Rosen, Discrete Mathematics and Its Applications, 8th edition Extra Examples Section 9.1—Relations and Their Properties

Extra — Page references correspond to locations of Extra Examples icons in the textbook.

p.604, icon at Example 10

#1. Let *R* be the following relation defined on the set $\{a, b, c, d\}$:

 $R = \{(a, a), (a, c), (a, d), (b, a), (b, b), (b, c), (b, d), (c, b), (c, c), (d, b), (d, d)\}.$

Determine whether R is:

(a) reflexive.

(b) symmetric.

(c) antisymmetric.

See Solution

p.604, icon at Example 10

#2. Let *R* be the following relation on the set of real numbers:

 $aRb \leftrightarrow \lfloor a \rfloor = \lfloor b \rfloor$, where $\lfloor x \rfloor$ is the floor of x.

Determine whether R is:

| (a) reflexive. | (b) symmetric. | (c) antisymmetric. |
|----------------|----------------|--------------------|
| | See Solution | |
| | | |
| | | |

p.604, icon at Example 10

#3. Let *A* be the set of all points in the plane with the origin removed. That is,

 $A = \{(x, y) | x, y \in \mathbf{R}\} - \{(0, 0)\}.$

Define a relation R on A by the rule:

 $(a, b)R(c, d) \leftrightarrow (a, b)$ and (c, d) lie on the same line through the origin.

| Determine whether R is: | | | | |
|---|--|--------------------|--|--|
| (a) reflexive. | (b) symmetric. | (c) antisymmetric. | | |
| | See Solution | | | |
| | | | | |
| | | | | |
| | | | | |
| | | | | |
| | | | | |
| | | | | |
| | | | | |
| p.604, icon at Example 10 #4. Let $A = \{(x, y) x, y \text{ integers}\}$. Define a relation <i>R</i> on <i>A</i> by the rule | | | | |
| | $(a,b)R(c,d) \leftrightarrow a \leq c \text{ and } b \leq d$ | | | |
| Determine whether R is: | | | | |
| (a) reflexive. | (b) symmetric. | (c) antisymmetric. | | |
| | See Solution | | | |
| | | | | |
| | | | | |
| | | | | |
| | | | | |
| | | | | |
| | | | | |
| p.604, icon at Example 10 #5. Let $A = \{(x, y) x, y \text{ integers}\}$. Define a relation <i>R</i> on <i>A</i> by the rule | | | | |
| $(a,b)R(c,d) \leftrightarrow a = c \text{ or } b = d.$ | | | | |
| Determine whether R is: | | | | |
| (a) reflexive. | (b) symmetric. | (c) antisymmetric. | | |



p.605, icon at Example 13

#1. Let *R* be the following relation defined on the set $\{a, b, c, d\}$:

 $R = \{(a, a), (a, c), (a, d), (b, a), (b, b), (b, c), (b, d), (c, b), (c, c), (d, b), (d, d)\}.$

Determine whether R is transitive.



p.605, icon at Example 13

#2. Let R be the following relation on the set of real numbers:

 $aRb \leftrightarrow \lfloor a \rfloor = \lfloor b \rfloor$, where $\lfloor x \rfloor$ is the floor of *x*.

Determine whether R is transitive.



p.605, icon at Example 13

#3. Let *A* be the set of all points in the plane with the origin removed. That is,

$$A = \{(x, y) | x, y \in \mathbf{R}\} - \{(0, 0)\}.$$

Define a relation on *A* by the rule:

 $(a, b)R(c, d) \leftrightarrow (a, b)$ and (c, d) lie on the same line through the origin.

Determine if R is transitive.



p.605, icon at Example 13 #4. Let $A = \{(x, y) | x, y \text{ integers}\}$. Define a relation *R* on *A* by the rule

 $(a,b)R(c,d) \leftrightarrow a \leq c \text{ and } b \leq d.$

Determine whether R is transitive.



p.605, icon at Example 13

#5. Let $A = \{(x, y) | x, y \text{ integers}\}$. Define a relation *R* on *A* by the rule

 $(a, b)R(c, d) \leftrightarrow a = c \text{ or } b = d.$

Determine whether R is transitive.

Rosen, Discrete Mathematics and Its Applications, 8th edition Extra Examples Section 9.2–*n*-ary Relations and Their Applications.

Extra Examples — Page references correspond to locations of Extra Examples icons in the textbook.

p. 613, icon at Example 5

#1. Consider the following table of planets of the solar system:

Name Mean Distance from the Sun (in AU) Number of Moons

Mercury 0.390

Venus 0.723 0

Earth 1 1

Mars 1.524 2

Jupiter 5.203 67

Saturn 9.539 62

Uranus 19.18 27

Neptun 30.06 13

Which one of the domains are primary keys for the relation displayed in the table?

