93, icon at Example 15

#1. Show that the statement "Every integer is less than its cube" is false by finding a counterexample.

See Solution

Show All Solutions

Rosen, Discrete Mathematics and Its Applications, 8th edition
Extra Examples
Section 1.8—Proof Methods and Strategy

Extra Examples — *Page references correspond to locations of Extra Examples icons in the textbook.*

---

p.97, icon at Example 1
#1. Prove that there is only one pair of positive integers that is a solution to $3x^2 + 2y^2 = 30$.

See Solution

---

p.98, icon at Example 3
#1. Prove that the square of every even integer ends in 0, 4, or 6.

See Solution

---

p.98, icon at Example 3
#2. Prove that the following is true for all real numbers x and y: $\max(x, y) = \frac{1}{2}(x + y + |x - y|)$.

See Solution

p.98, icon at Example 3

#3. Prove that the square of every odd integer ends in 1, 5, or 9.

See Solution

---

p.101, icon at Example 10

#1. Prove that there are numbers x and y whose sum is 5 and whose product is 2. (Note that we are only required to show that x and y exist; we are not required to find specific values for x and y.)

See Solution

p.104, icon at Example 13

#1. Show that if x is a nonzero rational number, then there is a unique rational number y such that $xy = 2$.

See Solution

p.105, icon at Example 14

#1. Prove that the square of every odd integer has the form $8k + 1$, where k is an integer.

See Solution

p.105, icon at Example 14

#2. Suppose a and b are integers such that $2a = b^2 + 3$. Prove that a is the sum of three squares.

See Solution

---

p.106, icon at Example 16

#1. Over the centuries, mathematicians have tried to adapt the proofs of others to obtain new results. A classic example of this is the proof of the Four Color Theorem. The Four Color Theorem states that the countries on every map can be colored with at most four colors so that two countries that share a common border have different colors.

In the nineteenth century it was proved that five colors are sufficient to color the countries on any map so that countries that share a common border receive different colors. No one was able to produce a map that required five colors, but no one was able to prove that four colors were sufficient to color every possible map. In 1976, two mathematicians, Kenneth Appel and Wolfgang Haken, were able to prove that four colors suffice to color the countries of every map so no countries that share a common border have the same color. This proof, complicated and very lengthy, was an adaptation of the much simpler proof that five colors always suffice. Map coloring problems will be discussed in detail in Section 9.8.

p.106, icon at Example 16

#2. (Adapted from Problem A4 from the 1988 William Lowell Putnam Mathematics Competition.)

(a) Suppose that every point of the plane is painted one of two colors, a or b. Must there be two points of the same color that are exactly one inch apart?

(b) Suppose that every point of the plane is painted one of three colors, a, b, or c. Must there be two points of the same color that are exactly one inch apart?

(c) Prove that if nine colors are allowed, a coloring is possible with the property that no two points one inch apart have the same color.

See Solution

p.107, icon at Example 17

1. Find a counterexample to the statement that the sum of two irrational numbers is also an irrational number.

See Solution

---

p. 107, icon BELOW Example 17

1. By examining the small powers of 2 and of 3, what conjectures can you make about how close a power of 2 can be to a power of 3?

See Solution

---

p.108, icon at Example 19

#1. A rectangular floor is tiled using two kinds of tiles, square 2×2 tiles and rectangular 1×4 tiles. Suppose that one tile is destroyed, but that one tile of the other kind is available. It is possible to tile the entire floor, using the original tiles with this one replacement, by rearranging the tiles?

See Solution

---

p.108, icon at Example 19
#2. Can you tile a 17×28 checkerboard using 4×7 tiles?

See Solution

---

p.108, icon at Example 19
#3. An *T-tetromino* consists of a row of three squares with a fourth square directly above the middle square. Show that a 14×14 checkerboard cannot be tiled with T-tetrominoes.

See Solution

---

p.108, icon at Example 19
#4. Show that in any tiling of an 8×8 checkerboard by tetrominoes, where any of the five different kinds of tetrominoes can be used, the number of T-tetrominoes must be even.

See Solution

---

p.108, icon at Example 19
#5. In a tiling of a checkerboard by dominoes, a *fault line* is a vertical or horizontal line that cuts the checkerboard into two pieces without passing through any of the dominoes. Show that whenever a 6×6 checkerboard is tiled with dominoes, the tiling has a fault line. That is, no matter how the 6×6 checkerboard is tiled with dominoes, it is possible to cut the checkerboard in two without passing through one of the dominoes.

See Solution

---

p.108, icon at Example 19

#6. Prove or disprove that there is a tiling of the 5×6 checkerboard that does not have a fault line, that is, a vertical or horizontal line that cuts the checkerboard in two without passing through one of the dominoes.

See Solution

---

p.108, icon at Example 19

#7. How many different pentominoes, that is, arrangements of five squares of a checkerboard joined along edges, are there, where two such arrangements are considered the same if one can be obtained from the other by a rotation or a flipping?

See Solution
