Show All Solutions

Rosen, Discrete Mathematics and Its Applications, 8th edition
Extra Examples
Section 8.1—Recurrence Relations

Extra Examples — *Page references correspond to locations of Extra Examples icons in the textbook.*

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p.528, icon at Example 1

#1. Find a recurrence relation (and initial condition) for each of the following:

(a) the number of strings of length n of letters of the alphabet.
(b) the number of strings of length n of letters of the alphabet, if no adjacent letters can be the same.
(c) the number of strings of length n of letters of the alphabet with no repeated letters.

See Solution

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p.528, icon at Example 1

#2. Find a recurrence relation for the sequence $1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \frac{1}{9}, \ldots$, which is given by the formula $a_n = \frac{1}{2n+1}$ for $n = 0, 1, 2, 3, \ldots$.

See Solution

p.528, icon at Example 1

#3. Suppose $b_n = 2b_{n-1} + n - 2^n$ and $b_0 = 5$.

(a) Find b_{n-1} in terms of b_{n-2}.
(b) Find b_n in terms of b_{n-2}.
(c) Find b_n in terms of b_{n-3}.
(d) Use parts (b) and (c) to conjecture a formula for b_n.

See Solution

p.528, icon at Example 1

#4. Solve: $a_n = 3a_{n-1} + 1$, $a_0 = 4$, by substituting for a_{n-1}, then a_{n-2}, etc.

See Solution

p.528, icon at Example 1

#5. Find a formula for the recurrence relation $a_n = 2a_{n-1} + 2^n$, $a_0 = 1$, using a recursive method.

See Solution

p.528, icon at Example 1

#6. You begin with \$1000. You invest it at 5% compounded annually, but at the end of each year you withdraw \$100 immediately after the interest is paid.

(a) Set up a recurrence relation and initial condition for the amount you have after n years.
(b) How much is left in the account after you have withdrawn \$100 at the end of the third year?
(c) Find a formula for a_n.
(d) Use the formula to determine how long it takes before the last withdrawal reduces the balance in the account to \$0.

See Solution

p.528, icon at Example 1
#7. Find a recurrence relation for the number of strings of letters of the ordinary alphabet that do not have adjacent vowels.

See Solution

p.528, icon at Example 1

#8. You have two distinct parallel lines L_1 and L_2. You keep adding additional lines, $L_3, L_4, \ldots$, with none parallel to L_1 or L_2 or to each other, and no three passing through the same point.

(a) Find a recurrence relation and initial condition(s) for r_n, which is defined to be the number of regions into which the plane is divided by the lines $L_1, L_2, \ldots, L_n$.

(b) Find a formula for the number of regions into which the plane is divided by $L_1, L_2, \ldots, L_n$.

See Solution

p.528, icon at Example 1

#9. This is a variation on Fibonacci's rabbit sequence. We begin with one pair of newborn rabbits. Once a pair is two months old, the pair has two pairs of offspring, and continues to have two pairs of offspring each month thereafter. Give a recurrence relation and initial condition(s) for the sequence f_n, where f_n is equal to the number of pairs of rabbits alive at the end of the nth month (after the offspring are born). Assume that the rabbits never die during the period being considered.

See Solution

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p.528, icon at Example 1

#10. Here is another variation on Fibonacci's rabbit sequence. We begin with one pair of newborn rabbits. At the end of each month a new pair of newborn rabbits is added to the population. Once any pair is two months old, the pair has one pair of offspring and continues to have one pair of offspring each month thereafter. Give a recurrence relation and initial condition(s) for the sequence f_n, where f_n is equal to the number of pairs of rabbits alive at the end of the nth month (after the rabbits have given birth and the newborn pair has been introduced). Assume that the rabbits never die during the period being considered.

See Solution

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p.528, icon at Example 1

#11. Here is a third variation on Fibonacci's rabbit sequence. We begin with one pair of newborn rabbits. Once the pair is three months old, the pair has one pair of offspring, and continues to have one pair of offspring every other month thereafter. Give a recurrence relation and initial condition(s) for the sequence f_n , where f_n is equal to the number of pairs of rabbits alive at the end of the nth month (just after any offspring are born). Assume that the rabbits never die during the period being considered.

See Solution

p.528, icon at Example 1
#12. (Problem A1 from the 1990 William Lowell Putnam Mathematics Competition)

Here are the first ten terms of an infinite sequence:

$$2, 3, 6, 14, 40, 152, 784, 5168, 40567, 363392.$$

(a) Find a formula for an infinite sequence $a_0, a_1, a_2, a_3, \ldots$ such that the first ten terms of the sequence are the ones given here. (Hint: consider the sum of two rapidly increasing sequences.)

(b) Show that the sequence in (a) satisfies the recurrence relation

$$a_n = (n+4)a_{n-1} - 4na_{n-2} + (4n-8)a_{n-3}.$$

See Solution

p.528, icon at Example 1

#13. Suppose a chess king is placed on the lower left square of an $m \times n$ chessboard (that is, a rectangular board with m rows and n columns). Let $M(m, n)$ be equal to the number of paths that a king can use moving from the lower left corner to the upper right corner of an $m \times n$ board, with the restriction that each move is either up, to the right, or diagonally up and to the right.

(a) Find a recurrence relation and initial condition(s) for $M(m, n)$.

(b) Find the number of ways in which the king can move from the lower left square to the upper right square on a 5×5 chessboard.

