Appendix 8A

The Maturity Model

book value accounting
Accounting method in which the assets and liabilities of the FI are recorded at historic values.

market value accounting
Accounting method in which the assets and liabilities of the FI are revalued according to the current level of interest rates.

marking to market
Valuing securities at their current market price.

As mentioned in the chapter, a weakness of the repricing model is its reliance on book values rather than market values of assets and liabilities. Indeed, in most countries, FIs report their balance sheets by using **book value accounting.** This method records the historic values of securities purchased, loans made, and liabilities sold. For example, for U.S. banks, investment assets (i.e., those expected to be held to maturity) are recorded at book values, while those assets expected to be used for trading (trading securities or available-for-sale securities) are reported according to market value.[1] The recording of market values means that assets and liabilities are revalued to reflect current market conditions. Thus, if a fixed-coupon bond had been purchased at \$100 per \$100 of face value in a low-interest rate environment, a rise in current market rates reduces the present value of the cash flows from the bond to the investor. Such a rise also reduces the price—say, to \$97—at which the bond could be sold in the secondary market today. That is, the **market value accounting** approach reflects economic reality, or the true values of assets and liabilities if the FI's portfolio were to be liquidated at today's securities prices rather than at the prices when the assets and liabilities were originally purchased or sold. This practice of valuing securities at their market value is referred to as **marking to market.** We discuss book value versus market value accounting and the impact that the use of the alternate methods has in measuring the value of an FI in more detail in Chapter 20. In the maturity and duration model, developed below and in Chapter 9, the effects of interest rate changes on the market values of assets and liabilities are explicitly taken into account. This contrasts with the repricing model, discussed in the body of the chapter, in which such effects are ignored.

EXAMPLE 8A–1
Fixed Income Assets and the Maturity Model

Consider the value of a bond held by an FI that has one year to maturity, a face value of \$100 (F) to be paid on maturity, one single annual coupon at a rate of 10 percent of the face value (C) and a current yield to maturity (R) (reflecting current interest rates) of 10 percent. The fair market price of the one-year bond, P_1^B, is equal to the present value of the cash flows on the bond:

$$P_1^B = \frac{F + C}{(1 + R)} = \frac{\$100 + \$10}{1.1} = \$100$$

Suppose the Federal Reserve tightens monetary policy so that the required yield on the bond rises instantaneously to 11 percent. The market value of the bond falls to:

$$P_1^B = \frac{\$100 + \$10}{1.11} = \$99.10$$

[1] More accurately, they are reported at the lower of cost or current market value (LOCOM). However, both the SEC and the Financial Accounting Standards Board (FASB) have strongly advocated that FIs switch to full market value accounting in the near future. Currently, FASB 115 requires FIs to value certain bonds at market prices but not loans.

Thus, the market value of the bond is now only $99.10 per $100 of face value, while its original book value was $100. The FI has suffered a capital loss (ΔP_1) of $0.90 per $100 of face value in holding this bond, or:

$$\Delta P_1^B = \$99.10 - \$100 = -\$0.90$$

Also, the percent change in the price is:

$$\%\Delta P_1^B = \frac{\$99.10 - \$100}{\$100} = -0.90\%$$

This example simply demonstrates the fact that:

$$\frac{\Delta P}{\Delta R} < 0$$

A rise in the required yield to maturity reduces the price of fixed-income securities held in FI portfolios. Note that if the bond under consideration were issued as a liability by the FI (e.g., a fixed-interest deposit such as a CD) rather than being held as an asset, the effect would be the same—the market value of the FI's deposits would fall. However, the economic interpretation is different. Although rising interest rates that reduce the market value of assets are bad news, the reduction in the market value of liabilities is good news for the FI. The economic intuition is illustrated in the following example.

EXAMPLE 8A–2
Fixed Rate Liabilities and the Maturity Model

Suppose the FI in the example above issued a one-year deposit with a promised interest rate of 10 percent and principal or face value of $100.[2] When the current level of interest rates is 10 percent, the market value of the liability is 100:

$$P_1^D = \frac{\$100 + \$10}{1.10} = \$100$$

Should interest rates on new one-year deposits rise instantaneously to 11 percent, the FI has gained by locking in a promised interest payment to depositors of only 10 percent. The market value of the FI's liability to its depositors would fall to $99.10; alternatively, this would be the price the FI would need to pay the depositor if it repurchased the deposit in the secondary market:

$$P_1^D = \frac{\$100 + \$10}{1.11} = \$99.10$$

That is, the FI gained from paying only 10 percent on its deposits rather than 11 percent if they were newly issued after the rise in interest rates.