See Solution

p.613, icon at Example 7

#1. Consider the table of planets of the solar system again:

Name Mean Distance from the Sun (in AU) Number of Moons

Mercury 0.39 0 Venus 0.723 0 Earth 1 1 Mars 1.524 2 Jupiter 5.203 67 Saturn 9.539 62

Uranus 19.18 27

Neptun 30.06 13

If C is the condition (Mean Distance from the Sun < $10 \land$ Number of Moons > 2), and R is the relation represented by the table, what is the output of the selection operator s_C applied to R?



p.617, icon at Example 14

#1. In a given hour, a grocery store recorded the following transactions:

Transaction Number Items

- 1 { avocados, strawberries, tomatoes, peppers, salt }
- 2 { red beans, avocados, tomatoes }
- 3 { red beans, white beans, peppers }
- 4 { avocados }
- 5 { peppers, tomatoes }
- 6 { salt }
- 7 { strawberries, blueberries }
- 8 { white beans, salt, avocados }
- 9 { peppers, red beans }
- 10 { avocados, tomatoes, peppers, salt }
- 1. Determine the count and the support of $A = \{ \text{ salt } \}$ and $B = \{ \text{ peppers, tomatoes } \}$.
- 2. If we set the support treshold to 0.4, which are the frequent items and itemsets?

p.619, icon at Example 15

#1. In a given hour, a grocery store recorded the following transactions:

Transaction Number Items

- 1 { avocados, strawberries, tomatoes, peppers, salt }
- 2 { red beans, avocados, tomatoes }
- 3 { red beans, white beans, peppers }
- 4 { avocados }
- 5 { peppers, tomatoes }
- 6 { salt }
- 7 { strawberries, blueberries }
- 8 { white beans, salt, avocados }
- 9 { peppers, red beans }
- 10 { avocados, tomatoes, peppers, salt }

For this set of transactions, find the support and the confidence of the association rule { avocados, tomatoes } \rightarrow { salt }.

Rosen, Discrete Mathematics and Its Applications, 8th edition Extra Examples Section 9.4—Closures of Relations

Extra — Page references correspond to locations of Extra Examples icons in the textbook.

p.629, icon at Example 2

#1. Let *R* be the relation on $\{1, 2, 3, 4\}$ such that

 $R = \{(1, 1), (1, 4), (2, 3), (3, 1), (3, 3), (4, 4)\}.$

Find:

- (a) the reflexive closure of R.
- (b) the symmetric closure of R.
- (c) the transitive closure of R.

Rosen, Discrete Mathematics and Its Applications, 8th edition Extra Examples Section 9.5—Equivalence Relations

Extra — Page references correspond to locations of Extra Examples icons in the textbook.

p.609, icon at Example 2 #1.

(a) Verify that the following is an equivalence relation on the set of real numbers:

 $aRb \leftrightarrow [a] = [b]$, where [x] is the floor of x.

(b) Describe the equivalence classes arising from the equivalence relation in part (a).

See Solution

p.609, icon at Example 2

#2. Let *A* be the set of all points in the plane with the origin removed. That is,

 $A = \{(x, y) | x, y \in \mathbf{R}\} - \{(0, 0)\}.$

Define a relation on *A* by the rule:

 $(a, b)R(c, d) \leftrightarrow (a, b)$ and (c, d) lie on the same line through the origin.

- (a) Prove that R is an equivalence relation.
- (b) Describe the equivalence classes arising from the equivalence relation R in part (a).
- (c) If A is replaced by the entire plane, is R an equivalence relation?

p.641, icon at Example 9

#1. Let A be the set of real numbers, and R be the equivalence relation R where xRy if and only if x-y is an integer. (Example 2 in the textbook confirms that R is an equivalence relation.)

Find the equivalence classes of 0, 1, 1.5 and 1.7. How many equivalence classes are there?

See Solution

p.641, icon at Example 9

#2. Let A be the set of real numbers, and R be the equivalence relation R where xRy if and only if x-y is a rational number.

- 1. Find the equivalence classes of 0, 1/2 and sqrt(2).
- 2. Are sqrt(2) and sqrt(3) in the same equivalence class or not?
- 3. How many equivalence classes are there?

Rosen, Discrete Mathematics and Its Applications, 8th edition Extra Examples Section 9.6—Partial Orderings

Extra — Page references correspond to locations of Extra Examples icons in the textbook.

p.650, icon at Example 1

#1. Let $A = \{(x, y) | x, y \text{ integers}\}$. Define a relation R on A by the rule

 $(a,b)R(c,d) \leftrightarrow a \leq c \text{ or } b \leq d.$

Determine whether R is a partial order relation on A.

See Solution

p.650, icon at Example 1

#2. Let $A = \{(x, y) | x, y \text{ integers}\}$. Define a relation R on A by the rule

 $(a, b)R(c, d) \leftrightarrow a = c \text{ or } b = d.$

Determine whether R is a partial order relation on A.

p.650, icon at Example 4

#1. Let *R* be the relation on the set of words in the English language where xRy if x precedes (that is, comes before) y in the dictionary. Show that *R* is not a partial ordering.



p.658, icon at Example 20

#1. Referring to this Hasse diagram of a partially ordered set, find the following:

- (a) all upper bounds of $\{d, e\}$.
- (b) the least upper bound of $\{d, e\}$.
- (c) all lower bounds of $\{a, e, g\}$.
- (d) the greatest lower bound of $\{a, e, g\}$.
- (e) greatest lower bound of $\{b, c, f\}$.
- (f) least upper bound of $\{h, i, j\}$.
- (g) greatest lower bound of $\{g, h\}$.
- (h) least upper bound of $\{f, i\}$.