See Solution

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Extra Examples
Section 8.2—Solving Linear Recurrence Relations

Extra Examples — *Page references correspond to locations of Extra Examples icons in the textbook.*

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p.543, icon at Example 3

#1. Solve: $a_n = 2a_{n-1} + 3a_{n-2}$, $a_0 = 0$, $a_1 = 1$.

See Solution

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p.543, icon at Example 3

#2. Solve: $a_n = -7a_{n-1} - 10a_{n-2}$, $a_0 = 3$, $a_1 = 3$.

See Solution

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p.516, icon at Example 3

#3. Solve: $a_n = 10a_{n-1} - 25a_{n-2}$, $a_0 = 3$, $a_1 = 4$.

See Solution

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p.543, icon at Example 3

#4. Suppose that the characteristic equation of a linear homogeneous recurrence relation with constant coefficients is

$$(r-3)^4(r-2)^3(r+6) = 0.$$

Write the general solution of the recurrence relation.

See Solution

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p.548, icon at Example 11

#1. Solve the recurrence relation $a_n = 3a_{n-1} + 2^n$, with initial condition $a_0 = 2$.

See Solution

p.548, icon at Example 11

#2. Solve the recurrence relation $a_n = 8a_{n-1} - 12a_{n-2} + 3n$, with initial conditions $a_0 = 1$ and $a_1 = 5$.

See Solution

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Extra Examples
Section 8.3—Divide-and-Conquer Algorithms and Recurrence Relations

Extra Examples — *Page references correspond to locations of Extra Examples icons in the textbook.*

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p.554, icon at Example 1
#1. Suppose $f(n) = 3f(n/2) + 4$ and $f(1) = 5$. Find $f(8)$.

See Solution

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p.554, icon at Example 1
#2. Suppose $f(n) = 2f(n/3) - 1$ and $f(1) = 2$. Find $f(9)$.

See Solution

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p.554, icon at Example 1
#3. Suppose $f(n) = 5f(n/2) + 2n - 1$ and $f(4) = 40$. Find $f(1)$.

See Solution

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p.557, icon at Example 6
#1. Suppose $f(n) = 2f(n/3) + 3$. Find a big-oh function for f.

See Solution

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p.557, icon at Example 6
#2. A recursive algorithm for finding the maximum of a list of numbers divides the list into three equal (or nearly equal) parts, recursively finds the maximum of each sublist, and then finds the largest of these three maxima. Let $f(n)$ be the total number of comparisons needed to find the maximum of a list of n numbers (n a power of 3). Set up a recurrence relation for $f(n)$ and give a big-oh estimate for f.

See Solution

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Extra Examples
Section 8.4

Extra Examples — *Page references correspond to locations of Extra Examples icons in the textbook.*

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p.569, icon at Example 10

#1. Find the number of solutions of $e_1 + e_2 + e_3 + e_4 = 15$, where e_1, e_2, e_3 and e_4 are integers with $1 \le e_1 \le 5$, $2 \le e_2 \le 6$, $3 \le e_3 \le 7$ and $4 \le e_4 \le 8$ by setting up and expanding a suitable product of polynomials. Use a computer algebra system. You will find the product of the polynomials easier to enter if you perform algebraic simplification first.

See Solution

#2. Find the number of solutions of $20 \le e_1 + e_2 + e_3 + e_4 \le 23$, where e_1, e_2, e_3 and e_4 are integers with $1 \le e_1 \le 5$, $2 \le e_2 \le 6$, $3 \le e_3 \le 7$ and $4 \le e_4 \le 8$.

See Solution

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Extra Examples
Section 8.5—Inclusion–Exclusion

Extra Examples — *Page references correspond to locations of Extra Examples icons in the textbook.*

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p.583, icon at Example 5
#1. How many positive integers less than or equal to 100 are divisible by 6 or 9?

See Solution

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p.583, icon at Example 5
#2. How many positive integers less than or equal to 100 are relatively prime to 15?

See Solution

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p.583, icon at Example 5
#3. Find the number of elements in $A_1 \cup A_2 \cup A_3 \cup A_4$ if each set has size 50, each intersection of two sets has size 30, each intersection of three sets has size 10, and the intersection of all four sets has size 2.

See Solution

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p.583, icon at Example 5
#4.

(a) Find the number of permutations of $1, 2, \dots, 8$ that begin with 52 or end with 387.
(b) Find the number of permutations of $1, 2, \dots, 8$ that begin with 52 or end with 327.

See Solution

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p.583, icon at Example 5
#5. Find the number of permutations of all 26 letters of the alphabet that contain at least one of the words FIGHT, BALKS, MOWER.

See Solution

p.583, icon at Example 5
#6. Find the number of permutations of all 26 letters of the alphabet that contain at least one of the words CAR, CARE, SCARE, SCARED.

See Solution

p.583, icon at Example 5
#7. Suppose $|U| = n$ and A and B are subsets of U such that $|A| > n/2$, and $B > n/2$. Prove that $A \cap B \neq \emptyset$.

See Solution

p.583, icon at Example 5
#8. Let n be an odd positive integer and let $a_1, a_2, \ldots, a_n$ represent an arbitrary arrangement of the integers $1, 2, 3, \ldots, n$. Prove that the product $(a_1 - 1)(a_2 - 2)\ldots(a_n - n)$ is an even integer.

See Solution