As a result, in a market value accounting framework, rising interest rates generally lower the market values of both assets and liabilities on an FI's balance sheet. Clearly, falling interest rates have the reverse effect: They increase the market values of both assets and liabilities.

[2] In this example we assume for simplicity that the promised interest rate on the deposit is 10 percent. In reality, for returns to intermediation to prevail, the promised rate on deposits would be less than the promised rate (coupon) on assets.

EXAMPLE 8A–3
Impact of Maturity on Change in Bond Value

In the preceding examples, both the bond and the deposit were of one-year maturity. We can easily show that if the bond or deposit had a two-year maturity with the same annual coupon rate, the same increase in market interest rates from 10 to 11 percent would have had a more *negative* effect on the market value of the bond's (and deposit's) price. That is, before the rise in required yield:

$$P_2^B = \frac{\$10}{1.10} + \frac{\$10 + \$100}{(1.10)^2} = \$100$$

After the rise in market yields from 10 to 11 percent:

$$P_2^B = \frac{\$10}{1.11} + \frac{\$10 + \$100}{(1.11)^2} = \$98.29$$

and

$$\Delta P_2^B = \$98.29 - \$100 = -\$1.71$$

The resulting percentage change in the bond's value is:

$$\%\Delta P_2^B = (\$98.29 - \$100)/\$100 = -1.71\%$$

If we extend the analysis one more year, the market value of a bond with three years to maturity, a face value of \$100, and a coupon rate of 10 percent is:

$$P_3^B = \frac{\$10}{1.10} + \frac{\$10}{(1.10)^2} + \frac{\$10 + \$100}{(1.10)^3} = \$100$$

After the rise in market rates from 10 to 11 percent, market value of the bond is:

$$P_3^B = \frac{\$10}{1.11} + \frac{\$10}{(1.11)^2} + \frac{\$10 + \$100}{(1.11)^3} = \$97.56$$

This is a change in the market value of:

$$\Delta P_3^B = \$97.56 - \$100 = -\$2.44$$

or

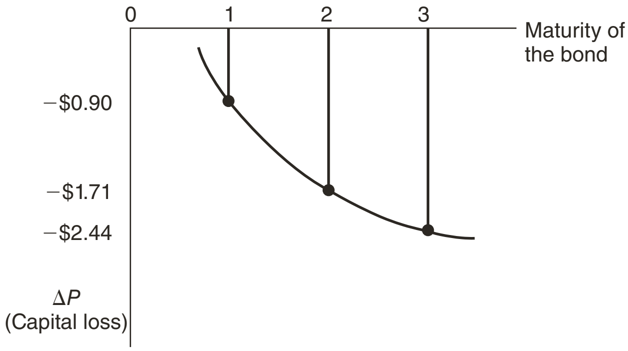
$$\%\Delta P_3^B = \frac{\$97.56 - \$100}{\$100} = -2.44\%$$

This example demonstrates another general rule of portfolio management for FIs: The *longer* the maturity of a fixed income asset or liability, the larger its fall in price and market value for any given increase in the level of market interest rates. That is:

$$\frac{\Delta P_1^B}{\Delta R} < \frac{\Delta P_2^B}{\Delta R} < \cdots < \frac{\Delta P_{30}^B}{\Delta R}$$

Note that while the two-year bond's fall in price is larger than the fall of the one-year bond's, the difference between the two price falls, $\%\Delta P_2^B - \%\Delta P_1^B$, is $-1.71\% - (-0.9\%) = -0.81\%$. The fall in the three-year, 10 percent coupon bond's price when yield increases to 11 percent is -2.44 percent. Thus, $\%\Delta P_3^B - \%\Delta P_2^B = -2.44\% - (-1.71\%) = -0.73\%$, This establishes an important result: While

FIGURE 8A–1
The Relationship between ΔR, Maturity, and ΔP (Capital Loss)



P_3^B falls more than P_2^B and P_2^B falls more than P_1^B, the size of the capital loss increases at a diminishing rate as we move into the higher maturity ranges. This effect is graphed in Figure 8A–1.

So far, we have shown that for an FI's fixed-income assets and liabilities:

1. A rise (fall) in interest rates generally leads to a fall (rise) in the market value of an asset or liability.
2. The longer the maturity of a fixed-income asset or liability, the larger the fall (rise) in market value for any given interest rate increase (decrease).
3. The fall in the value of longer-term securities increases at a diminishing rate for any given increase in interest rates.

