Show All Solutions

Rosen, Discrete Mathematics and Its Applications, 8th edition
Extra Examples
Section 2.1—Sets

Extra Examples — *Page references correspond to locations of Extra Examples icons in the textbook.*

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p.122, icon at Example 4
#1. Write the set $\{2, 3, 4\}$ (given in list notation) in set builder notation.

See Solution

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p.122, icon at Example 4
#2. Write the set $\{x \mid x \in \mathbf{R}, x^2 = 4 \text{ or } x^2 = 9\}$ in list form.

See Solution

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p.122, icon at Example 4
#3. Write the set $\{x \mid x \in \mathbf{R}, x \text{ is a solution to } x^2 = -1\}$ in list form.

See Solution

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p.122, icon at Example 4

#4. The same set can be written as a list in different ways. For example,

$$\{1, 2, 3, \ldots, 99\} = \{1, 2, 3, \ldots, 98, 99\} = \{1, 2, 3, 4, 5, \ldots, 97, 98, 99\}.$$

These three sets all describe the set of positive integers less than 100. As long as the pattern is clear, you can use any number of terms before and after the ellipsis.

Write this set in set-builder notation.

See Solution

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p.122, icon at Example 4

#5. Write the set $S = \{x \mid x \text{ is an even positive integer and } x \leq 64\}$ in list notation.

See Solution

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p.127, icon at Example 13

#1. Determine whether each set is finite or infinite:

(a) $\{1, 10, 100, 1000, 10000, \ldots\}$.
(b) $\{1, 3, 5, 7, 9, \ldots, 599\}$.
(c) The set of all real number solutions to $x + 3 + 2x = 3(x + 1)$.
(d) The set of telephone numbers of the form "(XXX) XXX-XXXX" in the United States.
(e) The set of real number solutions to the equation $x^2 = -4$.
(f) $\{x \mid x \text{ an integer}, x^2 - 9x + 14 < 0\}$.

See Solution

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p.127, icon at Example 13
#2. Let $S = \{\emptyset, a, \{a\}\}$. Determine whether each of these is an element of S, a subset of S, neither, or both.

(a) $\{a\}$
(b) $\{\{a\}\}$
(c) $\emptyset$
(d) $\{\{\emptyset\}, a\}$
(e) $\{\emptyset\}$
(f) $\{\emptyset, a\}$

See Solution

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p.127, icon at Example 13
#3.

(a) Prove that $\mathbf{P}(A) \cup \mathbf{P}(B) \subseteq \mathbf{P}(A \cup B)$ is true for all sets A and B.
(b) Prove that the converse of (a) is not true. That is, prove that $\mathbf{P}(A \cup B) \subseteq \mathbf{P}(A) \cup \mathbf{P}(B)$ is false for some sets A and B.

See Solution

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p.127, icon at Example 13
#4. Suppose that A and B are sets such that $\mathbf{P}(A \cup B) \subseteq \mathbf{P}(A) \cup \mathbf{P}(B)$. Prove that $A \subseteq B$ or $B \subseteq A$.

See Solution

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p.128, icon at Example 14
#1. What is the power set of the set $\{1, a, b\}$?

See Solution

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p.128, icon at Example 14
#2. What is the power set of the set $\{\emptyset, \{0\}\}$?

See Solution

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p.123, icon at Example 16

#1. The set $\mathbf{R} \times \mathbf{R} = \{(x, y) \mid x \in \mathbf{R}, y \in \mathbf{R}\} = \mathbf{R}^2$ is the xy-plane. Of particular interest is any subset of $\mathbf{R}^2$ defined by $\{(x, y) \mid x \in \mathbf{R}, y \in \mathbf{R}, y = f(x)\}$ where $f : \mathbf{R} \to \mathbf{R}$ is a function. This set of points is the graph of the function. For example, $\{(x, y) \mid x \in \mathbf{R}, y = x^2\} = \{(x, x^2) \mid x \in \mathbf{R}\}$ is the graph of a parabola.

