Show All Solutions

Rosen, Discrete Mathematics and Its Applications, 8th edition
Extra Examples
Section 5.1—Mathematical Induction

Extra Examples — *Page references correspond to locations of Extra Examples icons in the textbook.*

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p.337, icon at Example 1
#1. Use the Principle of Mathematical Induction to prove that

$$1 + 5 + 5^2 + 5^3 + \cdots + 5^n = \frac{5^{n+1} - 1}{4} \text{ for all } n \geq 0.$$

See Solution

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p.337, icon at Example 1

#2. Use the Principle of Mathematical Induction to prove the "generalized" distributive law

$$a(b_1 + b_2 + \cdots + b_n) = ab_1 + ab_2 + \cdots + ab_n$$

for all integers $n \geq 2$.

See Solution

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p.337, icon at Example 1

#3. Use the Principle of Mathematical Induction to prove that

$$\sum_{i=1}^{n} (2i + 3) = n(n + 4) \quad \text{for all } n \geq 1.$$

See Solution

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p.337, icon at Example 1
#4. Find a formula for

$$\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{4^2}\right)\cdots\left(1-\frac{1}{n^2}\right)$$

for $n \geq 2$, and use the Principle of Mathematical Induction to prove that the formula is correct.

See Solution

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p.341, icon at Example 5

#1. Use the Principle of Mathematical Induction to show that the following inequality is true for all integers $n \geq 2$:

$$\sum_{i=1}^{n} \frac{1}{\sqrt{i}} > \sqrt{n}.$$

See Solution

p.341, icon at Example 5

#2. Use the Principle of Mathematical Induction to prove that $n^2 - 5n + 3 > 0$ for all $n \geq 5$.

See Solution

p.343, icon at Example 8
#1. Prove that 6 is a divisor of $4^n + 7^n + 1$ for all positive integers n.

See Solution

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p.343, icon at Example 8
#2. Use the Principle of Mathematical Induction to prove that $2 \mid (n^2 - n)$ for all $n \geq 0$.

See Solution

p.343, icon at Example 8

#3. Use mathematical induction to prove for all positive integers n, then $3^{2^n} - 1$ is divisible by 2^{n+2}.

See Solution

p.343, icon at Example 8

#4. Use mathematical induction to show that every positive integer not exceeding $n!$ can be expressed as the sum of at most n distinct divisors of $n!$

See Solution

p.343, icon at Example 9

1. Use mathematical induction to prove that $5^n + 9^n + 6$ is divisible by 4 whenever n is a positive integer.

See Solution

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p.347, icon at Example 13

#1. Prove that for all positive integers n, $\frac{(2n)!}{2^n n!}$ is odd.

See Solution

p.347, icon at Example 13

#2. *Muddy Children Puzzle*: The teacher of a group of children tells these children to play in their schoolyard without getting dirty. However, while playing, exactly n children get mud on their foreheads. When the children come back to the classroom after playing, the teacher states: “At least one of you has a muddy forehead” and then asks the children to answer “Yes” or “No” to the question: “Do you know whether you have a muddy forehead?”

The teacher asks this question over and over. What will the children with the muddy foreheads answer each time this question is asked, assuming that a child can see whether other children have muddy foreheads, but cannot see his or her own forehead? Furthermore, we assume that each child is honest, intelligent, and perceptive, and that all children answer each question simultaneously. (Note that the reason that the answer can differ when the question is asked repeatedly is that the children might learn from the previous answers of the other children.)

See Solution

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Extra Examples
Section 5.2—Strong Induction and Well-Ordering

Extra Examples — *Page references correspond to locations of Extra Examples icons in the textbook.*

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p.357, icon at Example 2

#1. Consider an infinite checkerboard of squares, where all squares are white other than an initial set B_0 of n black squares; we call B_0 the initial generation of black squares. We define new generations of black squares recursively. Subsequent generations of black squares $B_1, B_2, \dots$ are defined by the rule that a square is in B_k if and only if at least two of this square itself, the square directly above it, and the square directly to its right are in B_{k-1}. That is, a square on the checkerboard is in a new generation of black squares, if in the previous generation of black squares, there are more black squares than white squares among the square itself, the square above it, and the square to its right. Use strong induction to prove that $B_n = \emptyset$, that is, after n steps (where n is the number of initial black squares), no squares are black.