The Maturity Model with a Portfolio of Assets and Liabilities

The preceding general rules can be extended beyond an FI holding an individual asset or liability to a portfolio of assets and liabilities. Let M_A be the weighted-average maturity of an FI's assets and M_L the weighted-average maturity of an FI's liabilities such that:

$$M_i = W_{i1}M_{i1} + W_{i2}M_{i2} + \cdots + W_{in}M_{in}$$

where

M_i = Weighted-average maturity of an FI's assets (liabilities), $i = A$ or L
W_{ij} = Importance of each asset (liability) in the asset (liability) portfolio as measured by the market value of that asset (liability) position relative to the market value of all the assets (liabilities)
M_{ij} = Maturity of the jth asset (or liability), $j = 1 \ldots n$

This equation shows that the maturity of a portfolio of assets or liabilities is a weighted average of the maturities of the assets or liabilities that constitute that portfolio. In a portfolio context, the same three principles prevail as for an individual security:

1. A rise in interest rates generally reduces the market values of an FI's asset and liability portfolios.
2. The longer the maturity of the asset or liability portfolio, the larger the fall in value for any given interest rate increase.
3. The fall in value of the asset or liability portfolio increases with its maturity at a diminishing rate.

TABLE 8A–1
The Market Value Balance Sheet of an FI

Assets	Liabilities
Long-term assets (*A*)	Short-term liabilities (*L*)
	Net worth (*E*)

TABLE 8A–2
Initial Market Values of an FI's Assets and Liabilities (in millions of dollars)

Assets	Liabilities
A = \$100 (M_A = 3 years)	\$ 90 = *L* (M_L = 1 year)
	10 = *E*
\$100	\$100

maturity gap Difference between the weighted-average maturity of the FI's assets and liabilities.

Given the preceding, the net effect of rising or falling interest rates on an FI's balance sheet depends on the extent and direction in which the FI mismatches the maturities of its asset and liability portfolios. That is, the effect depends on whether its **maturity gap,** $M_A - M_L$, is greater than, equal to, or less than zero.

Consider the case in which $M_A - M_L > 0$ (as shown in Table 8A–1); that is, the maturity of assets is longer than the maturity of liabilities. This is the case of most commercial banks and thrifts. These FIs tend to hold large amounts of relatively longer-term fixed-income assets such as conventional mortgages, consumer loans, commercial loans, and bonds,[3] while issuing shorter-term liabilities, such as certificates of deposit with fixed interest payments promised to the depositors.

Consider the simplified portfolio of a representative FI in Table 8A–2 and notice that all assets and liabilities are marked to market; that is, we are using a market value accounting framework. Note that in the real world, reported balance sheets differ from Table 8A–2 because historic or book value accounting rules are used. In Table 8A–2 the difference between the market value of the FI's assets (*A*) and the market value of its liabilities such as deposits (*L*) is the net worth or true equity value (*E*) of the FI. This is the economic value of the FI owners' stake in the FI. In other words, it is the money the owners would get if they could liquidate the FI's assets and liabilities at today's prices in the financial markets by selling off loans and bonds and repurchasing deposits at the best prices. This is also clear from the balance sheet identity:

$$E = A - L$$

As was demonstrated earlier, when interest rates rise, the market values of both assets and liabilities fall. However, in this example, because the maturity on the asset portfolio is longer than the maturity on the liability portfolio, for any given change in interest rates, the market value of the asset portfolio (*A*) falls by more than the market value of the liability portfolio (*L*). For the balance sheet identity to hold, the difference between the changes in the market value of its assets and liabilities must be made up by the change in the market value of the FI's equity or net worth:

$$\underset{\text{(change in FI net worth)}}{\Delta E} = \underset{\text{(change in market value of assets)}}{\Delta A} - \underset{\text{(change in market value of liabilities)}}{\Delta L}$$

[3] These assets generate periodic interest payments such as coupons that are fixed over the asset's life. In Chapter 9 we discuss interest payments fluctuating with market interest rates, such as on an adjustable rate mortgage.