More generally, any "relation" between real numbers x and y can be described as a subset of the Cartesian product $\mathbf{R} \times \mathbf{R}$. For example, $\{(x, y) \mid x \in \mathbf{R}, y \in \mathbf{R}, x^2 + y^2 = 1\}$ is the graph of the circle of radius 1 with center at the origin. The set $\{(x, y) \mid x \in \mathbf{R}, y \in \mathbf{R}, x < y\}$ is the portion of the plane above the diagonal line $y = x$.

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Extra Examples
Section 2.2

Extra Examples — *Page references correspond to locations of Extra Examples icons in the textbook.*

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p.140, icon at Example 16

#1. For $i = 1, 2, \ldots$ let $A_i = \left[0, \frac{1}{i}\right)$. If n is a positive integer, find the union and the intersection of these sets from $i = 1$ to n.

See Solution

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p.140, icon at Example 16

#2. For $i = 1, 2, \ldots$ let $A_i = [i, i + 1)$. If n is a positive integer, find the union and the intersection of these sets from $i = 1$ to n.

See Solution

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Extra Examples
Section 2.3—Functions

Extra Examples — *Page references correspond to locations of Extra Examples icons in the textbook.*

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p.148, icon at Example 3
#1. Determine if the following describes a function with the given domain and codomain.

$f : \mathbf{N} \to \mathbf{N}$ where $f(n)$ is equal to the sum of the digits in n.

See Solution

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p.148, icon at Example 3
#2. Determine if each of the following describes a function with the given domain and codomain.

(a) $f : \mathbf{N} \to \mathbf{N}$ where $f(n) = 7 - n$.
(b) $f : \mathbf{N} \to \mathbf{Z}$ where $f(n) = 7 - n$.

See Solution

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p.148, icon at Example 3
#3. Determine if each of the following describes a function with the given domain and codomain.

(a) $f : \mathbf{N} \to \mathbf{N}$ where $f(n) = \frac{1}{n-\pi}$.
(b) $f : \mathbf{N} \to \mathbf{R}$ where $f(n) = \frac{1}{n-\pi}$.
(c) $f : \mathbf{R} \to \mathbf{R}$ where $f(n) = \frac{1}{n-\pi}$.

See Solution

p.148, icon at Example 3

#4. Determine if the following describes a function with the given domain and codomain.

$f : \mathbf{N} \to \mathbf{N}$ where $f(n) = \begin{cases} x+4, & \text{if } n < 7 \\ x^2, & \text{if } n > 11. \end{cases}$

See Solution

p.148, icon at Example 3

#5. Determine if the following describes a function with the given domain and codomain.

$f : \mathbf{N} \to \mathbf{N}$ where $f(n) = \begin{cases} x+4, & \text{if } n < 7 \\ x^2, & \text{if } n > 4. \end{cases}$

See Solution

p.150, icon at Example 8

#1. Let $f : \mathbf{N} \to \mathbf{Z}$ be defined by the two-part rule $f(n) = \begin{cases} n/2, & \text{if } n \text{ is even} \\ -(n+1)/2, & \text{if } n \text{ is odd.} \end{cases}$

Determine whether f is one-to-one.

See Solution

p.152, icon at Example 13

#1. Let $f : \mathbf{N} \to \mathbf{Z}$ be defined by the two-part rule $f(n) = \begin{cases} n/2, & \text{if } n \text{ is even;} \\ -(n+1)/2, & \text{if } n \text{ is odd.} \end{cases}$

Determine whether f is onto $\mathbf{Z}$.

See Solution

p.152, icon at Example 13

#2. Find a function $f : \mathbf{Z} \to \mathbf{N}$ that is one-to-one but not onto.

See Solution

p.152, icon at Example 13
#3. Find a function $f : \mathbf{Z} \to \mathbf{N}$ that is one-to-one and onto.

See Solution

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p.159, icon at Example 31
#1. Let $f : \mathbf{R} \to \mathbf{R}$ have the rule $f(x) = \lfloor 3x \rfloor - 1$. Find $f(S)$ where $S = [1, 3]$.