See Solution

p.362, icon at Example 5

#1. Use the well-ordering property directly to show that if you can reach the first rung of an infinite ladder and if for every positive integer, if you can reach the nth rung, then you can reach the $(n + 1)$st rung, then you can reach every rung.

See Solution

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Extra Examples
Section 5.3—Recursive Definitions and Structural Induction

Extra Examples — *Page references correspond to locations of Extra Examples icons in the textbook.*

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p.367, icon at Example 1

#1. Suppose $f(n+1) = \left\lfloor \dfrac{n^2 f(n) + 2}{n+1} \right\rfloor$ and $f(0) = 2$. Find $f(1)$, $f(2)$, $f(3)$, $f(4)$.

See Solution

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p.367, icon at Example 1

#2. Suppose

$$f(n) = \begin{cases} f(n-2) & \text{if } n \text{ is even} \\ f(n-2) + 3 & \text{if } n \text{ is odd.} \end{cases}$$

Also suppose that $f(0) = 1$ and $f(1) = 4$. Find $f(7)$.

See Solution

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p.367, icon at Example 1

#3. Prove that the following proposed recursive definition of a function on the set of nonnegative integers fails to produce a well-defined function.

$$f(n) = \begin{cases} f(n-2) & \text{if } n \text{ is even} \\ 3f(n-2) & \text{if } n \text{ is odd} \end{cases}$$
with $f(0) = 4$.

See Solution

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p.367, icon at Example 1
#4. Prove that the following proposed recursive definition of a function on the set of nonnegative integers fails to produce a well-defined function.
$f(n) = f(f(n-1)) + 5, f(0) = 1$.

See Solution

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p.368, icon at Example 4
#1. For the sequence of Fibonacci numbers $f_0, f_1, f_2, \ldots (0, 1, 1, 2, 3, 5, 8, 13, \ldots)$, prove that

$$f_0 + f_2 + f_4 + f_6 + \cdots + f_{2n} = f_{2n+1} - 1$$

for all $n \geq 0$.

See Solution

p.368, icon at Example 4

#2. For the sequence of Fibonacci numbers $f_0, f_1, f_2, \ldots$ $(0, 1, 1, 2, 3, 5, 8, 13, \ldots)$, prove for all nonnegative integers n:

$$f_0^2 + f_1^2 + f_2^2 + \cdots + f_n^2 = f_n f_{n+1}.$$

See Solution

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p.370, icon at Example 5

#1. Give a recursive definition for the set $S = \{4, 7, 10, 13, 16, 19, \ldots\}$.

See Solution

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Extra Examples
Section 5.4—Recursive Algorithms

Extra Examples — *Page references correspond to locations of Extra Examples icons in the textbook.*

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p.382, icon at Example 1
#1.

(a) Write a recursive algorithm for finding the sum of the first n even positive integers.
(b) Use mathematical induction to prove that the algorithm in (a) is correct.

See Solution

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Extra Examples
Section 5.5—Program Correctness

Extra Examples — *Page references correspond to locations of Extra Examples icons in the textbook.*

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p.394, icon at Example 1

#1. Show that the program segment S

$a := 5$
$c := a + 2b$

is correct with respect to the initial assertion $p\colon b = 3$ and the final assertion $q\colon c = 11$.

See Solution

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p.396, icon at Example 4

#1. Use a loop invariant to prove that this program segment for computing nx (x a real number), where n is a positive integer, is correct:

multiple := 0
$i := 1$
while $i \leq n$
begin
  multiple := *multiple* + x
  $i := i + 1$
end

See Solution