TABLE 8A–3
An FI's Market Value Balance Sheet after a Rise in Interest Rates of 1 Percent (in millions of dollars)

Assets	Liabilities
A = \$97.56	L = \$89.19
	E = 8.37
\$97.56	\$97.56

or $\Delta E = \Delta A - \Delta L$

$-\$1.63 = (-\$2.44) - (-\$0.81)$

To see the effect on FI net worth of having longer-term assets than liabilities, suppose that initially the FI's balance sheet looks like the one in Table 8A–2. The \$100 million of assets is invested in three-year, 10 percent coupon bonds, and the liabilities consist of \$90 million raised with one-year deposits paying a promised interest rate of 10 percent. We showed earlier that if market interest rates rise 1 percent, from 10 to 11 percent, the value of three-year bonds falls 2.44 percent while the value of one-year deposits falls 0.9 percent.[4] Table 8A–3 depicts this fall in asset and liability market values and the associated effects on FI net worth.

Because the FI's assets have a three-year maturity compared with its one-year maturity liabilities, the value of its assets has fallen by more than has the value of its liabilities. The FI's net worth declines from \$10 million to \$8.37 million, a loss of \$1.63 million, or 16.3 percent! Thus, it is clear that with a *maturity gap* of two years:

$$\underset{(3)}{M_A} - \underset{(1)}{M_L} = 2\text{years}$$

a 1 percentage point rise in interest rates can cause the FI's owners or stockholders to take a big hit to their net worth. Indeed, if a 1 percent rise in interest rates leads to a fall of 16.3 percent in the FI's net worth, it is not unreasonable to ask how large an interest rate change would need to occur to render the FI economically insolvent by reducing its owners' equity stake or net worth to zero. That is, what increase in interest rates would make E fall by \$10 million so that all the owners' net worth would be eliminated? For the answer to this question, look at Table 8A–4. If interest rates were to rise a full 7 percent, from 10 to 17 percent, the FI's equity (E) would fall by just over \$10 million, rendering the FI economically insolvent.[5]

[4] The market value of deposits (in millions of dollars) is initially:

$$P_1^D = \frac{\$9 + \$90}{1.10} = \$90$$

When rates increase to 11 percent, the market value decreases:

$$P_1^D = \frac{\$9 + \$90}{1.11} = \$89.19$$

The resulting change is:

$$\%\Delta P_1^D = \frac{\$89.19 - \$90}{\$90} = -0.90\%$$

[5] Here we are talking about economic insolvency. The legal and regulatory definition may vary, depending on what type of accounting rules are used. In particular, under the Federal Deposit Insurance Corporation Improvement Act (FDICIA) (November 1991), a DI is required to be placed in conservatorship by regulators when the book value of its net worth falls below 2 percent. However, the true or market value of net worth may well be less than this figure at that time.

TABLE 8A–4
An FI Becomes Insolvent after a 7 Percent Rate Increase (in millions of dollars)

Assets	Liabilities
A = \$84.53	L = \$84.62
	E = −0.09
\$84.53	\$84.53

or $\Delta E = \Delta A - \Delta L$

$-\$10.09 = -\$15.47 - (-\$5.38)$

TABLE 8A–5
An FI with an Extreme Maturity Mismatch (in millions of dollars)

Assets	Liabilities
A = \$100 (M_A = 30 years)	L = \$90 (M_L = 1 year)
	E = 10
\$100	\$100

TABLE 8A–6
The Effect of a 1.5 Percent Rise in Interest Rates on the Net Worth of an FI with an Extreme Asset and Liability Mismatch (in millions of dollars)

Assets	Liabilities
A = \$87.45	L = \$88.79
	E = −1.34
\$87.45	\$87.45

or $\Delta E = \Delta A - \Delta L$

$-\$11.34 = (-\$12.55) - (-\$1.21)$

EXAMPLE 8A–4
Extreme Maturity Mismatch

Suppose the FI had adopted an even more extreme maturity gap by investing all its assets in 30-year fixed-rate bonds paying 10 percent coupons while continuing to raise funds by issuing one-year deposits with promised interest payments of 10 percent, as shown in Table 8A–5. Assuming annual compounding and a current level of interest rates of 10 percent, the market price of the bonds (in millions of dollars) is initially:

$$P_{30}^{B} = \frac{\$10}{1.10} + \frac{\$10}{(1.10)^2} + \cdots + \frac{\$10}{(1.10)^{29}} + \frac{\$10 + \$100}{(1.10)^{30}} = \$100$$

If interest rates were to rise by 1.5 percent to 11.5 percent, the price (in millions of dollars) of the 30-year bonds would fall to:

$$P_{30}^{B} = \frac{\$10}{1.115} + \frac{\$10}{(1.115)^2} + \cdots + \frac{\$10}{(1.115)^{29}} + \frac{\$10 + \$100}{(1.115)^{30}} = \$87.45,$$

a drop of \$12.55, or as a percentage change, $\%\Delta P_{30}^{B} = (\$87.45 - \$100)/\$100 = -12.55\%$. The market value of the FI's one-year deposits would fall to:

$$P_{1}^{D} = \frac{\$9 + \$90}{1.115} = \$88.79$$

a drop of \$1.21 or $(\$88.79 - \$90)/\$90 = -1.34\%$.