See Solution

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p.159, icon at Example 31
#2. Let $f : \mathbf{R} \to \mathbf{R}$ have the rule $f(x) = \lfloor 3x \rfloor - 1$. Find $f^{-1}(S)$ where $S = \{0\}$

See Solution

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p.159, icon at Example 31
#3. Let $f : \mathbf{R} \to \mathbf{R}$ have the rule $f(x) = \lfloor 3x \rfloor - 1$ and $g : \mathbf{R} \to \mathbf{R}$ have the rule $g(x) = x/3$. Find $f \circ g(T)$ where $T = [-3, 3.5]$.

See Solution

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p.159, icon at Example 31
#4. Let $f : \mathbf{R} \to \mathbf{R}$ have the rule $f(x) = \lfloor 3x \rfloor - 1$.

(a) Find $(f \circ f)(1)$.
(b) Find $(f \circ f)(U)$ where $U = [2, 3]$.

See Solution

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p.159, icon at Example 31
#5. Let $f : \mathbf{R} \to \mathbf{R}$ have the rule $f(x) = \lfloor 3x \rfloor - 1$ and $g : \mathbf{R} \to \mathbf{R}$ have the rule $g(x) = x/3$.

(a) Find $(f \circ g)^{-1}(\{2.5\})$.
(b) Find $(f \circ g)^{-1}(\{2\})$.

See Solution

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p.159, icon at Example 31
#6. Find all solutions to $\lceil x \rceil + \lfloor x \rfloor = 2x$.

See Solution

p.159, icon at Example 31
#7. Find all solutions to $\lfloor x \rfloor \lceil x \rceil = x^2$.

See Solution

p.159, icon at Example 31
#8. Use the floor and/or ceiling function to find a formula for computing the units' digit of a positive integer n.

See Solution

Show All Solutions

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Extra Examples
Section 2.4—Sequences and Summations

Extra Examples — *Page references correspond to locations of Extra Examples icons in the textbook.*

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p.169, icon at Example 11
#1. Find a rule that produces a sequence $a_1, a_2, a_3, \ldots$ with the first terms $5, 7, 9, 11, 13, \ldots$.

See Solution

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p.169, icon at Example 11
#2. Find a formula for an infinite sequence $a_1, a_2, a_3, \ldots$ that begins with the terms $1/3, 1/4, 1/5, 1/6, \ldots$.

See Solution

---

p.169, icon at Example 11
#3. Find a formula for an infinite sequence $a_1, a_2, a_3, \ldots$ that begins with the terms $7, 11, 15, 19, 23, \ldots$.

See Solution

---

p.169, icon at Example 11

#4. Find a formula for an infinite sequence $a_1, a_2, a_3, \ldots$ that begins with the terms $1, 2, 1, 2, 1, 2, 1$ and continues this alternating pattern.

See Solution

---

p.169, icon at Example 11

#5. Find a formula for an infinite sequence $a_1, a_2, a_3, \ldots$ that begins with the terms $0, 2, 6, 12, 20, 30, 42, \ldots$.

See Solution

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p.169, icon at Example 11

#6. Find a rule that produces a sequence $a_1, a_2, a_3, \ldots$ with the first terms $3, 6, 12, 24, 48, \ldots$.

See Solution

---

p.169, icon at Example 11

#7. Find a rule that produces a sequence $a_1, a_2, a_3, \ldots$ with the first terms $1, 1, 2, 2, 3, 3, 4, 4, 5, 5, 6, 6, \ldots$.

See Solution

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p.170, icon at Example 12

#1. Find a recurrence relation (and initial condition) for each of the following:

(a) the number of strings of length n of letters of the alphabet.
(b) the number of strings of length n of letters of the alphabet, if no adjacent letters can be the same.
(c) the number of strings of length n of letters of the alphabet with no repeated letters.

See Solution

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p.170, icon at Example 12

#2. Find a recurrence relation for the sequence $1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \frac{1}{9}, \ldots$, which is given by the formula $a_n = \frac{1}{2n+1}$ for $n = 0, 1, 2, 3, \ldots$

See Solution

p.170, icon at Example 12

#3. Suppose $b_n = 2b_{n-1} + n - 2^n$ and $b_0 = 5$.