Look at Table 8A–6 to see the effect on the market value balance sheet and the FI's net worth after a rise of 1.5 percent in interest rates. It is clear from Table 8A–6 that when the mismatch in the maturity of the FI's assets and liabilities is extreme (29 years), a mere 1.5 percent increase in interest rates completely eliminates the FI's \$10 million in net worth and renders it completely and massively insolvent (net worth is −\$1.34 million after the rise in rates). In contrast, a smaller maturity gap (such as the two years from above) requires a much larger change in interest rates (i.e., 7 percent) to wipe out the FI's equity. Thus, interest rate risk increases as the absolute value of the maturity gap increases.

Given this example, it is not surprising that savings associations with 30-year fixed-rate mortgages as assets and shorter-term CDs as liabilities suffered badly during the 1979–82 period, when interest rates rose so dramatically (see Figure 8–1). At the time, depository institutions measured interest rate risk almost exclusively according to the repricing model, which captures the impact of interest rate changes on net interest income only. Regulators monitoring this measure only, rather than a market value–based measure, were unable to foresee the magnitude of the impact of rising interest rates on the market values of these FIs' assets and thus on their net worth.

immunize
Fully protect an FI's equity against interest rate risk.

From the preceding examples, you might infer that the best way for an FI to **immunize,** or protect, itself from interest rate risk is for its managers to match the maturities of its assets and liabilities, that is, to construct its balance sheet so that its maturity gap, the difference between the weighted-average maturity of its assets and liabilities, is zero ($M_A - M_L = 0$). However, as we discuss next, maturity matching does not always protect an FI against interest rate risk.

WEAKNESSES OF THE MATURITY MODEL

The maturity model has two major shortcomings: (1) It does not account for the degree of leverage in the FI's balance sheet and (2) it ignores the timing of the cash flows from the FI's assets and liabilities. As a result of these shortcomings, a strategy of matching asset and liability maturities moves the FI in the direction of hedging itself against interest rate risk, but it is easy to show that this strategy does not always eliminate all interest rate risk for an FI.

To show the effect of leverage on the ability of the FI to eliminate interest rate risk using the maturity model, assume that the FI is initially set up as shown in Table 8A–7. The $100 million in assets is invested in one-year, 10 percent coupon bonds, and the $90 million in liabilities are in one-year deposits paying 10 percent. The maturity gap ($M_A - M_L$) is now zero. A 1 percent increase in interest rates results in the balance sheet in Table 8A–8. In Table 8A–8, even though the maturity gap is zero, the FI's equity value falls by $0.10 million. The drop in equity value is due to the fact that not all the assets (bonds) are financed with deposits. Rather, equity is used to finance a portion of the FI's assets. As interest rates increase, only $90 million in deposits are directly affected, while $100 million in assets are directly affected.

TABLE 8A–7
Initial Market Values of an FI's Assets and Liabilities with a Maturity GAP of Zero (in millions of dollars)

Assets	Liabilities
A = \$100 (M_A = 1 year)	L = \$ 90 (M_L = 1 year)
	E = 10
\$100	\$100

TABLE 8A–8
FI's Market Value Balance Sheet after a 1 Percent Rise in Interest Rates (in millions of dollars)

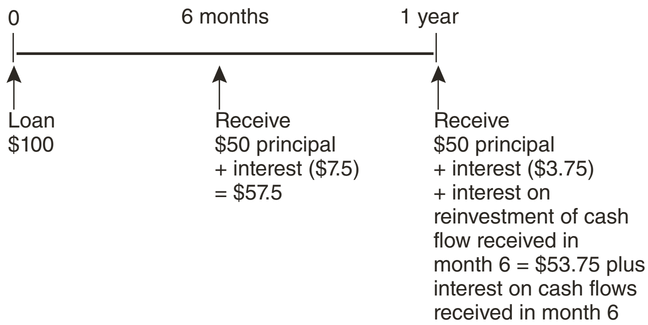
Assets	Liabilities
A = \$99.09	L = \$89.19
	E = 9.90
\$99.09	\$99.09

or $\Delta E = \Delta A - \Delta L$

$-0.10 = -0.91 - (-0.81)$

FIGURE 8A–2
One-Year CD Cash Flows

FIGURE 8A–3
One-Year Loan Cash Flows



We show next, using a simple example, that an FI choosing to directly match the maturities and values of its assets and liabilities (so that $M_A = M_L$ and $\$A = \$L$) does not necessarily achieve perfect immunization, or protection, against interest rate risk. Consider the example of an FI that issues a one-year CD to a depositor. This CD has a face value of $100 and an interest rate promised to depositors of 15 percent. Thus, on maturity at the end of the year, the FI has to repay the borrower $100 plus $15 interest, or $115, as shown in Figure 8A–2.

Suppose the FI lends $100 for one year to a corporate borrower at a 15 percent annual interest rate (thus, $\$A = \$L$). However, the FI contractually requires half of the loan ($50) to be repaid after six months and the last half to be repaid at the end of the year. Note that although the maturity of the loan equals the maturity of the deposit of 1 year and the loan is fully funded by deposit liabilities, the cash flow earned on the loan may be greater or less than the $115 required to pay off depositors, depending on what happens to interest rates over the one-year period. You can see this in Figure 8A–3.

At the end of the first six months, the FI receives a $50 repayment in loan principal plus $7.5 in interest ($100 \times 1/2$ year $\times$ 15 percent), for a total midyear cash flow of $57.5. At the end of the year, the FI receives $50 as the final repayment of loan principal plus $3.75 interest ($\$50 \times 1/2$ year $\times$ 15 percent) plus the reinvestment income earned from relending the $57.5 received six months earlier. If interest rates do not change over the period, the FI's extra return from its ability to reinvest part of the cash flow for the last six months will be ($\$57.5 \times 1/2 \times 15$ percent) $= 4.3125$. We summarize the total cash flow on the FI's one-year loan in Table 8A–9.

As you can see, by the end of the year, the cash paid in on the loan exceeds the cash paid out on the deposit by $0.5625. The reason for this is the FI's ability to reinvest part of the principal and interest over the second half of the year at 15 percent. Suppose that interest rates, instead of staying unchanged at 15 percent throughout the whole one-year period, had fallen to 12 percent over the last six months in the year. This fall in rates would affect neither the promised deposit rate of 15 percent

TABLE 8A–9
Cash Flow on a Loan with a 15 Percent Interest Rate

Cash Flow at 1/2 Year	
Principal	$ 50.00
Interest	7.50
Cash Flow at 1 Year	
Principal	$ 50.00
Interest	3.75
Reinvestment income	4.3125
	$115.5625

TABLE 8A–10
Cash Flow on the Loan When the Beginning Rate of 15 Percent Falls to 12 Percent

Cash Flow at 1/2 Year	
Principal	$ 50.00
Interest	7.50
Cash Flow at 1 Year	
Principal	$ 50.00
Interest	3.75
Reinvestment income	3.45
	$114.70

nor the promised loan rate of 15 percent because they are set at time 0 when the deposit and loan were originated and do not change throughout the year. What is affected is the FI's *reinvestment income* on the $57.5 cash flow received on the loan at the end of six months. It can be re-lent for the final six months of the year only at the new, lower interest rate of 12 percent (see Table 8A–10).

The only change to the asset cash flows for the bank comes from the reinvestment of the $57.50 received at the end of six months at the lower interest rate of 12 percent. This produces the smaller reinvestment income of $3.45 ($57.5 × ½ × 12 percent) rather than $4.3125 when rates stayed at 15 percent throughout the year. Rather than making a profit of $0.5625 from intermediation, the FI loses $0.3. Note that this loss occurs as a result of interest rates changing, even when the FI had matched the maturity of its assets and liabilities ($M_A = M_L = 1$ year), as well as the dollar amount of loans (assets) and deposits (liabilities) (i.e., $\$A = \$L$).

Despite the matching of maturities, the FI is still exposed to interest rate risk because the *timing* of the *cash flows* on the deposit and loan are not perfectly matched. In a sense, the cash flows on the loan are received, on average, earlier than cash flows are paid out on the deposit, where all cash flows occur at the end of the year. Chapter 9 shows that only by matching the average lives of assets and liabilities—that is, by considering the precise timing of arrival (or payment) of cash flows—can an FI immunize itself against interest rate risk.