(a) Find b_{n-1} in terms of b_{n-2}.
(b) Find b_n in terms of b_{n-2}.
(c) Find b_n in terms of b_{n-3}.
(d) Use parts (b) and (c) to conjecture a formula for b_n.

See Solution

p.170, icon at Example 12

#4. Solve: $a_n = 3a_{n-1} + 1$, $a_0 = 4$, by substituting for a_{n-1}, then a_{n-2}, etc.

See Solution

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p.170, icon at Example 12

#5. Find a formula for the recurrence relation $a_n = 2a_{n-1} + 2^n$, $a_0 = 1$, using a recursive method.

See Solution

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p.170, icon at Example 12

#6. You begin with \$1000. You invest it at 5% compounded annually, but at the end of each year you withdraw \$100 immediately after the interest is paid.

(a) Set up a recurrence relation and initial condition for the amount you have after n years.
(b) How much is left in the account after you have withdrawn \$100 at the end of the third year?
(c) Find a formula for a_n.
(d) Use the formula to determine how long it takes before the last withdrawal reduces the balance in the account to \$0.

See Solution

p.170, icon at Example 12
#7. Find a recurrence relation for the number of strings of letters of the ordinary alphabet that do not have adjacent vowels.

See Solution

p.170, icon at Example 12
#8. You have two distinct parallel lines L_1 and L_2. You keep adding additional lines, $L_3, L_4, \ldots$, with none parallel to L_1 or L_2 or to each other, and no three passing through the same point.

(a) Find a recurrence relation and initial condition(s) for r_n, which is defined to be the number of regions into which the plane is divided by the lines $L_1, L_2, \ldots, L_n$.
(b) Find a formula for the number of regions into which the plane is divided by $L_1, L_2, \ldots, L_n$.

See Solution

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p.170, icon at Example 12

#9. This is a variation on Fibonacci's rabbit sequence. We begin with one pair of newborn rabbits. Once a pair is two months old, the pair has two pairs of offspring, and continues to have two pairs of offspring each month thereafter. Give a recurrence relation and initial condition(s) for the sequence f_n, where f_n is equal to the number of pairs of rabbits alive at the end of the nth month (after the offspring are born). Assume that the rabbits never die during the period being considered.

See Solution

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p.170, icon at Example 12

#10. Here is another variation on Fibonacci's rabbit sequence. We begin with one pair of newborn rabbits. At the end of each month a new pair of newborn rabbits is added to the population. Once any pair is two months old, the pair has one pair of offspring and continues to have one pair of offspring each month thereafter. Give a recurrence relation and initial condition(s) for the sequence f_n, where f_n is equal to the number of pairs of rabbits alive at the end of the nth month (after the rabbits have given birth and the newborn pair has been introduced). Assume that the rabbits never die during the period being considered.

See Solution

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p.170, icon at Example 12

#11. Here is a third variation on Fibonacci's rabbit sequence. We begin with one pair of newborn rabbits. Once the pair is three months old, the pair has one pair of offspring, and continues to have one pair of offspring every other month thereafter. Give a recurrence relation and initial condition(s) for the sequence f_n, where f_n is equal to the number of pairs of rabbits alive at the end of the nth month (just after any offspring are born). Assume that the rabbits never die during the period being considered.

See Solution

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p.170, icon at Example 12

#12. (Problem A1 from the 1990 William Lowell Putnam Mathematics Competition)

Here are the first ten terms of an infinite sequence:

$$2, 3, 6, 14, 40, 152, 784, 5168, 40567, 363392.$$

(a) Find a formula for an infinite sequence $a_0, a_1, a_2, a_3, \ldots$ such that the first ten terms of the sequence are the ones given here. (Hint: consider the sum of two rapidly increasing sequences.)

(b) Show that the sequence in (a) satisfies the recurrence relation

$$a_n = (n+4)a_{n-1} - 4na_{n-2} + (4n-8)a_{n-3}.$$

See Solution

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p.170, icon at Example 12

#13. Suppose a chess king is placed on the lower left square of an $m \times n$ chessboard (that is, a rectangular board with m rows and n columns). Let $M(m, n)$ be equal to the number of paths that a king can use moving from the lower left corner to the upper right corner of an $m \times n$ board, with the restriction that each move is either up, to the right, or diagonally up and to the right.

(a) Find a recurrence relation and initial condition(s) for $M(m, n)$.

(b) Find the number of ways in which the king can move from the lower left square to the upper right square on a 5×5 chessboard.

See Solution

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p.173, icon at Example 17

#1. Express in sigma notation the sum of the first 50 terms of the series $4 + 4 + 4 + 4 + 4 + \ldots$.

See Solution

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p.173, icon at Example 17
#2. Find the value of each of these sums

(a) $\displaystyle\sum_{j=1}^{4}(j^2 - 1).$

(b) $\displaystyle\sum_{k=1}^{4}(k^2 - 1).$

(c) $\displaystyle\sum_{j=1}^{4}(k^2 - 1).$

See Solution

---

p.173, icon at Example 17
#3. Find the value of each of these sums:

(a) $\displaystyle\sum_{k=1}^{4}(k^2 - 1).$

(b) $\displaystyle\sum_{k=1}^{4}(k^2 - 1).$

See Solution

---

p.174, icon at Example 20

#1. Express in sigma notation the sum of the first 50 terms of the series $3 + 6 + 9 + 12 + 15 + \ldots$.

See Solution

---

p.174, icon at Example 20

#2. The following is a geometric series: $\sum_{i=0}^{10} 2^i$. Identify a, r, and n, and then find the sum of the series.

See Solution

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p.174, icon at Example 20

#3. The following is a geometric series: $4 + 2 + 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots + \frac{1}{64}$. Identify a, r, and n, and then find the sum of the series.

See Solution

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p.174, icon at Example 20

#4. Find the sum of the series $2^4 + 2^5 + 2^6 + \cdots + 2^{17}$.

See Solution

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p.176, icon at Example 24

#1. Find $1 + x^2 + x^4 + x^6 + x^8 + \cdots$ assuming $|x| < 1$.

See Solution

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p.176, icon at Example 24

#2. Prove that $\displaystyle\sum_{i=1}^{\infty} \frac{1}{4i} = 2 \sum_{i=1}^{\infty} \frac{1}{7^i}$.

See Solution

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p.176, icon at Example 24

#3. Find the sum of each of these infinite series:

(a) $\displaystyle\sum_{i=1}^{\infty} \frac{1}{2^i}$.

(b) $\displaystyle\sum_{i=1}^{\infty} (-1)^i \frac{1}{2^i}$.

See Solution

Show All Solutions

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Extra Examples
Section 2.5—Cardinality of Sets

Extra Examples — *Page references correspond to locations of Extra Examples icons in the textbook.*

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p.183, icon at Example 5
#1. We know that the set of rational numbers is countable. Are the irrational numbers (the real numbers that cannot be written as fractions a/b where a and b are integers and $b \neq 0$) also countable, or are they uncountable?

See Solution

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p.183, icon at Example 5
#2. Show that the set $\{x \mid 0 < x < 1\}$ is uncountable by showing that there is a one-to-one correspondence between this set and the set of all real numbers.

See Solution

Show All Solutions

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Extra Examples
Section 2.6—Matrices

Extra Examples — *Page references correspond to locations of Extra Examples icons in the textbook.*

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p.190, icon at Example 3

#1. Let $\mathbf{A} = \begin{pmatrix} 2 & 7 \\ -1 & 5 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 1 & -3 \\ 4 & 2 \end{pmatrix}$. Find the products **AB** and **BA**.

See Solution

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p.190, icon at Example 3

#2. Determine whether the following is true for all 2×2 matrices: $(\mathbf{A}+\mathbf{B})^2 = \mathbf{A}^2 + 2\mathbf{AB} + \mathbf{B}^2$.

See Solution

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p.190, icon at Example 3

#3. Suppose a matrix $\mathbf{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ with real numbers as entries commutes under multiplication with all real 2×2 matrices. That is, $\mathbf{AB} = \mathbf{BA}$ all 2×2 matrices **B** with real numbers as entries. What form must **A** have?

See Solution